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Mathematics is a gentleman's art: Analysis and synthesis in American college geometry teaching, 1790-1840

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Mathematics is a gentleman's art:

Analysis and synthesis in American college geometry teaching, 1790-1840

by

Amy K. Ackerberg-Hastings

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: History of Technology and Science

Major Professor: David B. Wilson

Iowa State University

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LIST OF ABBREVIATIONS

ANB   *American National Biography*

DAB   *Dictionary of American Biography*

DNB   *Dictionary of National Biography*

DSB   *Dictionary of Scientific Biography*

NCAB   *The National Cyclopedia of American Biography*

NUC   *The National Union Catalog: Pre-1956 Imprints*

ACKNOWLEDGEMENTS

ABSTRACT

The story of geometry education in the American college has been subject to neglect, with most historians assuming that all available information was published in secondary sources around the turn of the twentieth century. However, recent trends in the history of science include the revelation of the development of the scientific community before the Civil War and an interest in the study of textbooks. Additionally, the literature lacks attempts to place geometry education and mathematics professors within the scientific community. There are also no modern biographies of the three principal actors, Jeremiah Day, John Farrar, and Charles Davies. Finally, mathematicians in the early nineteenth century often framed their discussions according to various understandings of two key terms, "analysis" and "synthesis."

This study, therefore, seeks to address these gaps. Day, Farrar, and Davies were the first three American authors to write series of mathematical textbooks, and their volumes on geometry were the most popular in nineteenth-century American colleges. As these facts are explored, the existence of a significant community of mathematics professors is demonstrated. These professors made incremental adjustments to the traditional liberal arts curriculum while carrying out "normal science" and publicizing European mathematics in colleges which were themselves friendly to mathematics. Day, Farrar, and Davies weighed British and French influences, had much in common with their contemporaries in Scotland, and formed an essential step between elite colonial amateur mathematicians and university research mathematics.

The dissertation is presented in six chapters. The first reviews the literature on the history of American mathematics and science between 1790 and 1840. This chapter also establishes French mathematics and the history of analysis and

synthesis as "givens" in the background of the story of American college geometry education. The second chapter evaluates the Scottish experience with geometry textbooks, paying special attention to the manifestation of analysis and synthesis as mathematical styles, method of proof, and educational techniques in John Playfair's *Elements of Geometry*. Then, the third, fourth, and fifth chapters lay out the biographies and careers of Day, Farrar, and Davies, and these chapters discuss the professors' geometry textbooks with respect to analysis and synthesis. Finally, the conclusion ties together the themes raised above and outlines the history of American geometry education after 1840.

Introduction/Chapter One

Sketching the Givens: Influences Shaping

Mathematics in Republican America

Doors opened to John Farrar (1779-1853) from the day of his arrival in

England in 1836 to visit his wife's family. His reputation as a Harvard astronomer

and mathematician preceded him, resulting in this letter from the Cambridge

Professor of Mathematics: "Come to Cambridge; you need bring no letters of

introduction; we all know you, and we want to see you."[1] Farrar also examined the

Royal Observatory at Greenwich and the Observatory at Armagh, where he solicited

ideas from Thomas Romney Robinson for the observatory Farrar wished to build

back at Harvard; the two men discussed recent discoveries in the polarization of

light as well. Farrar's visit to the popularizer and expositor of Laplace's *Mécanique*

céleste, Mary Somerville, led additionally to dinner with the four-time president of

the Astronomical Society, Francis Bailey. Farrar also spent time in the household of

Maria Edgeworth, another of the significant women in nineteenth-century British

science.[2]

Yet, even though John Farrar was a prominent intellectual and author in his

day, the stories which illustrate the extent of his interaction with other scientists

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[1] Quoted in Eliza Ware (Rotch) Farrar, *Recollections of Seventy Years*, 2d ed. (Boston: Ticknor and Fields, 1866), p. 184. The actual letter apparently no longer exists, so it is not clear whether the author was Charles Babbage, Lucasian Professor of Mathematics at Cambridge in 1836; James Challis, Plumian Professor of Astronomy and Experimental Philosophy; William Lax, Lowndean Professor of Astronomy and Geometry; or William Farish, Professor of Natural and Experimental Philosophy.

[2] Eliza Farrar, *Recollections* (cit. n. 1), pp. 118-127, 184-194. For a popularized introduction to these women, see Margaret Alic, *Hypatia's Heritage* (Boston: Beacon Press, 1988), pp. 174-175, 181-190.

survive only because his wife was influential enough in her own right to publish her memoirs. Similarly, Farrar's generation has been largely overlooked in the history of American mathematics and science. Later nineteenth-century American scientists themselves urged that their predecessors be played down in importance, making remarks such as Simon Newcomb's famous statement that: "For half a century [after Benjamin Franklin and David Rittenhouse] there was nothing worthy of the name of national science, nothing on which the public could look, and say with pride that it was a product of our educational system or of our effort to promote the knowledge of science."[3] Accepting this claim of backwardness at face value, historians of American physical science and astronomy at first studied only colonial natural philosophy and the development of the scientific research community after 1876.[4] They generally characterized American science of the intervening period as completely utilitarian and only practiced by elite amateurs outside institutions, such as Thomas Jefferson and Nathaniel Bowditch. They further claimed that the nature of American colleges inherently retarded the progress of science by showing interest only in the classical languages.

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[3] Simon Newcomb, "Abstract Science in America, 1776-1876," *North American Review* 122 (1876): 88-123, on p. 96. Newcomb is quoted in, among others, George H. Daniels, *American Science in the Age of Jackson* (New York: Columbia University Press, 1968), p. 6.

[4] Some of the earlier studies of colonial science include: Frederick E. Brasch, "The Newtonian Epoch in the American Colonies (1680-1783)," *Proceedings of the American Antiquarian Society*, n.s., 49 (1939): 314-332; Dirk J. Struik, *Yankee Science in the Making* (Boston: Little, Brown, and Company, 1948); Brooke Hindle, *The Pursuit of Science in Revolutionary America 1735-1789* (Chapel Hill: The University of North Carolina Press, 1956); and Brooke Hindle, ed., *Early American Science* (New York: Science History Publications, 1976), which is a collection of essays originally published between 1939 and 1973. For an example of earlier work on American science after 1876, see I. Bernard Cohen, *Some Early Tools of American Science: An Account of the Early Scientific Instruments and Mineralogical and Biological Collections in Harvard University* (New York: Russell & Russell, 1950). On the beginnings of the historiography of American science, see Arnold Thackray, "The Pre-History of an Academic Discipline: The Study of the History of Science in the United States, 1891-1941," *Minerva* 18 (1980): 448-473; and Nathan Reingold, "History of Science Today, 1. Uniformity as Hidden Diversity: History of Science in the United States, 1920-1940," *British Journal for the History of Science* 19 (1986): 243-262.

The history of American mathematics historiography differed from that of

American science in that certain founders of the history of mathematics in the

United States encyclopedically covered the mathematics and mathematical

textbooks of the early republic as part of efforts to list every know fact about

American mathematics.[5] By the time of the last of these publications, though,

mainstream historians of American mathematics had already shifted their full

attention to university research mathematics as practiced in institutions founded in

the late nineteenth century, such as the Johns Hopkins University and the University

of Chicago.[6] The same was true when historians essentially resurrected the

discipline of the history of mathematics after a nearly barren period throughout

World War II and the middle of the twentieth century — studies by Americans of

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[5] For an overview of the first American historians of mathematics, see Uta C. Merzbach, "The Study of the History of Mathematics in America: A Centennial Sketch," in *A Century of Mathematics in America*, ed. Peter Duren, vol. 3, (Providence, RI: American Mathematical Society, 1989), pp. 639-666. One of these founders, Florian Cajori, prepared the most thorough study ever of the history of mathematics education in the United States, *The Teaching and History of Mathematics in the United States* (Washington, DC: Government Printing Office, 1890); while Louis C. Karpinski painstakingly catalogued American mathematics publications from earliest settlement up to 1850 in *Bibliography of Mathematical Works Printed in America Through 1850* (Ann Arbor: The University of Michigan Press, 1940). See also Lao Genevra Simons, *Introduction of Algebra into American Schools in the Eighteenth Century*, Bureau of Education Bulletin No. 18 (Washington, DC: Government Printing Office, 1924); Lao Genevra Simons, "Algebra at Harvard College in 1730," *American Mathematical Monthly* 32 (1925): 63-70; Lao Genevra Simons, "The Influence of French Mathematicians at the End of the Eighteenth Century Upon the Teaching of Mathematics in American Colleges," *Isis* 15 (1931): 104-123; Lao Genevra Simons, "The Adoption of the Method of Fluxions in American Schools," *Scripta Mathematica* 4 (1936): 207-219; Lao Genevra Simons, *Bibliography of Early American Textbooks on Algebra*, Scripta Mathematica Studies No. 1 (New York: Scripta Mathematica, 1936); and Lao Genevra Simons, "Short Stories in Colonial Geometry," *Osiris* 1 (1936): 584-605. Simons was a student of the giant, David Eugene Smith, and her biography is in Carolyn Eisele, "Lao Genevra Simons," *Scripta Mathematica* 16 (1950): 22-30.

[6] For example, David Eugene Smith and Jekuthiel Ginsburg included brief accounts of mathematics in the colonies and early republic, but they called the early nineteenth century merely a time of "preparation for action" and devoted half of their text to the period between 1850 and 1900; David Eugene Smith and Jekuthiel Ginsburg, *A History of Mathematics in America Before 1900*, Carus Mathematical Monographs No. 5 (Mathematical Association of America, 1934), p. 65.

Americans were all about recent higher mathematics.[7] There have been only

scattered articles in the history of mathematics to provide evidence that there was an

influential group of mathematics professors within the antebellum American

scientific community.[8]

Over the past forty years, meanwhile, historians have reappraised the history

of American science and revealed the gradual development of a scientific

community before the Civil War.[9] These scholars saw the War of 1812 as a turning

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[7] In this, they followed one of the major tendencies in mathematics historiography listed by Ivor Grattan-Guinness, a preference for pure mathematics to the exclusion of any other mathematical endeavors. This is a favorite theme of Grattan-Guinness; see, for example, Ivor Grattan-Guinness, "Talepiece: The History of Mathematics and Its Own History," in *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, ed. Ivor Grattan-Guinness, vol. 2 (London and New York: Routledge, 1994), pp. 1665-1675. Note also that four-fifths of the pages in Dalton Tarwater, ed., *The Bicentennial Tribute to American Mathematics, 1776-1976* (Mathematical Association of America, 1977) are devoted to American mathematics after 1891; that J. Dalton Tarwater, John T. White, and John D. Miller, ed., *Men and Institutions in American Mathematics*, Graduate Studies No. 13, Texas Tech University (Lubbock: Texas Tech Press, 1976) is also weighted toward living mathematicians; and that there is a similar emphasis on the twentieth century in Duren, *Century of Mathematics* (cit. n. 5). Even Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community, 1876-1900: J. J. Sylvester, Felix Klein, and E. H. Moore*, History of Mathematics Vol. 8 (American Mathematical Society, 1994), p. 2, discounted mathematics in the United States before the time period of their book, stating that Americans were essentially closed out of European mathematics until 1850.

[8] Most notable are a series of articles by Edward Hogan which have improved in quality over time: Edward R. Hogan, "The Beginnings of Mathematics in a Howling Wilderness," *Historia Mathematica* 1 (1974): 151-166; Edward R. Hogan, "George Baron and the *Mathematical Correspondent*," *Historia Mathematica* 3 (1976): 403-415; Edward R. Hogan, "Robert Adrain: American Mathematician," *Historia Mathematica* 4 (1977): 157-172; Edward R. Hogan, "Theodore Strong and Ante-Bellum American Mathematics," *Historia Mathematica* 8 (1981): 439-455; Edward R. Hogan, "*The Mathematical Miscellany* (1836-1839)," *Historia Mathematica* 12 (1985): 245-257; Edward R. Hogan, "The Fading Amateur: William Lenhart and Nineteenth-Century American Mathematics," *Historia Mathematica* 17 (1990): 6-15; and Edward R. Hogan, "'A Proper Spirit is Abroad': Peirce, Sylvester, Ward, and American Mathematics, 1829-1843," *Historia Mathematica* 18 (1991): 158-172. See also Dirk J. Struik, "Mathematics in Colonial America," and Judith V. Grabiner, "Mathematics in America: The First Hundred Years," in Tarwater, *Bicentennial Tribute* (cit. n. 7), pp. 1-7, 9-24. Meanwhile, for the story of an American outside institutions toward the middle of the nineteenth century, see Pat Allaire, "Artemas Martin: An Amateur Mathematician of the Nineteenth Century," in *Combined Proceedings of the 1996 and 1997 Midwest Mathematical History Conferences*, pp. 1-12.

[9] The author's sense of how scientists work was originally shaped by Thomas S. Kuhn, *The Structure of Scientific Revolutions*, 2d ed. (Chicago: The University of Chicago Press, 1970). The more recent viewpoint in the history of American science was laid out, for example, by Nathan Reingold,

point for reawakened American interest in European ideas because the end to wartime curbs on shipping allowed the importation of Continental mathematics, astronomy, and physical science. The broadened historiographical perspective included a demonstration that, between the War of 1812 and the Civil War, a class of professional scientists who practiced "normal science" in specialized disciplines formed alongside the several talented amateurs who were previously known to scholars.[10] These scientists were employed in colleges, which were shown to be friendly to an increased role for science in the curriculum — after all, even before the supposed new opportunities after the War of 1812, the twenty-one full-time jobs in science existent in the United States in 1802 were all in academia.[11] These professors

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ed. and intro., *Science in Nineteenth-Century America: A Documentary History* (New York: Hill and Wang, 1964); Daniels, *American Science* (cit. n. 3); and George H. Daniels, ed., *Nineteenth-Century American Science: A Reappraisal* (Evanston: Northwestern University Press, 1976). A. Hunter Dupree also demonstrated that the history of American science was inseparable from the social and intellectual history of the United States in *Science and the Federal Government: A History of Policies and Activities to 1940* (Cambridge: Harvard University Press, 1957), and in *Asa Gray* (Cambridge: Harvard University Press, 1959). Studies following the reappraisal methodology include Simon Baatz, "'Squinting at Silliman': Scientific Periodicals in the Early American Republic, 1810-1833," *Isis* 82 (1991): 223-244; Stanley M. Guralnick, "Sources of Misconception on the Role of Science in the Nineteenth-Century American College," *Isis* 65 (1974): 352-366; Stanley M. Guralnick, *Science and the Antebellum American College* (Philadelphia: American Philosophical Society, 1975); John C. Greene, *American Science in the Age of Jefferson* (Ames: Iowa State University Press, 1984); Nathan Reingold, ed., *The Sciences in the American Context: New Perspectives* (Washington, DC: Smithsonian Institution Press, 1979); and Jurgen Herbst, "American Higher Education in the Age of the College," *History of Universities* 7 (1988): 37-59. The reappraisal came together with an emphasis on social history in the history of science in studies such as Hyman Kuritz, "The Popularization of Science in Nineteenth-Century America," *History of Education Quarterly* 21 (1981): 259-274; Patricia Cline Cohen, *A Calculating People: The Spread of Numeracy in Early America* (Chicago and London: The University of Chicago Press, 1982); and Sally Gregory Kohlstedt, "Parlors, Primers, and Public Schooling: Education for Science in Nineteenth-Century America," *Isis* 81 (1990): 425-445.

[10] George H. Daniels, *American Science* (cit. n. 3), named 56 leaders of American science (Theodore Strong, who is mentioned in Chapter Three, was the only mathematician listed), while Donald deB. Beaver, "The American Scientific Community, 1800-1860: A Statistical-Historical Study," (Ph.D. diss., Yale University, 1966), pp. 93-135, counted a community of 138 active scientists, with almost two thousand more people who contributed at least one paper to scientific journals.

[11] Douglas Sloan, *The Scottish Enlightenment and the American College Ideal* (Teachers College, Columbia University: Teachers College Press, 1971), pp. 225-247.

formulated some of the problems explored by the researchers of the late nineteenth century. As historians took a fresh look at the early to mid-nineteenth century, they urged that science in the American college be studied within its own context.[12] In the meantime, though, mathematics had come to be omitted completely from histories of American science.[13]

In other words, historians of mathematics currently tend to ignore the early American republic, while historians of American science recognize the importance of this time period but ignore mathematics despite the subject's centrality in the liberal arts college curriculum. Yet, if the American scientific community was developing in the colleges during this period, certainly mathematics professors were a part of the transformation. Active participants in international philosophical and intellectual debates, these men were aware of European mathematics, including a contemporary emphasis on analysis and synthesis, French achievements in mathematics, and Scottish discussion of the changing mathematical world. In addition, by the late eighteenth century, American colleges had made the transition from teaching all mathematics in the last year of a student's education to requiring mathematics throughout a four-year course. This meant in part that geometry was introduced as a separate college subject around 1790. As professors explored

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[12] In this regard, see especially Guralnick, "Sources of Misconception," (cit. n. 9); Nathan Reingold, "American Indifference to Basic Research: A Reappraisal," in Daniels, *Nineteenth-Century American Science* (cit. n. 9), pp. 38-62; and Nathan Reingold, "The Peculiarities of the Americans, or Are There National Styles in the Sciences?" *Science in Context* 4, no. 2 (1991): 347-366.

[13] The archetypal example of this is the first volume of the new series of *Osiris*, which was devoted to American science but contained no article on mathematics. This work was also published as Sally Gregory Kohlstedt and Margaret W. Rossiter, ed., *Historical Writing on American Science: Perspectives and Prospects* (Baltimore: The John Hopkins University Press, 1986). History of science and history of mathematics in general have been separate enterprises in the second half of the twentieth century, as Ivor Grattan-Guinness has also repeatedly stated. See Ivor Grattan-Guinness, "Does History of Science Treat of the History of Science? The Case of Mathematics," *History of Science* 28 (1990): 149-173.

different ways to present the essential material, they prepared a variety of textbooks in a flurry of activity that began to abate after 1844, when geometry became an entrance requirement at Harvard.[14]

In addition to two Scottish editions of Euclid's *Elements*, Farrar, Jeremiah Day, and Charles Davies produced the geometry textbooks printed the most often in the United States between 1790 and 1840, according to the bibliography compiled by Louis Karpinski.[15] Robert Simson's 1756 *The Elements of Euclid* went through twelve American printings and John Playfair's 1795 *Elements of Geometry* appeared in thirty-three printings in the United States, while there were twelve printings of Jeremiah Day's 1816 *A Practical Application of the Principles of Geometry to the Mensuration of Superficies and Solids*, John Farrar's 1819 *Elements of Geometry* went through ten printings, and Charles Davies's 1828 *Elements of Geometry and Trigonometry* appeared in thirty-three printings. (The latter two texts began as translations of Adrien-Marie

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[14] Yet, historians of American education have largely been uninterested in mathematics education in the early republic. For example, Frederick Rudolph, *Curriculum: A History of the American Undergraduate Course of Study Since 1636* (San Francisco: Jossey-Bass Publishers, 1977); and Frederick Rudolph, *The American College and University: A Study* (New York: A. Knopf, 1962; reprint, intro. John R. Thelin, Athens and London: The University of Georgia Press, 1990) portrayed the early nineteenth century as stale and unchanging, while Elmer Ellsworth Brown, *The Making of Our Middle Schools: An Account of the Development of Secondary Education in the United States*, 3d ed. (New York: Longman, Green, and Co., 1907); Robert Middlekauff, *Ancients and Axioms: Secondary Education in Eighteenth-Century New England* (New Haven and London: Yale University Press, 1963; reprint, New York: Arno Press and The New York Times, 1971); and Lawrence A. Cremin, *American Education: The National Experience 1783-1876* (New York: Harper & Row, 1980) are typical of studies concerned with the development of primary and secondary schooling merely for demonstrating the origins of universal public education. Most sources on the general development of geometry education are quite dated, such as Alva Walker Stamper, *A History of the Teaching of Elementary Geometry*, Columbia Teachers College Series No. 23 (New York: Teachers College Press, 1909); Florian Cajori, "Attempts Made During the Eighteenth and Nineteenth Centuries to Reform the Teaching of Geometry," *American Mathematical Monthly* 17 (1910): 181-201; and F. V. Kokomoor, "The Teaching of Elementary Geometry in the Seventeenth Century," *Isis* 10 (1928): 21-32, 369-415; 11 (1928) 85-110. Finally, the treatments of American geometry textbooks have been superficial and descriptive only. See John Donald Wilson, "An Analysis of the Plane Geometry Content of Geometry Textbooks Published in the United States Before 1900" (Ed.D. diss., University of Pittsburgh, 1959); and John A. Nietz, *The Evolution of American Secondary School Textbooks* (Rutland, VT: Charles E. Tuttle Company, 1966).

[15] Karpinski, *Bibliography* (cit. n. 5), pp. 149-150, 163-165, 183-185, 189-190, and 292-293.

Legendre's 1794 *Éléments de Géometrie*.) The only books used to teach geometry and comparable to these in popularity were by English mathematics professors: John Bonnycastle's *An Introduction to Mensuration and Practical Geometry* (twenty printings from the tenth London edition, beginning in 1812), and Charles Hutton's *A Course of Mathematics* (eight printings of Robert Adrain's 1812 revised edition), which was a compendium rather than an individual text on geometry. These two textbooks were also generally used in academies and for private study rather than in colleges.

Thus, an examination of geometry textbooks taught in colleges in the United States is a worthwhile enterprise. French scholars have set a precedent for looking to textbooks for help with understanding the making of science or mathematics and the scientific community.[16] Likewise, a consideration of Day, Farrar, and Davies aids with filling in the overlooked story of mathematics education and the mathematical community in the early American republic by bringing attention to the importance of mathematics education and textbooks to the professors who established the scientific community during that time period. The careers of the three professors all illustrate the friendly relationship between the American college and science and mathematics. Furthermore, the professors' works show the role of analysis and synthesis in geometry teaching, including but transcending the similarities between the usage of the terms and the long-held belief that at least one role of the American college lay in the creation of gentlemen. Lastly, an examination of the production of geometry textbooks by Day, Farrar, and Davies demonstrates that American

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[16] See Jean G. Dhombres, "French Textbooks in the Sciences 1750-1850," *History of Education* 13 (1984): 153-161; Jean G. Dhombres, "French Mathematical Textbooks from Euler to Cauchy," *Historia Scientiarum* 28 (1985): 91-137; Bernadette Bensaude-Vincent, "A View of the Chemical Revolution Through Contemporary Textbooks: Lavoisier, Fourcroy and Chaptal," *British Journal for the History of Science* 23 (1990): 435-460; Pierre Lamandé, "Trois Traités Français de Géométrie à l'Oré du XIXᵉ Siécle: Legendre, Peyrard et Lacroix," *Physis* 30 (1993): 243-302.

mathematical developments connected to the contemporary European context and most specifically to influences from Scotland.

Furthermore, as a first step toward meeting a need pointed out by the historian Chandos Michael Brown, who stated in a cultural biography of Day's colleague, Benjamin Silliman, that, "Historians are greatly in need of modern biographies of several early American men of learning,"[17] the dissertation will devote more space to biography than might ordinarily be expected in a textual study. For, despite the popularity of the three American professors' textbooks during their lifetimes, many details of the careers and lives of Day, Farrar, and Davies have not heretofore been published. The three men do not all appear in any of the major biographical dictionaries.[18] They have only been mentioned sporadically in histories of American science.[19] The first historians of American

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[17] Chandos Michael Brown, *Benjamin Silliman: A Life in the Young Republic* (Princeton: Princeton University Press, 1989), p. 336n. Indeed, research on this dissertation has illustrated the need for a historian to draw together the varied cast of characters of early nineteenth-century American science into a monograph.

[18] Scientific encyclopedias containing only a biography of Farrar are Brooke Hindle, "Farrar, John," in *Dictionary of Scientific Biography*, ed. Charles Coulston Gillispie, vol. 4 (New York: Charles Scribner's Sons, 1970), pp. 546-547 (hereinafter cited *DSB*); and "Farrar, John," in Clark A. Elliott, *Biographical Dictionary of American Science: The Seventeenth Through the Nineteenth Centuries* (Westport, CT: Greenwood Press, 1979), p. 91. The usually comprehensive *Dictionary of American Biography* (hereinafter cited *DAB*) omitted Davies despite the unprecedented distribution of his entire mathematical textbook series; Harris Elwood Starr, "Day, Jeremiah," and David Eugene Smith, "Farrar, John," in *DAB*, ed. Allen Johnson, vols. 5-6 (New York: Charles Scribner's Sons, 1927-1936), pp. 161-162, 292-293. The recent *American National Biography* (hereinafter cited *ANB*) contains articles on Day (which, although generally correct, errs seriously by claiming Day had no children rather than four) and Farrar's wife, Eliza; John S. Whitehead, "Day, Jeremiah," and Joonok Huh, "Farrar, Eliza Ware Rotch," in *ANB*, ed. John A. Garraty and Mark C. Carnes, vols. 6-7 (New York and Oxford: Oxford University Press, 1999), pp. 272-273, 737-738. Day and Davies, along with Eliza Farrar, were also covered in *The National Cyclopedia of American Biography* (hereinafter cited *NCAB*), "Davies, Charles," "Day, Jeremiah," and "Farrar, Eliza Ware (Rotch)," in *NCAB*, (New York and Clifton, NJ: James T. White and Company, 1898-1984), vol. 3, p. 26; vol. 1, pp. 169-170; vol. 13, pp. 317-318.

[19] See, for example, Guralnick, *Science* (cit. n. 9).

mathematics similarly included Day, Farrar, and Davies only incidentally within the larger stories they wanted to tell.[20]

Yet, there are at least some manuscript materials available to assist with biographies of each of these professors.[21] The Yale University Library maintains sixteen boxes of Day's notes, records, and drafts of textbooks, sermons, and theological treatises, as well as two boxes and one folder of correspondence and miscellaneous biographical material. There are also twenty-one volumes of letters to Day (including approximately twenty-five drafts of letters by Day) in the Beinecke Rare Book and Manuscript Library at Yale; the majority of these, however, are one-time requests made of Day during the years he was president of Yale. The materials for Farrar and Davies are considerably smaller in volume but do contain pieces of relevance to their authorship of mathematical textbooks. The two folders of correspondence and newspaper clippings related to Farrar held by the Harvard University Archives are significantly augmented by letters and meeting reports contained in Harvard records, especially the Harvard College Papers and Harvard Corporation Records. Nearly sixty additional letters by Farrar and his wife, Eliza Ware Rotch Farrar, are available in various collections at the Boston Public Library, Massachusetts Historical Society, and Old Dartmouth Historical Society. The United States Military Academy Archives holds two folders of biographical materials and letters related to Davies and has further assembled a folder from its own and other collections of photocopies of nearly forty letters by or to Davies and his family members. Combined with isolated letters in collections from other American

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[20] For instance, consult Cajori, "Attempts" (cit. n. 14), Simons, "Influence" (cit. n. 5), and Simons, "Short Stories" (cit. n. 5).

[21] Full details on the archival sources consulted during this study are provided in the Bibliography.

professors and intellectuals, there are enough manuscripts to shed a new level of light upon Day, Farrar, Davies, and their textbooks.

Perhaps the most substantial previous treatment of all three men was contained in Florian Cajori's detailed *The Teaching and History of Mathematics in the United States*, where Cajori explained the origin of each man's textbook and further traced the use of each volume at institutions beyond Yale, Harvard, and the United States Military Academy, the respective colleges at which Day, Farrar, and Davies taught when they first prepared their geometries.[22] Cajori, however, fit each man's story within his viewpoint that American mathematics professors were originally guided only by British textbooks until French works completely displaced them around 1820 and then served indefinitely as the central influence on American textbook authors.[23] The lone voice to question Cajori's "British, then French" framework has been Helena Pycior, who examined Day's, Farrar's, and Davies's algebra textbooks, which were used as widely as their geometries, along with one written by Benjamin Peirce.[24] Pycior concluded that Day's algebra textbook was "British" in style, Farrar's translation introduced the "French" influence, Davies

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[22] Cajori, *Teaching and History* (cit. n. 5).

[23] This is not an entirely fair summary of Cajori's argument as he did note that "we must guard against the impression that French authors and methods entirely displaced the English" since some nineteenth-century Americans did continue to use English textbooks and the English systems of weights and measures and of trigonometry were retained; Cajori, *Teaching and History* (cit. n. 5), p. 100. Still, Cajori's model as presented here has been very popular in the history of American mathematics and science. For example, John C. Greene differentiated between British and French methods of teaching and argued that the "French" way was introduced before 1820 only at Harvard and the Military Academy; Greene, *American Science* (cit. n. 9), pp. 128-157.

[24] Helena M. Pycior, "British Synthetic Vs. French Analytic Styles of Algebra in the Early American Republic," in *The History of Modern Mathematics*, ed. David E. Rowe and John McCleary, vol. 1 (San Diego: Academic Press, Inc., 1989), pp. 125-154. Pycior also argued that, rather than solving the problem of instruction, the introduction of French algebra textbooks brought Americans into the debate over analysis and synthesis, on p. 145. Peirce is not included in this study beyond brief mentions in Chapters Four and Six because he began to teach considerably later than the others and because his geometry textbook was unread outside of Harvard.

mixed "British" and "French" approaches, and Peirce broke with the tradition of weighing British and French textbooks against each other to independently compose an algebra textbook free of rhetoric about the mental discipline afforded by the study of mathematics or about the meaning of negative numbers. This study seeks to extend Pycior's reinterpretation of the American problem of mathematics instruction and its role in the international debate over analysis and synthesis to the geometry textbooks prepared by Day, Farrar, and Davies. In doing so, the author also has discovered that Day, too, mixed textbook influences from Great Britain and France and that some aspects of the British influence can be separated into factors originating from England or from Scotland.

Besides being grouped by the popularity of their textbooks and by Pycior's article, there are several reasons for looking at Day, Farrar, and Davies within one study. All three men were interested in French mathematics, understood it in their own individual ways, and tried to make it known to others: Day as part of a treatise on mensuration which was to be accompanied by Playfair's *Elements* and Farrar and Davies through translations of Legendre's *Éléments*. At the same time, the three professors had much in common intellectually with Scottish mathematicians. In all, the three were among American science and mathematics professors practicing an active form of reception early in the nineteenth century.[25] Two of the terms they learned were "analysis" and "synthesis," which surfaced, among other areas, in American discussions of liberal education.[26] All three professors believed in a form

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[25] Another reception study with respect to Euclidean geometry and education is Gregg De Young, "Euclidean Geometry in the Mathematical Tradition of Islamic India," *Historia Mathematica* 22 (1995): 138-153. India is certainly a neglected area historiographically, but there is still much to learn about the United States, as well.

[26] A study of analysis and synthesis considering several of the same issues as this project but with a different methodology is Massimo Mazzotti, "The Geometers of God: Mathematics and Reaction in the Kingdom of Naples," *Isis* 89 (1998): 674-701.

of liberal education which included substantial amounts of science and mathematics, and they hoped their students would truly understand mathematics and find it useful in their lives, as well as appreciate the logical structure of mathematics, the paradigm of which was Euclidean geometry.[27] Day, Farrar, and Davies each helped make small modifications to the classical curriculum that presaged the American college reform movement historians usually date only as far back as the 1820s. In all of their activities, the three men were members of a bridge generation involved in republic- and university-building which formed an essential step between the colonial amateurs and the professional university researchers. Thus, after introducing French influences in geometry and the background to the terms of "analysis" and "synthesis," the dissertation will examine in detail the Scottish origins of American geometry textbooks before turning to studies of Day, Farrar, Davies, and their geometry textbooks. Finally, the project will summarize the path of American college geometry education after these men's careers.

Background Material — French Mathematics

Two aspects of mathematical culture were so pervasive around 1800 that they are relied upon as standards or mathematical "givens" in this study. First, Scots and Americans both appreciated French mathematics, albeit from the outside.[28] Toward the beginning of the eighteenth century, they would have observed the dominant

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[27] Patricia Cline Cohen argued that American appreciation of the value of mathematics for mental training began with Warren Colburn's 1820 *Intellectual Arithmetic*, but this dissertation will show that this attitude was already in place by the 1810s. Cohen, *Calculating People* (cit. n. 9), pp. 116-149.

[28] It does not appear that either group tried to influence mathematical practice in France, nor were French intellectuals interested in their ideas for the most part. For instance, Maurice Crosland has stated that French scientists generally read only French publications in the nineteenth century; Maurice P. Crosland, *Science Under Control: The French Academy of Sciences, 1795-1914* (Cambridge: Cambridge University Press, 1992), pp. 11-49. One exception was French interest in John Leslie's 1809 *Elements of Geometry*, which is noted in Chapter Two.

educational philosophy to be that developed by teachers based in Port Royal. These educators emphasized the elimination of competition in the classroom, the study of French language and culture, and the intuitionist approach to logic, all set down in the influential *Logique de Port-Royal*.[29] Their Cartesian view of mathematics, natural philosophy, and metaphysics shaped eighteenth-century university study in France, which led to the three professions, theology, medicine, and law. The Port Royalists also began to gradually replace the French tradition of dictation by teachers with textbooks the students could peruse both in class and on their own at all levels of learning.[30] In geometry, the prevailing voice was the lingering influence of Antoine Arnauld. He had urged French educators to put aside Euclid's *Elements* in 1668 because he believed that one could rely on intuition to arrange the propositions of geometry into "natural order," from the least to the greatest levels of abstraction. He thought it was possible to ignore the fine details of geometry and to avoid *reductio ad absurdum*, proof by contradiction, and yet retain the mental clarity which gave geometry its chief intellectual benefit.[31] In addition, André Tacquet published a Latin edition of Euclid's *Elements* in 1654 in which he omitted propositions he considered useless and introduced algebra into the content of the work, assuring readers that clear reasoning was compatible with the discipline of algebra.[32]

Although the influence of the Port Royal educationalists began to wane with the expulsion of the Jesuits in 1761 and with the gradual acceptance in France of

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[29] Cajori, "Attempts" (cit. n. 14).

[30] L. B. Brockliss, *French Higher Education in the Seventeenth and Eighteenth Centuries: A Cultural History* (Oxford: Clarendon Press, 1987), pp. 52-104, 111-177.

[31] Lamandé, "Trois Traités" (cit. n. 16); Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 130.

[32] John L. Heilbron, *Geometry Civilized: History, Culture, and Technique* (Oxford: Clarendon Press, 1998), pp. 14-15.

Newtonian physics after Voltaire and Emilie du Châtelet popularized the *Principia* in 1738, the Port Royalist emphasis on textbooks survived and deepened in the secondary schools (*collèges*) but also at least as much so in the military engineering schools (*écoles*) founded in the middle of the eighteenth century.[33] In mathematics, the professors Charles Bossut (1730-1814) and Étienne Bézout (1739-1783), set a standard for complete textbooks ranging from arithmetic and geometry to calculus and engineering, by writing courses intended to contain the material which should be expected of every French student.[34] Indeed, each man's *Cours de Mathématiques* appeared in multiple editions.[35] Bézout's course included a separate volume on elementary geometry. In the 1780s, Gaspard Monge (1746-1818) expanded the mathematics curriculum when he introduced the study of analytical geometry and created descriptive geometry, the practice of drawing three-dimensional objects in two dimensions, as a separate discipline.[36] Meanwhile, many of the spectacular developments stemming from the exploitation of the calculus had been accomplished by eighteenth-century mathematicians who were employed by the French state.[37] For example, Alexis Clairaut and others measured the obloid shape of

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[33] Brockliss, *French Higher Education* (cit. n. 30), pp. 337-390.

[34] See C. Stewart Gillmor, "Bossut, Charles," and Judith V. Grabiner, "Bezout, Étienne," in *DSB* (cit. n. 18), vol. 2, pp. 111-114, 334-335.

[35] Etienne Bézout, *Cours de Mathématiques à l'usage des Gardes du Pavillon et de la Marine*, 6 vols., Paris, 1764-1769; Charles Bossut, *Cours de Mathématiques à l'usage des Ecoles Royales Militaires*, Paris, 1782.

[36] For an evaluation of Monge's contributions to mathematics, see Eduard Glas, "On the Dynamics of Mathematical Change in the Case of Monge and the French Revolution," *Studies in History and Philosophy of Science* 17 (1986): 249-268.

[37] On the unique union of science and state in France, see Gerald L. Geison, ed., *Professions and the French State, 1700-1900* (Philadelphia: University of Pennsylvania Press, 1984); and Roger Hahn, *Anatomy of a Scientific Institution: The Paris Academy of Sciences, 1666-1803* (Berkeley and Los Angeles: University of California Press, 1971).

the earth and ascertained that the motion of the moon also corresponded to the requirements of Newton's gravitational theory. New disciplines such as differential equations and probability theory were created and applied.[38] In fact, the powerful techniques of differential equations gave birth themselves to integration by parts, Lagrange's principle of virtual velocities, and the calculus of variations, which is used to compute the maximum and minimum areas of planar figures. From hard bodies to the vibrating string to *vis viva*, mathematicians sought to extend the triumphant new mathematics to the explanation of all physical phenomena.[39] However, despite the expanded range of mathematics, there were limits to the thoroughness of mathematics education in the *ancien régime* — mathematics was still not taught in the universities beyond the context of its connection to logic and reasoning, while technical education in France remained "haphazard," to use Frederick Artz's word.[40]

[38] On the Enlightenment and probability, see Lorraine J. Daston, *Classical Probability in the Enlightenment* (Princeton: Princeton University Press, 1988).

[39] Some of the more thorough textbook accounts of eighteenth-century mathematics are in Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics*, 2d ed. (New York: John Wiley & Sons, 1989); John Fauvel and Jeremy Gray, ed., *The History of Mathematics: A Reader* (Milton Keynes: The Open University, 1987); and Victor Katz, *The History of Mathematics: An Introduction* (New York: Addison-Wesley, 1993). Another good explanation of the invention of calculus is in Richard S. Westfall, *Never at Rest: A Biography of Isaac Newton* (Cambridge: Cambridge University, 1980). One of the standard sources on the eighteenth century mathematization of natural philosophy is Thomas L. Hankins, *Science and the Enlightenment* (Cambridge: Cambridge University Press, 1985). See also Tore Frängsmyr, John L. Heilbron, and Robin E. Rider, ed., *The Quantifying Spirit in the 18th Century* (Berkeley: University of California Press, 1990).

[40] Frederick B. Artz, *The Development of Technical Education in France, 1500-1850* (Cambridge: MIT Press and Society for the History of Technology, 1966), pp. 60-111; H. C. Barnard, *Education and the French Revolution* (Cambridge: Cambridge University Press, 1969), pp. 1-15. An overview of French scientific activity immediately before the Revolution is Charles Coulston Gillispie, *Science and Polity in France at the End of the Old Regime* (Princeton: Princeton University Press, 1980). Older but still of interest is Emile Durkheim, *The Evolution of Educational Thought: Lectures on the Formation and Development of Secondary Education in France* (Presses Universitaires de France, 1938; trans. Peter Collins, London: Routledge & Kegan Paul, 1977).

As is well known, French institutions were then thrown into upheaval by the Revolution. Among the first to feel the effects were the various schools, as they closed in the early stages of the Revolution under financial hardship and with the forced resignation of the clergy, who made up the majority of teachers.[41] Although the succession of French governmental bodies from 1789 continued to pass laws mandating compulsory primary education, these schools were not successfully established until the middle of the nineteenth century.[42] In the upper levels of education, the Ideologues controlled the philosophy of education, advocating rationalism, practical courses, service to society, and a national system which did not emphasize the *Université*, an institution with little prestige during and after the French Revolution.[43] Ironically for the anti-elitist Revolutionary society, the educational institutions which survived and were functional were the recreated military *écoles*, which provided specialized instruction to the top echelon of students,

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[41] L. Pearce Williams, "Science, Education, and the French Revolution," *Isis* 44 (1953): 311-330; Jonathan E. Helmreich, "The Establishment of Primary Schools in France Under the Directory," *French Historical Studies* 2 (1961-1962): 189-208. Specific information on French education, especially French mathematics education, is difficult to locate for periods before the Revolution. In contrast, the story of French Revolutionary education is far more popular. In addition to the sources cited in this section, see Margaret Bradley, "Scientific Education Versus Military Training: The Influence of Napoleon Bonaparte on the École Polytechnique," *Annals of Science* 32 (1975): 415-449; Margaret Bradley, "Scientific Education for a New Society: The École Polytechnique 1795-1830," *History of Education* 5 (1976): 11-24; and the primary account in Maurice P. Crosland, ed. and intro., *Science in France in the Revolutionary Era, Described by Thomas Bugge, Danish Astronomer Royal and Member of the International Commission on the Metric System (1798-1799)* (Cambridge: The MIT Press and Society for the History of Technology, 1969). Barnard, *Education and the French Revolution* (cit. n. 40), may be the most comprehensive account of the full range of educational institutions and their experiences throughout the Revolution and First Empire.

[42] Helmreich, "Establishment" (cit. n. 41). On the lack of attention to primary education and late improvement of these institutions, see also Barnard, *Education and the French Revolution* (cit. n. 40), pp. 210-222; Artz, *Development of Technical Education* (cit. n. 40), pp. 182-268; Joseph N. Moody, *French Education Since Napoleon* (Syracuse: Syracuse University Press, 1978), pp. 17-85; and Raymond Grew and Patrick J. Harrigan, *School, State, and Society: The Growth of Elementary Schooling in Nineteenth-Century France – A Quantitative Analysis* (Ann Arbor: The University of Michigan Press, 1991).

[43] On the domination by the *écoles* from the Revolution through the Empire and Restoration and into the Second Republic, see Ivor Grattan-Guinness, "Grandes écoles, petite Université: Some Puzzled Remarks on Higher Education in France, 1795-1840," *History of Universities* 7 (1988): 197-225.

who had been imbued in science, engineering, and mathematics at the new *École Polytechnique* (which opened in 1794).

The remodeled educational institutions involved France's leading scientists and mathematicians in the fundamentals of education.[44] These researchers, usually members of the *Académie des Sciences*, adapted themselves to Revolutionary demands for utility in science and mathematics and even enthusiastically set up curriculum programs.[45] In addition to teaching courses, they responded to an ever-increasing demand for textbooks in the *écoles* and the central schools, the institutions for secondary education established during the Directory.[46] Geometry was included among the mathematical material at the secondary level, with its capability for showing students how to think clearly valued over a detailed mastery of its content (although a substantial knowledge of geometry with its applications was also necessary to excel on the competitive entrance examinations at the *École Polytechnique*). For instance, Pierre-Simon Laplace (1749-1827) ambitiously devoted the seventh of his ten lectures at the short-lived *École Normale* to elementary

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[44] For differing views on the extent to which the Terror disrupted scientific activity, contrast Hahn, *Anatomy* (cit. n. 37), pp. 159-251; and Robert Fox, "Science, the University, and the State in Nineteenth-Century France," in Geison, *Professions* (cit. n. 37), pp. 66-145; with Dorinda Outram, "Politics and Vocation: French Science, 1793-1830," *British Journal for the History of Science* 13 (1980): 27-43; and Dorinda Outram, "The Ordeal of Vocation: The Paris Academy of Sciences and the Terror, 1793-95," *History of Science* 21 (1983): 251-273.

[45] See, for example, Jean G. Dhombres, "L'enseignement des Mathématiques par la 'méthode révolutionnaire.' Les Leçons de Laplace à l'Ecole Normale de l'An III," *Revue d'Histoire des Sciences* 33 (1980): 315-348. For simplicity's sake, "*Académie des Sciences*" is used here to refer to the state institution which recognized scientists and mathematicians and supported French science and mathematics but which closed and reopened under different names during the French Revolution and under Napoleon. For the full story of this body, see Hahn, *Anatomy* (cit. n. 37).

[46] The administration at the *École Polytechnique* also required instructors to write down their lectures, a requirement soon copied by other schools. See Dhombres, "French Textbooks" (cit. n. 16), p. 157. Lecture notes were initially printed as well in the *Journal of the École Polytechnique*, but these were replaced by research articles within a decade, most notably in mathematics. See Grattan-Guinness, "Grandes écoles" (cit. n. 43), p. 200.

geometry in addition to the notion of the limit, trigonometry and spherical trigonometry, the application of the limit to areas and volumes, and regular polyedrons.[47]

Adrien-Marie Legendre (1752-1833) published *Éléments de Géometrie* in 1794, as this environment was forming. Legendre had established himself as an intellectual before 1789, but he lost his small fortune in the Revolution and needed both employment and publications which would sell well. He may also have been inspired by Condorcet's 1791 call for elementary textbooks in *Mémoires sur l'instruction publique*, and furthermore he and Lagrange were commissioned by the Committee for Public Instruction in 1793 to write a geometry and calculus textbook, by which time Legendre's *Éléments* was already nearly completed.[48] Legendre was concerned that the Port Royalist attempts to make geometry more readily understandable had resulted in the loss of proper Euclidean rigor, but he also believed that geometry as systematized in Euclid's *Elements* was so problematic that a rewritten presentation of the elements of geometry, putting the propositions into a new and more logical order, was needed.[49] Thus, Legendre organized *Éléments* into

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[47] Dhombres, "L'enseignement" (cit. n. 45), p. 331.

[48] Jean Itard, "Legendre, Adrien-Marie," in *DSB* (cit. n. 18), vol. 8, pp. 136-137. See also Parisot, "Legendre (Adrien-Marie)," in *Bibliographie Universelle: Ancienne et Moderne*, ed. J. Fr. Michaud, new ed., vol. 23 (Paris, 1854), pp. 610-615. On various bits and pieces of Legendre's career, beyond the secondary literature listed by Itard, consult David Eugene Smith and Vera Sanford, "A Great Mathematician as a School Boy," *Mathematics Teacher* 14 (1921): 362-366; E. H. Neville, "A Bibliographical Note," *Mathematical Gazette* 17 (1933): 200-201; E. H. Neville, "Legendre Again," *Mathematical Gazette* 18 (1934): 195-196; "Anniversary Meeting, November 30, 1833," *Proceedings of the Royal Society of London* 3 (1830-1831): 230-232; Ruth Inez Champagne, "The Role of Five Eighteenth-Century French Mathematicians in the Development of the Metric System" (Ph.D. diss., Columbia University, 1979); and Duncan M. Y. Sommerville, "Note on Legendre's and Bertrand's Proofs of the Parallel-Postulate by Infinite Areas," *Proceedings of the Edinburgh Mathematical Society* 30 (1911-1912): 31-36.

[49] Adrien-Marie Legendre, *Éléments de Géométrie* (Paris: Firmin Didot, 1794), pp. v-vii; Vera Sanford, "Adrien-Marie Legendre," *Mathematics Teacher* 28 (1935): 182-184; Lamandé, "Trois Traités" (cit. n. 16).

eight books: the principles of geometry, the circle and the measurement of angles

(followed by eighteen problems related to the first two books), the proportions of

figures (followed by nineteen problems for Book III), regular polygons and the

measurement of the circle (with an appendix on the maxima and minima of areas,

isoperimetry), planes and solid angles, polyedrons, the sphere (with an appendix on

regular polygons), and the three round bodies. Perhaps the most notable aspect of

writing the *Éléments* was that Legendre began a search for a satisfactory proof of the

parallel postulate — in other words, he wanted to reduce the postulate to the status of

a theorem — which would continue for the rest of his career.[50]

The detailed endnotes Legendre prepared on parallels and other subjects

overshadowed the rest of the content of *Éléments* enough that the book was not

chosen in a 1794 competition for elementary textbooks because others saw it as too

advanced.[51] There were also several features of *Éléments* which initially

distinguished it to French readers as a work of geometrical research. Legendre

replaced the incommensurable magnitudes in the Euclidean theory of proportion

with arithmetical rational and irrational numbers for an algebraic treatment of

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[50] Legendre catalogued his efforts in "Réflexions sur différentes manières de démontrer la théorie des parallèles ou le théorème sur la somme des trois angles du triangle," *Mémoires de l'Académie des Sciences* 12 (1833): 367–410. Legendre's attempts, doomed to failure because he uncritically believed in absolute space, included a proof in 1823 that the sum of the angles in a triangle cannot be less than two right angles, which rested on the assumption that a line cutting both sides of an angle could be drawn through any point in the circumscribing circle, and a proof in 1833 that if the angles in any triangle are equal to two right angles, then all triangles would have angles summing to two right angles; however, Legendre could not demonstrate the existence of such a triangle. "Legendre" in *DSB*; Florian Cajori, *A History of Elementary Mathematics with Hints on Methods of Teaching*, rev. and enl. ed. (New York: The Macmillan Company, 1925), p. 271. For a summary of the importance of Legendre's concern with the parallel postulate in the history of non-Euclidean geometry, see Jeremy Gray, *Ideas of Space: Euclidean, Non-Euclidean, and Relativistic*, 2d ed. (Oxford: Clarendon Press, 1989), pp. 78–82. Roberto Bonola's classic work omits Legendre's role; Roberto Bonola, *Non-Euclidean Geometry: A Critical and Historical Study of Its Developments*, trans. H. S. Carslaw (Chicago: Open Court, 1912; reprint, Dover, 1955).

[51] Barnard, *Education and the French Revolution* (cit. n. 40), pp. 124–135.

planar figures.[52] He also provided his own, more extensive, theorems for Book VII, on spherical triangles, and he more generally provided fuller coverage of solid geometry.[53] Rather than direct the reader to the construction of the five Platonic solids, as Euclid had, Legendre's aim was to report original results on the geometry of the sphere. Further, Legendre added new material on quadrature, or the measurement of the circle and computation of π, and on isoperimetry, maximizing and minimizing plane figures. He separated plane geometry from solid geometry and, more significantly, problems from theorems. Legendre did not comment on this action, but he clearly had a different view of the relative status of mathematical statements than was common in other treatises on geometry, whether authors of those works were bound to categories of mathematical statements, like Euclid, or avoided naming "axioms" or "theorems" to ease the path of examiners and examinees, like Bézout. For instance, Legendre proved the fundamental theorems of Book I, from "All right angles are equal to each other" to "The two diagonals AC, DB of a parallelogram divide each other into equal parts" without relying upon any of the basic constructions, such as bisecting a line or angle, which were intermingled with the theorems in Euclid's *Elements*.[54] For these propositions, Legendre additionally prepared new demonstrations while he admitted hypothetical constructions as a proof technique for these figures. In addition, when he combined *Éléments* with a treatise on trigonometry in 1799, Legendre adopted the decimal division of the circle which stemmed from the projects developed in order to

[52] Legendre, *Éléments* (cit. n. 49), p. vii; Cajori, "Attempts" (cit. n. 14), p. 185.

[53] Sanford, "Legendre" (cit. n. 49); Lamandé, "Trois Traités" (cit. n. 16).

[54] Adrien-Marie Legendre, *Éléments de Géométrie, avec des Notes*, 9th ed. (Paris: Firmin Didot, 1812), pp. 1-32, 49-59.

decimalize French society by the Committee on Weights and Measures, to which

Legendre belonged.[55]

The method of presentation in *Éléments* was shaped as well by Legendre's

opinions on mathematics pedagogy. Legendre rejected the Port Royalist claims of a

"natural way" to do geometry and adherents' reliance on intuition and discovery.[56]

Instead, he wanted to restore systematic rigor to treatises on geometry as a mark of

quality. He situated himself alongside creative mathematicians such as Laplace and

Joseph Louis Lagrange (1736-1813), who advocated formal rigor to establish

fundamentals and to return researchers' attention from the empirical sciences to the

autonomy of pure mathematics.[57] Legendre thus described his book as constructed

in the "synthetic method," indicating that his style of proof in *Éléments* was

demonstrative rather than leading to the invention of new mathematics.[58] By setting

out the propositions and proofs, Legendre was explicit about the axiomatic structure

of elementary geometry; he also allowed *reductio ad absurdum*.[59] At the same time,

however, Legendre's dense and concise treatment was not unduly tied to what he

considered an old-fashioned view of pure geometry. For instance, Legendre was not

interested in elementary geometry as an instrument for developing mental

discipline. In addition, he was willing to intermix algebra, geometry, and

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[55] See Champagne, "Metric System" (cit. n. 48), pp. 121-176.

[56] Cajori, "Attempts" (cit. n. 14), p. 185; Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 130.

[57] Joan L. Richards, "Rigor and Clarity: Foundations of Mathematics in France and England, 1800-1840," *Science in Context* 4, no. 2 (1991): 297-319; Glas, "Monge and the French Revolution" (cit. n. 36), p. 256.

[58] Legendre, *Éléments*, 1794 (cit. n. 49), p. viii; Lamandé, "Trois Traités" (cit. n. 16).

[59] Jean G. Dhombres, "Mathématisation et Communauté Scientifique Française (1775-1825)," *Archives Internationales d'Histoire des Sciences* 36 (1986): 249-293.

trigonometry and to incorporate practical examples within the body of propositions, which he believed reflected the modern study of geometry but which would also assist students in following the text.

Indeed, Legendre's *Éléments* soon emerged alongside two other works as one of the significant geometry textbooks in a newly active French publishing atmosphere. Although Legendre's *Éléments* was one of only eleven mathematics books published in France between 1790 and 1794, the number of books prepared then exploded, in part because printers began to embrace Revolutionary freedom of the press and in part because teachers at the central schools were initially free to choose their textbooks, causing writers to compete for this market.[60] Seventy-five mathematics books were published between 1795 and 1799, including the 1799 second edition of *Éléments*, an adaptation of Bézout's course also issued in 1799 by François Peyrard (1759-1822), and the 1798 *Élémens de géométrie* by Sylvestre François Lacroix (1765-1843). Despite the preponderance of mathematics books (ninety-three more were published from 1800 to 1804) available in general, these three authors dominated geometry teaching in France by 1805, with fifteen editions of *Éléments* in print by 1847, twenty-three printings of Peyrard's revision of the geometry volume by Bézout by 1833, and fourteen editions of Lacroix's *Élémens de géométrie* by 1820 with seven more published by 1880.[61] One major reason for the popularity of these

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[60] A table of French mathematics publications from 1775 to 1825 appears on Dhombres, "Mathématisation" (cit. n. 59), p. 267. See also Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 105. On the end of *ancien régime* censorship, see Robert Darnton and Daniel Roche, ed., *Revolution in Print: The Press in France, 1775-1800* (Berkeley: University of California Press, 1989).

[61] Before 1854, a professor named Blanchet revised Legendre's *Éléments* and published it in another twenty-four editions by 1879; Lamandé, "Trois Traités" (cit. n. 16), p. 250; *The National Union Catalog* (hereinafter cited as *NUC*), vol. 310 (London: Mansell, 1976), pp. 655-656. The dominance of the three textbooks is an occurrence Jean Dhombres finds "curious." However, although he promised to explore the reasons for the dominance of these textbooks in "Evolution des contenus des manuels mathematiques," this article was never published. See Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 136; and Dhombres, "French Textbooks" (cit. n. 16), p. 159.

authors was that textbook decisions for the *lycées* Napoleon created after
suppressing the central schools in 1802 were guided by official lists — which
contained Lacroix's, Bézout's, and Legendre's books.[62] The *lycées* marked France's
return to a traditional style of secondary education, emphasizing the Latin language
and mathematics, although a knowledge of mathematics for its own sake remained
necessary to gain entrance into the *écoles*.[63] Peyrard's adaptation of Bézout's
geometry fit into these schools especially well because Peyrard had replaced much
of the original content with propositions from Euclid's *Elements*, including the
Euclidean theory of proportion. He provided a means of highlighting the reasoning
process of geometry while retaining Bézout's assignation of geometry as the first
subject for beginners, before algebra.[64]

On the other hand, Lacroix's *Élémens de géométrie* was appealing to French
teachers in the *écoles* because it was written in the intuitionist tradition of the Port
Royalists as well as in the analytical pedagogical style he took from Clairaut. Unlike
Peyrard and Legendre, who both believed that systematic rigor was the mark of
quality mathematics, Lacroix maintained the didactic tradition of an intuitively
"natural way" to present geometry.[65] He followed mathematicians such as Jean-le-
Rond d'Alembert and Louis Bertrand, who advocated organizing geometry
textbooks according to the order in which the propositions were originally

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[62] Admittedly, on the first list prepared by Laplace, Monge, and Lacroix, only books by
Lacroix were included. Bézout's *Cours* and Legendre's *Éléments* were soon added, though. Dhombres,
"French Mathematical Textbooks" (cit. n. 16), p. 105, 127n.

[63] Williams, "Science, Education" (cit. n. 41); Dhombres, "L'enseignement" (cit. n. 45), p. 160.

[64] "Bézout" in *DSB* (cit. n. 34), p. 112. Peyrard served on the Council of Instruction and as the
librarian at the *École Polytechnique*. He later translated works of ancient mathematics, including
Euclid's *Elements* from the Vatican manuscript. Lamandé, "Trois Traités" (cit. n. 16) 1993.

[65] Jean Itard, "Lacroix, Sylvestre François," in *DSB* (cit. n. 18), vol. 7, pp. 549-551; Lamandé,
"Trois Traités" (cit. n. 16); Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 130.

discovered and dividing the presentation into sections on straight lines and circles, surfaces, and solids. This school of thought argued that there should be no axioms, and that definitions should be introduced only as they became necessary.[66] In an essay on the order and manner of writing *Élémens de géométrie* placed at the beginning of the textbook, therefore, Lacroix urged readers to rely on sensation when forming geometrical judgments.[67] Lacroix also prepared an essay on mathematical method for the preface of *Élémens de géométrie*. To him, the method of mathematics was synthesis, or composition, and analysis, or resolution. He used these words as directions in the process of reasoning, either forwards or backwards. Although Lacroix believed analysis and synthesis were both necessary and both led to certain knowledge, he admitted that the importance of analysis had been lost in geometry in the sense that analysis was also a method of invention.[68] Like Legendre, Lacroix separated plane from solid geometry. He provided the essential theorems for working with lines and circles, planar figures, planes and solids, and round bodies, respectively. When he demonstrated the measurement of areas and volumes, Lacroix provided the relationships as algebraic formulas.[69] Unlike Legendre, Lacroix omitted any treatment of the proportional relationships between planar figures or of regular polygons and the approximation of π. Further, Lacroix did not go into dense detail or expect students to follow the reasoning without additional explanation, as

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[66] Cajori, "Attempts" (cit. n. 14), pp. 183-184.

[67] Sylvestre François Lacroix, *Élémens de géométrie, precedés de réflexions sur l'order à suivre dans ces Élémens, sur la manière de les écrire, et sur la méthode en Mathématiques* (Paris: De Crapelet, 1798).

[68] Lacroix, *Élémens de géométrie* (cit. n. 67); Lamandé, "Trois Traités" (cit. n. 16). Lacroix appealed to understandings of "analysis" and "synthesis" which will continue to appear throughout this dissertation, as directions of reasoning in proof and as a method of invention, but *Élémens de géométrie* was never translated into English; *NUC* (cit. n. 61), vol. 310, pp. 655-656.

[69] Lacroix, *Élémens de géométrie* (cit. n. 67).

Legendre did, which seemed to make Lacroix's book more suitable for future engineers. Indeed, Lacroix himself referred the reader to Legendre's *Éléments* for more information.[70] Approximately 1500 copies each of *Élémens de géométrie*, Legendre's *Éléments*, and Peyrard's version of Bézout's course were sold each year until about 1825.[71]

By the turn of the nineteenth century, British and American intellectuals had renewed their awareness of the rapid changes in French mathematics which reached a zenith with the researches of Laplace and Lagrange. Yet, Ivor Grattan-Guinness has noted that these researchers continued to follow eighteenth-century problem-solving procedures: natural philosophers chose a physical problem and made it mathematizable, constructed mathematics which were suitable for the problem, solved the resulting differential equation, and interpreted the solution back into the original physical problem.[72] Their conception of mathematics as a closed system and attempts to consolidate existing recent mathematical results were viewed as bringing Newtonian mathematics and science to completion.[73] They defined mathematics as a system of operations on symbols according to a set of rules, which meant that an analytical or algebraic result provided a correctly ordered set of symbols, while a geometric proof merely illustrated an argument which could be expressed analytically. Indeed, Lagrange's career was typical of the Enlightenment

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[70] Lacroix, *Élémens de géométrie* (cit. n. 67), pp. 1-2.

[71] Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 100, 103–105.

[72] Ivor Grattan-Guinness, "Mathematical Physics in France, 1800-1840: Knowledge, Activity, and Historiography," in *Mathematical Perspectives: Essays on Mathematics and Its Historical Development*, ed. Joseph W. Dauben (New York: Academic Press, 1981), pp. 95-138. See also Ivor Grattan-Guinness, *Convolutions in French Mathematics, 1800-1840: From the Calculus and Mechanics to Mathematical Analysis and Mathematical Physics*, 3 vols. (Basel: Birkhäuser Verlag, 1990).

[73] Glas, "Monge and the French Revolution" (cit. n. 36), p. 266; Robert Fox, "The Rise and Fall of Laplacian Physics," *Historical Studies in the Physical Sciences* 4 (1974): 89-136, on p. 90.

celebration of algebra which helped lead to the nineteenth-century program of algebraization. He did all of his research with an eye for how the mathematics involved could be expressed by algebraic equations, culminating with his attempt to base calculus on the expansion of power series. His work was informed throughout by the belief that algebra and geometry were wholly separate disciplines with the traditional treatises of geometry now rendered rather superfluous.[74]

Their status as outsiders helped lead British and American observers to overemphasize Lagrange's algebraic statements and thus miss at least one feature of French mathematical life after the turn of the nineteenth century. This was that other French mathematicians remained interested in synthetic geometry. Lazare Carnot and Monge, followed by Michel Chasles and Jean-Victor Poncelet, presented the discipline as an alternative to algebraic analysis with its fragile foundations, which lacked satisfactory demonstration until Augustin Louis Cauchy's ideas began to be accepted in the 1820s. The French geometers viewed geometry as united with mechanics and also believed that geometry was indispensable to pedagogy.[75] For a variety of reasons, though, and despite the French tendency to refer to mathematicians as *géomètres* throughout the nineteenth century, these men were never influential in the *Académie des Sciences* or the Institut, and so their research

[74] On Lagrange's algebraic method, see Robin Rider Hamburg, "The Theory of Equations in the 18th Century: The Work of Joseph Lagrange," *Archive for History of Exact Sciences* 16 (1976): 17-36; Craig G. Fraser, "Joseph Louis Lagrange's Algebraic Vision of the Calculus," *Historia Mathematica* 14 (1987): 38-53; Craig G. Fraser, "Lagrange's Analytical Mathematics, Its Cartesian Origins and Reception in Comte's Positive Philosophy," *Studies in History and Philosophy of Science* 21 (1990): 243-256; Judith V. Grabiner, "Changing Attitudes Toward Mathematical Rigor: Lagrange and Analysis in the Eighteenth and Nineteenth Centuries," in *Epistemological and Social Problems of the Sciences in the Early Nineteenth Century*, ed. Hans Wiels Jahnke and Michael Otte (Dordrecht: D. Reidel Publishing, 1981). On Laplace's and Lagrange's activities within the reconstituted Institut, see Crosland, *Science Under Control* (cit. n. 28), pp. 50-90.

[75] For the full story of French synthetic geometry, see Lorraine J. Daston, "The Physicalist Tradition in Early Nineteenth Century French Geometry," *Studies in History and Philosophy of Science* 17 (1986): 269-295.

program faded away after exciting only a brief debate among French

mathematicians.[76] Meanwhile, no geometry textbooks replaced those by Bézout and

Peyrard, Legendre, and Lacroix until Cauchy's textbooks were accepted after 1830.[77]

Perhaps any observations of the school of French synthetic geometry would

have interfered with the belief in British decline relative to French mathematics

which was already becoming entrenched by 1800. Although the excellent

reputations of French mathematicians were certainly deserved, British observers

made developments in France the model for good mathematics at the expense of

confidence in their own potential and of their admiration for eighteenth-century

figures, most notably Colin Maclaurin. As a result, British mathematicians and

professors began to call for reform of the status quo in mathematics and

mathematics education. John Playfair (1748-1819) was one of the first people to voice

the so-called "British decline" thesis, with his papers appearing in the widely-read

Edinburgh Review.[78] His discontent over the separate, geometrical style of

mathematics practiced in Britain—attributed to the acrimonious Newton-Leibniz

priority controversy over invention of the calculus and exacerbated by political and

social differences which accompanied the French Revolution—joined the views of

others who believed that Continental mathematicians had become superior to

researchers in Newton's homeland. For example, Robert Woodhouse used Leibniz's

differential notation in his courses in the first decade of the nineteenth century,

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[76] Crosland, *Science Under Control* (cit. n. 28), pp. 124-166.

[77] Dhombres, "French Mathematical Textbooks" (cit. n. 16), p. 137. On the objections in the 1810s and 1820s that Cauchy was making mathematics too difficult, see Grattan-Guinness, "Grandes écoles" (cit. n. 43), pp. 213-215.

[78] See for example, [John Playfair], review of *Traité de Méchanique Céleste*, by Pierre-Simon Laplace, *Edinburgh Review* 11 (1808): 249-284; and [John Playfair], review of *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry*, by John Leslie, *Edinburgh Review* 20 (1812): 79-100. A complete list of Playfair's papers on this subject appears in Chapter Two.

although the problems he wrote for the Mathematical Tripos at Cambridge were expressed in Newton's fluxional notation. As Cambridge students in the 1810s, Charles Babbage, John Herschel, George Peacock, and William Whewell formed the Analytical Society to propagate differential notation and to advocate for the inclusion of Lagrangian calculus in the curriculum.[79] Although Whewell later backed off on his commitment to analytical mathematics and came to support the traditional format of the Tripos, which treated pure mathematics as a tool for mixed mathematics, Babbage continued to rail against the overall intellectual stagnation he saw throughout his career, most notably with *Reflections on the Decline of Science in England*, published in 1830. Although British mathematics in the eighteenth century was not as far removed from French mathematics as commentators asserted and although there were limits to the features of French mathematics adopted by British practitioners, the concept of "British decline" soon became a truism in the history of mathematics.[80] Even in the 1990s, Ivor Grattan-Guinness wrote, "[T]he effect of

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[79] In fact, most tellings of the "British decline" focus only on the significance of the Cambridge Analytical Society. See Harvey W. Becher, "William Whewell and Cambridge Mathematics," *Historical Studies in the Physical Sciences* 11 (1980): 1–48; Philip C. Enros, "Cambridge University and the Adoption of Analytics in Early Nineteenth-Century England," in *Social History of Nineteenth Century Mathematics*, ed. Herbert Mehrtens, Henk Bos, and Ivo Schneider (Boston: Birkhäuser, 1981), pp. 135-148; Philip C. Enros, "The Analytical Society (1812-1813): Precursor of the Renewal of Cambridge Mathematics," *Historia Mathematica* 10 (1983): 24–47; Harvey W. Becher, "Radicals, Whigs and Conservatives: The Middle and Lower Classes in the Analytical Revolution at Cambridge in the Age of Aristocracy," *British Journal for the History of Science* 28 (1995): 405–426; and William J. Ashworth, "Memory, Efficiency, and Symbolic Analysis: Charles Babbage, John Herschel, and the Industrial Mind," *Isis* 87 (1996): 629-653.

[80] Two historians of mathematics who made an oversimplification of eighteenth-century British and French mathematics acceptable were Walter William Rouse Ball and Florian Cajori. See Walter William Rouse Ball, *A History of the Study of Mathematics at Cambridge* (Cambridge: Cambridge University Press, 1889); Walter William Rouse Ball, *A Short Account of the History of Mathematics*, 4th ed. (London: Macmillan and Co., Limited, 1908); and Cajori, *Teaching and History* (cit. n. 5). Compare their views with the French declinists who had a vested interest in disparaging French science after 1830 by focusing on the glorious period between 1799 and 1815 as described by Fox, "Rise and Fall" (cit. n. 73); Terry Shinn, "The French Science Faculty System: Institutional Change and Research Potential in Mathematics and the Physical Sciences," *Historical Studies in the Physical Sciences* 10 (1979): 271-332; and Robert Fox and George Weisz, ed., *The Organization of Science and Technology in France*

Newton's achievements in mathematics . . . [was] rather unfortunate for British

mathematics in general."[81]

Background Material — Analysis and Synthesis

This first mention of the British mathematical insecurity related to the staying

power of Newtonian fluxions evokes the second "given" of this study, the

pervasiveness of the terms "analysis" and "synthesis" around 1800.[82] These words

experienced an extraordinarily long historical development. Rooted in the

mathematical practice of ancient Greece, the terms then denoted a process for

discovering the solution of construction problems now called geometrical analysis.[83]

In general, geometers would assume the problem could be solved and make

deductions from that assumption until a problem known to be solvable was

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1808-1914 (Cambridge and Paris: Cambridge University Press and Editions de la Maison des Sciences de l'Homme, 1980).

[81] Ivor Grattan-Guinness, "The British Isles," in *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, ed. Ivor Grattan-Guinness, vol. 2 (London and New York: Routledge, 1994), pp. 1484-1494, on p. 1485.

[82] For a broader consideration of various uses of analysis and synthesis in the historiography of science, see Gerald Holton, "Analysis and Synthesis as Methodological Themata," *Methodology and Science* 10 (1977): 3-33. An overview different from the following account of the development of analysis and synthesis from Plato to Descartes is provided by Marco Panza, "Classical Sources for the Concepts of Analysis and Synthesis," in *Analysis and Synthesis in Mathematics: History and Philosophy*, ed. Michael Otte and Marco Panza (Dordrecht: Kluwer Academic Publishers, 1997), pp. 365-414. Both Panza's account and this summary are only very preliminary steps toward the two-thousand-year history of analysis and synthesis in mathematics, philosophy, and early modern science called for by Hardy Grant; Hardy Grant, review of *Vita Mathematica: Historical Research and Integration with Teaching*, ed. by Ronald Calinger, *American Mathematical Monthly* 104 (1997): 471-478, on p. 473.

[83] Although geometrical analysis is a subject still under contention by historians of ancient mathematics and philosophy (for a review of many of the influential sources, including those by Michael Mahoney and Jaakko Hintikka and Unto Remes, see W. Rehder, "Die Analysis und Synthesis bei Pappus," *Philosophia Naturalis* 19 (1982): 350-370), the intent of the following account is to provide as much of a "synthesis" of the different interpretations as possible, since this project is more concerned with how analysis and synthesis were understood near the turn of the nineteenth century than with ascertaining what Greek authors really meant.

reached.[84] Apparently a common technique of pre-Alexandrian mathematics, most examples of analysis were lost after Euclid's *Elements* became influential.[85] Greek compilers recorded only the explanation showing that the problem was indeed solved, or what they called the synthesis. They covered up their analyses in order to emphasize the logical dependence of the entire system rather than the importance of individual propositions.[86]

One consequence of their preferences was that written evidence of analysis and synthesis which survived was scanty. Besides the semi-apocryphal Book XIII of the *Elements*, definitions of the terms appear in Book VII of the *Collection* by Pappus:

> Now *analysis* is a method of taking that which is sought as though it were admitted and passing from it through its consequences in order to something which is admitted as a result of synthesis; for in analysis we suppose that which is sought to be already done, and we inquire what it is from which this comes about, and again what is the antecedent cause of the latter, and so on until, by retracing our steps, we light upon something already known or ranking as a first principle; and such a method we call analysis, as being a reverse solution.

> But in *synthesis*, proceeding in the opposite way, we suppose to be already done that which was last reached in the analysis, and arranging in their natural order as consequents what were formerly

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[84] Michael S. Mahoney, "Another Look at Greek Geometrical Analysis," *Archive for History of Exact Sciences* 5 (1968-1969): 318-348, on p. 329.

[85] For reconstructions of ancient Greek analysis, consult Wilbur Richard Knorr, *The Ancient Tradition of Geometrical Problems* (New York: Dover Publishers, Inc., 1986), pp. 1-99.

[86] See Knorr, *Ancient Tradition* (cit. n. 85), pp. 101-149.

antecedents and linking them with another, we finally arrive at the construction of what was sought; and this we call synthesis.[87]

These definitions have often led commentators to focus on the directions of analysis and synthesis when they try to understand the meaning of the concepts.[88] In essence, this theoretical analysis (in contrast to the systematic body of techniques encompassing the problematical analysis described at the beginning of this section) was a descriptive process for developing the logical proof for a theorem. The analysis was a series of deductive steps logically reversible by the synthesis.[89] Geometers needed to demonstrate that the theorem under consideration was valid and to find the steps which would bridge from the analytical to the synthetic parts of the proof. Proof by contradiction, or *reductio ad absurdum*, was considered a variant and perhaps precursor type of analysis.

Post-Euclidean geometers — most notably Archimedes — engaged in a sophisticated practice of analysis and synthesis as a method of proof.[90] However, although medieval treatises on analysis and synthesis are as rare as their ancient counterparts, it appears that the Islamic inheritors of Greek mathematics did not continue this aspect of the tradition. For instance, Ibrahim Ibn Sinan (909-946) classified problems according to the sufficiency of their hypotheses and conditions.

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[87] Quoted in Fauvel and Gray, *History of Mathematics Reader* (cit. n. 39), p. 209, italics in source. On Book XIII, see Euclid, *The Thirteen Books of the Elements*, trans. and intro. Thomas L. Heath, 2d ed., vol. 1 (New York: Dover Publications, 1956), pp. 137-142.

[88] For example, direction is the central issue in Jaakko Hintikka and Unto Remes, *The Method of Analysis: Its Geometrical Origins and Its General Significance* (Dordrecht: D. Reidel Publishing Company, 1974); Ali Behboud, "Greek Geometrical Analysis," *Centaurus* 37 (1994): 52-86 (even though Behboud claims to be redirecting attention to the structure of analysis and synthesis); and Petri Mäenpään, "From Backward Reduction to Configurational Analysis," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 201-226.

[89] Mahoney, "Another Look" (cit. n. 84), pp. 329-330.

[90] Mahoney, "Another Look" (cit. n. 84), p. 337.

Then, he showed how to provide a complete analysis and synthesis for examples of solvable problems, in which the mathematical object sought was either determined or found to be determinable in the analysis and the object proven to solve the problem with syllogisms in the synthesis. Ibn Sinan's writings were significant because his techniques were immediately applicable to algebraic problems.[91] Similarly, later in the tenth century, Ibn Al-Haytham accommodated the analysis/synthesis distinction to concerns of contemporary practice, such as the study of motion.[92]

Early modern algebraists were the next to pick up the threads of analysis and synthesis. As part of his justification for symbolic computation, François Viète appealed to his own interpretation of Pappus' definitions. If he needed to find magnitudes in a problem, Viète assumed the magnitudes as given and represented them by letters. This enabled him to translate the problem into an equation solvable by arithmetic. Then, he had to verify that the solutions were correct and to reinterpret the solutions as geometrical magnitudes, which comprised the synthetic stage to Viète.[93]

Viète's successors devoted their attention to two outgrowths of Viète's research: the application of algebra to geometry and the propagation of the "new

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[91] Hélène Bellosta, "Ibrahim Ibn Sinan: On Analysis and Synthesis," *Arabic Sciences and Philosophy* 1 (1991): 211-232.

[92] Roshdi Rashed, "Analysis and Synthesis According to Ibn al-Haytham," trans. Mathieu Marion, in *Artifacts, Representations and Social Practice: Essays for Marx Wartofsky*, ed. Carol C. Gould and Robert S. Cohen (Dordrecht: Kluwer Academic Publishers, 1994), pp. 121-140. On p. 122, Rashed issues a call for historians and philosophers to study analysis and synthesis by examining sources other than the writings of Pappus.

[93] On Viète and analysis and synthesis, see Marco Panza, "Classical Sources," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 401-405; and Jean G. Dhombres, "The Analysis of the Synthesis of the Analysis . . . Two Moments of a Chiasmus: Viète and Fourier," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 147-176, on pp. 149-154.

algebra." Justifiably the most famous name connected to the relationship between algebra and geometry, René Descartes framed his research in part in terms of ancient methods. One of the first to criticize Greek mathematicians for covering up their analyses, Descartes claimed to restore analysis to its proper place, together with the demonstration of procedures characteristic of synthesis. His geometry was about the thorough construction of figures through deduction but expressed in coordinates with algebra as a universal language. Algebra helped Descartes resolve a problem back to its causes, which were then composed in the synthesis to highlight the effects. In other words, Descartes appealed to analysis and synthesis almost entirely as a methodology.[94] Mathematics was all one discipline to him, and he used algebra applied to geometry because this was helpful with the task of classifying curves as geometrical objects. The curves Descartes expressed with formulas could still be represented synthetically as well.[95]

[94] Giorgio Israel, "The Analytical Method in Descartes' Géométrie," in Otte and Panza, Analysis and Synthesis (cit. n. 82), pp. 3-34; Howard Duncan, "Descartes and the Method of Analysis and Synthesis," in An Intimate Relation: Studies in the History and Philosophy of Science Presented to Robert E. Butts on His Sixtieth Birthday, ed. James Robert Brown and Jürgen Mittelstrass (Dordrecht: Kluwer Academic Publishers, 1989), pp. 65-80; Timothy Lenoir, "Descartes and the Geometrization of Thought: The Methodological Background of Descartes's Géométrie," Historia Mathematica 6 (1979): 355-379. Although these authors appear to think that the concepts of analysis and synthesis informed all of Descartes's intellectual pursuits, Daniel Garber and Leslie Cohen argued that analysis and synthesis were not influential when Descartes wrote Principles; Daniel Garber and Leslie Cohen, "A Point of Order: Analysis, Synthesis, and Descartes's Principles," Archiv für Geschichte der Philosophie 64 (1982): 136-147.

[95] This limitation is one reason why Descartes is no longer considered a direct founder of analytical geometry. See evaluations of what Descartes actually did and why (to bring order to geometry) in Andrew G. Molland, "Shifting the Foundations: Descartes's Transformation of Ancient Geometry," Historia Mathematica 3 (1976): 21-49; and Henk J. M. Bos, "On the Representation of Curves in Descartes' Géométrie," Archive for History of Exact Sciences 24 (1981): 295-338. For the claim that Descartes was vital to the creation of analytical geometry, see Eric G. Forbes, "Descartes and the Birth of Analytic Geometry," Historia Mathematica 4 (1977): 141-151. The classic histories of analytical geometry include Carl B. Boyer, History of Analytic Geometry (New York: Scripta Mathematica, 1956); and Julian Lowell Coolidge, A History of Geometrical Methods (Oxford: Clarendon Press, 1940).

Later in the seventeenth century, as algebra became more and more useful as a problem-solving tool, mathematicians debated the extent to which algebra was appropriate for analytical methods. At the center of the argument were three English mathematicians, John Wallis, Isaac Barrow, and Thomas Hobbes.[96] While Wallis defended the new algebra and contended that Cavalieri's method of indivisibles was equivalent to the geometrical method of exhaustion, Barrow was concerned that indivisibles were unreliable despite his willingness to adopt the analytical method for its brevity. Hobbes, in contrast, completely rejected algebraization and the concept of infinity. He also contended that algebra, which did not hold the status of a mathematical discipline, caused arithmetic and geometry to become conflated with each other. He gave his own definitions of analysis and synthesis, saying that synthesis revealed the connectedness between propositions and that the analytical approach replaced magnitudes with empty symbols, resulting in a step backward in certainty. In general, the three mathematicians represented seventeenth-century concerns with the traditional methods of analysis and synthesis, supplemented by an interest in rigor and measured by the method of exhaustion, as they engaged in debates over the relative importance of algebra and geometry, which were beginning to be perceived as separate disciplines.[97]

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[96] Historians have been fascinated by what motivated Hobbes's attack on algebra, so several have delved into what analysis and synthesis meant to him, including Richard A. Talaska, "Analytic and Synthetic Method According to Hobbes," *Journal of the History of Philosophy* 26 (1988): 207-237; and Douglas M. Jesseph, "Of Analytics and Indivisibles: Hobbes on the Methods of Modern Mathematics," *Revue d'Histoire des Sciences* 46 (1993): 153-193. On Barrow, see Chikara Sasaki, "The Acceptance of the Theory of Proportion in the Sixteenth and Seventeenth Centuries: Barrow's Reaction to the Analytical Mathematics," *Historia Scientiarum* 29 (1985): 83-116. For a comparison of Wallis, Hobbes, and Barrow and an argument that they were direct influences on George Berkeley's philosophy of mathematics, consult Helena M. Pycior, "Mathematics and Philosophy: Wallis, Hobbes, Barrow, and Berkeley," *Journal of the History of Ideas* 48 (1987): 265-286.

[97] For the general philosophy of seventeenth-century mathematics, see Douglas M. Jesseph, "Philosophical Theory and Mathematical Practice in the Seventeenth Century," *Studies in History and Philosophy of Science* 20 (1989): 215-244; and Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (New York and Oxford: Oxford University Press, 1996). See also the

Analysis and synthesis also influenced experimental science in the seventeenth century. For Galileo, analysis encompassed experiment, mathematical study of the experimental phenomena, and the search for a physical principle which could explain the mathematical results, while the synthesis involved the physical explanation of the phenomena.[98] Newton, in contrast, emphasized the two-part structure of analysis and synthesis, pursued in the proper order to establish general causes through observation and experiment.[99]

The varied understandings of analysis and synthesis which were emerging continued to evolve in the eighteenth century. Most notably, analysis was equated with algebra more than ever, reaching a zenith in the French Enlightenment and Revolution. The revolutionary event, of course, was the essentially simultaneous invention of the calculus by Newton and Gottfried-Wilhelm Leibniz. In extending the calculus and applying it to natural phenomena, eighteenth-century mathematicians gradually found Leibniz's differential notation more helpful than Newtonian fluxions and in fact understood the calculus as an extension of algebra.[100] For example, Euler's work showed a new awareness of the differences between algebraic and geometrical methods as well as the belief that analytical techniques were better for calculus proofs.[101] By the end of the century, Joseph-Louis Lagrange

work by Newton's master editor, Derek Thomas Whiteside, "Patterns of Mathematical Thought in the Later Seventeenth Century," *Archive for History of Exact Sciences* 1 (1960-1962): 179-388.

[98] Ronald H. Naylor, "Galileo's Method of Analysis and Synthesis," *Isis* 81 (1990): 695-707.

[99] Erik Lars Sageng, "Colin MacLaurin and the Foundations of the Method of Fluxions" (Ph.D. diss., Princeton University, 1989); Hintikka and Remes, *Method of Analysis* (cit. n. 88); Giogio Tonelli, "Analysis and Synthesis in XVIIIth Century Philosophy Prior to Kant," *Archiv für Begriffsgeschichte* 20 (1976): 178-213, on p. 180.

[100] Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), pp. 400-401.

[101] Craig Fraser, "The Background to and Early Emergence of Euler's Analysis," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 47-78, on p. 63.

was attempting to place the calculus on a wholly algebraic foundation. He had already removed geometrical diagrams from the study of mechanics in the 1788 *Mécanique analytique*, a work viewed as the epitome of eighteenth-century algebra.[102]

Enlightenment writers added their own twist to the view of analysis as algebra by adopting "analysis" as a central theme.[103] Impressed by the broad utility of algebra in its eighteenth-century applications, writers such as the Marquis de Condorcet and Abbé de Condillac tried to adapt mathematical laws to human reasoning.[104] To Condorcet and Condillac, analysis symbolized a process of discovering general natural truths about any field of knowledge from chemistry to man's moral being, which were then classified in a logical order. They recognized algebra's relatively new role as the fundamental language of mathematics and embraced algebra as the unambiguous model for all communication, effectively dismissing synthetic geometry as antiquated. To British observers including Edmund Burke, however, the French rationalists were engaged in empty speculation divorced from sensory experience, which was as detrimental to government as this approach was to natural philosophy.[105]

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[102] On Lagrange's mathematics, see Grabiner, "Changing Attitudes" (cit. n. 74); Fraser, "Algebraic Vision" (cit. n. 74); and Fraser, "Lagrange's Analytical Mathematics" (cit. n. 74).

[103] Fundamental sources on the Enlightenment and science include Robert E. Schofield, *Mechanism and Materialism: British Natural Philosophy in an Age of Reason* (Princeton: Princeton University Press, 1970); Gillispie, *Science and Polity* (cit. n. 40); and Hankins, *Science and the Enlightenment* (cit. n. 39). On quantitative thinking, natural philosophy, and the Enlightenment, see Frängsmyr, Heilbron, and Rider, *Quantifying Spirit* (cit. n. 39).

[104] For instance, see *Philosophical Writings of Etienne Bonnot, Abbé de Condillac*, trans. Franklin Philip and Harlan Lane (Hillsdale, NJ: Laurence Eribaum Associates, 1982); and Isabel F. Knight, *The Geometric Spirit: The Abbé de Condillac and the French Enlightenment* (New Haven and London: Yale University Press, 1968).

[105] Edmund Burke, *Reflections on the Revolution in France*, ed. and intro. L. G. Mitchell (London, 1790; reprint, Oxford and New York: Oxford University Press, 1993), pp. 58, 61.

Yet, discussions of analysis and synthesis as procedures for determining

scientific truth were widespread in the eighteenth century, with at least two

additional consequences. First, encyclopedia authors brought the notions to a

popular level, often quoting directly from intellectuals such as d'Alembert and

Condillac.[106] As another example, Ephraim Chambers listed logical, mathematical,

and chemical definitions for the terms by 1753.[107] Second, philosophers began to

separate analysis from synthesis. Christian Wolff, like many contemporary

mathematicians and philosophers, spoke of geometry as being demonstrated only

synthetically, while algebra was proven analytically. Perhaps in the future, he

conjectured, someone would discover a process for demonstrating geometry

analytically or algebra synthetically.[108] Similarly, Gaspard Monge, even though he

coined the term "analytic geometry," understood algebra and geometry as

autonomous disciplines. He believed one had to make translations to communicate

between the two because they were fundamentally and logically distinct.

Popular and philosophical appeals to "analysis" and "synthesis" came

together in another role in which the terms were employed by the end of the

eighteenth century, education. The common belief that the Greeks had made their

mathematical discoveries through analyses which were removed in the final telling

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[106] Tonelli, "Analysis and Synthesis" (cit. n. 99), p. 191. Tonelli's article is an encyclopedia itself of mentions of analysis and synthesis in eighteenth-century publications. For guidelines on the historiographical usefulness of encyclopedias and dictionaries, see Jonathan R. Topham, "Beyond the 'Common Context': The Production and Reading of the Bridgewater Treatises," *Isis* 89 (1998): 233-262; Arthur Hughes, "Science in English Encyclopaedias, 1704-1875," *Annals of Science* 7 (1951): 340-370; 8 (1952): 323-367; 9 (1953): 233-264; and 11 (1955): 74-92; John Leonard Clive, *Scotch Reviewers: The Edinburgh Review, 1802-1815* (Cambridge: Harvard University Press, 1957); Herman Kogan, *The Great EB: The Story of the Encyclopaedia Britannica* (Chicago: The University of Chicago Press, 1958); and Richard Yeo, "Reading Encyclopaedias: Science and the Organization of Knowledge in British Dictionaries of Arts and Sciences, 1730-1850," *Isis* 82 (1991): 24-49.

[107] Tonelli, "Analysis and Synthesis" (cit. n. 99), p. 181.

[108] Tonelli, "Analysis and Synthesis" (cit. n. 99), p. 198.

had led intellectuals to associate analysis with invention and synthesis with explanation and teaching in the seventeenth century.[109] Meanwhile, chairs of mathematics were founded at universities, and mathematics instruction began to become more systematized. Mathematicians in teaching positions often voiced their beliefs about what content should be taught and how students should learn mathematics in the language of analysis and synthesis.[110] Their opinions affected which textbooks were written and used and especially affected how geometry was treated in a university curriculum which was aimed toward training up gentlemen in the traditional liberal arts.[111]

Finally, although he was more influential on philosophers than on mathematicians, Immanuel Kant classified mathematical judgments as *a priori* and synthetic.[112] According to his definitions, mathematics could not be analytical and *a priori* because its truths could not be proved merely with general laws and definitions. Rather, a mathematics of which Euclidean geometry was the paradigm depended as well on constructions. To Kant, analytical reasoning established concepts, was necessarily true, and rested on the principle of contradiction. He thus adapted the traditional directional meanings of analysis and synthesis because he

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[109] See Enrico Pasini, "Arcanum Artis Inveniendi: Leibniz and Analysis," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 35-46, on p. 37.

[110] The classic illustration of mathematicians' belief in the educative value of mathematics for training the mind is Florian Cajori, *Mathematics in Liberal Education: A Critical Examination of the Judgments of Prominent Men of the Ages* (Boston: The Christopher Publishing House, 1928), in which Cajori compiled a scorecard of which learned men were for and against this use of mathematics.

[111] Indeed, Whiteside, "Patterns of Mathematical Thought" (cit. n. 97), pp. 270-289, argued that the subtlety of Euclidean geometry was lost as it was increasingly adopted into a major role in English universities.

[112] Kant will not appear in the story which follows. For instance, he apparently had little or no influence on John Playfair. Playfair never mentioned Kant in his writings, and he did not own Kant's major philosophical works; see *Catalogue of the Library of the Late John Playfair, Esq.* (Edinburgh: James Ballantyne and Co., 1820), p. 45.

desired to separate mathematics from philosophy. If philosophy consisted of knowledge gained through concepts, mathematics consisted merely of knowledge gained through the construction of concepts.[113]

In summary, by 1800, understandings of analysis and synthesis could vary almost from mathematician to mathematician. Still, these philosophical concepts were a part of mathematics in a number of concrete ways. Analysis and synthesis denoted the direction taken in proof and also indicated the type of proof in some cases. In other circumstances, the words "analysis" and "synthesis" could indicate whether algebra or geometry was being appealed to, especially with respect to the analytical problem-solving techniques of differential calculus. In both usages, mathematicians revealed their concern for rigor by giving reasons for preferring analysis, synthesis, or a combination of the two. Further, mathematicians associated analysis with discovery — and the Greeks' hiding of how they arrived at their techniques — and synthesis with teaching. Although historians who study analysis and synthesis have generally concentrated on the highest levels of mathematics and philosophy, these different understandings were also pervasive in the mathematical lives of those who were more educators than creators of mathematics.

In the story of American college geometry teaching in the early nineteenth century, then, these two "givens" of French mathematics and the concepts of analysis and synthesis were interrelated. The perceived analytical or algebraic style of French mathematics was at that time taken to be the archetype of good mathematics throughout Europe and North America. Roughly simultaneously with

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[113] Willem R. de Jong, "Kant's Theory of Geometrical Reasoning and the Analytic-Synthetic Distinction. On Hintikka's Interpretation of Kant's Philosophy of Mathematics," *Studies in History and Philosophy of Science* 28 (1997): 141-166; Jean-Michel Salanskis, "Analysis, Hermeneutics, Mathematics," in Otte and Panza, *Analysis and Synthesis* (cit. n. 82), pp. 227-241, on pp. 231-235. For a differing view, see Michael Friedman, "Kant's Theory of Geometry," *The Philosophical Review* 94 (1985): 455-506.

the British mathematicians who asked how and whether to pursue the French program, American mathematics professors discussed the benefits and disadvantages of the different ways mathematics was done in the two European nations. Although institutional mathematics research was not a part of the American landscape until late in the nineteenth century, even later than in Great Britain, the Americans did more readily adopt the differential and integral calculus and were not as averse to the reductionist or the rigorization programs. The authors of geometry textbooks in this study played a role in this weighing out of French and British mathematics, especially with respect to ascertaining the relative status of algebra and geometry as mathematical disciplines.

Similarly, French textbooks and educational institutions were seen as advocating a type of analytical practice in teaching students. Whereas the prevailing view in liberal education had been that the memorization of geometrical proofs trained young men to become gentlemanly reasoners, some of the French mathematicians who wrote textbooks argued that beginners ought to trace mathematics along the path in which it had been originally discovered. This would reveal the inner logic and structure of mathematics to students, and they would be prepared to continue to higher mathematics. Because this process of retracing the steps by which mathematical results were obtained was similar to that believed to be practiced by the Greeks in their two-sided proofs, it became associated with "analysis." The following chapters will show that Scottish and American professors were interested in this method of teaching, sometimes evolving their own forms of early techniques of "discovery learning."

One understanding of analysis and synthesis which did not much merge with appreciation of French mathematics for Americans was that of analysis and synthesis as parts of mathematical proof. There, they tended either to follow British

mathematicians who tried to restore the missing elements of ancient mathematics or to state their own concerns about whether analysis and synthesis were necessarily the correct processes for arriving at a correct proof. The validity of the theory of parallels also was not very important to Americans during this time. Further, geometry remained a subject generally expressed in synthetical form throughout this time period — even French textbooks such as Legendre's *Éléments* were in the synthetic style — so that the professors in this study did not raise this understanding of analysis and synthesis as often as they discussed the others.

Overall, the issues raised by the "givens" assist with providing the context for and with illustrating the geometry textbooks widely used in the United States by 1840. French authors provided textbooks when Americans began to look to the Continent for inspiration, and Americans also looked at French mathematics as they voiced their own opinions about "British decline" and any perceived effects the phenomenon had on American potential in mathematics. At the same time, Americans viewed geometry textbooks in terms of analysis and synthesis. French authors have already been seen to have appealed to these concepts, and those interpretations will resurface throughout this project with respect to the content of the textbooks, including the theory of proportion and issues specific to each author, and with respect to reception of the textbooks. The "givens" therefore will play an essential role in taking a fresh and more complete look at mathematics in the American college in the early nineteenth century. Specifically, chapters on Scottish geometry textbooks, such as Simson's *The Elements of Euclid* and Playfair's *Elements*, Day and *Mensuration*, Farrar and *Elements of Geometry*, and Davies and *Elements of Geometry and Trigonometry* will demonstrate the Scottish and French influences on American geometry teaching, the relationship between American colleges and

mathematics, the gradual changes in the aims and structure of the college curriculum, and the lives of the professors who taught in the colleges.

Chapter Two

The Whole is the Sum of Diverse Parts:

The Scottish Legacy in Geometry Education[*]

As the American scientific community began to form in the early nineteenth

century, professors at the center of activity collected foreign publications for

educating themselves and their students in recent developments. Thanks to open

importation routes from Great Britain and the lack of international copyright laws,

Scottish writers became one of the favorite American sources for information and

textbooks. For example, the *Edinburgh Review* began to be reprinted in New York

immediately after it appeared in Scotland as early as 1810.[1] John Playfair's *Elements*

of Geometry, however, was the Scottish mathematics book which really took on a life

of its own in the United States. Published at least thirteen times in Scotland between

1795 and 1875, the textbook was printed in the United States in thirty-nine of the

sixty-five years between 1806 and 1871 in two versions, one originally overseen by

Francis Nichols, a bookseller in Philadelphia, and one prepared by James Ryan in

New York and eventually known as the "Dean edition." Additionally, variant

editions of Playfair's textbook included one by an anonymous American editor

printed in Boston in 1814; combinations of *Elements* with Robert Simson's 1756 *The*

Elements of Euclid by John D. Craig in Baltimore in 1819 and by Martin Roche in

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[*] An early version of this chapter, "Analysis and Synthesis in John Playfair's *Elements of Geometry*," was presented at the Annual Meeting of the History of Science Society, Kansas City, MO, 25 October 1998.

[1] The University of Maryland (College Park) library holds issues of *Edinburgh Review* which were printed in the United States; see, for example, *Edinburgh Review* 16 (1810).

Philadelphia in 1829; and an abbreviated *Elements* with half the pages of the original text published by Nichols in 1829. A revision of Playfair's Book V was printed in Cambridge in 1812 and 1815 for the use of Harvard students.[2]

Indeed, several aspects of the Scottish tradition of liberal education with a pragmatic cast, as they were portrayed in geometry textbooks, were attractive to Americans, perhaps none more so than Playfair's perspective on analysis, synthesis, and the structure of elementary geometry in his 1795 version of Euclid's *Elements*. Scottish geometry teaching drew upon different "parts" based upon the textbooks published over a seventy-five-year period roughly between 1750 and 1825. To some extent, these parts combine into a whole, but the piece contributed by Playfair loomed the largest both in the entirety of Great Britain and in the United States. Thus, although the following surveys the Scottish mathematics professors and textbook writers which provided the background of influence on their counterparts in the United States, most attention will be paid to Playfair and *Elements of Geometry*. This book was the most widely disseminated geometry textbook in American colleges, and, viewed together with Playfair's other publications, it further provides an excellent example of the three main different understandings of analysis and synthesis which coexisted in geometry education around 1800.

Robert Simson and *The Elements of Euclid*

The standard British geometry textbook for much of the eighteenth century was Robert Simson's 1756 *The Elements of Euclid*. The author was born on October 14,

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[2] *The National Union Catalog: Pre-1956 Imprints* (hereinafter cited *NUC*), vol. 461 (London: Mansell, 1976), pp. 412–417. The other work by Playfair which was republished in the United States was his "Dissertation Second: Exhibiting a General View of the Progress of Mathematical and Physical Science," a supplement to the fourth and fifth editions of the *Encyclopaedia Britannica*, reprinted separately in Boston in 1817, 1819, 1820, and 1821.

1687, to a wealthy Glasgow merchant and his wife.[3] Simson entered the University of Glasgow, where his maternal uncle was professor of divinity, in 1701 to be trained for the Church. He studied Latin, Greek, logic, and natural philosophy — there were no formal mathematics classes at that time because the chair of mathematics, Robert Sinclair, had neglected his duties. After completing the course in the faculty of arts, Simson remained in Glasgow to study theology and the Semitic languages. He also became interested in mathematics, perhaps from reading a treatise in Latin by Sinclair's father, George, the previous holder of the Glasgow chair of mathematics. Simson then mastered Euclid's *Elements* by studying a 1572 Latin translation by Commandinus. He soon gained a reputation as a mathematician as well as a botanist, and the university senate approached him about replacing Robert Sinclair. Simson felt he needed more mathematical training, however, and he spent the academic year from 1710 to 1711 in London, where he met a number of prominent mathematicians, including Edmund Halley and James Jurin. While he was gone from Glasgow, Sinclair resigned and Simson was elected to the chair, which he assumed on November 20, 1711, after a brief mathematical examination.

As Glasgow's professor of mathematics, Simson developed a course of five hours of lectures per week which students attended for two years. Although he was already absorbed in researching ancient geometry, Simson also taught fluxions,

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[3] For biographies of Simson, see Ian N. Sneddon, "Simson, Robert," in *Dictionary of Scientific Biography* (hereinafter cited *DSB*), ed. Charles Coulston Gillispie, vol. 12 (New York: Charles Scribner's Sons, 1970), pp. 445-447; E. Irving Carlyle, "Simson, Robert," in *Dictionary of National Biography* (hereinafter cited *DNB*), ed. George Smith, vol. 52 (London: Oxford University Press, 1885-1901), pp. 287-288; "Simson, Robert," in *Encyclopaedia Britannica*, 9th ed., vol. 22 (Edinburgh, 1875-1889), pp. 87-88; "Simson, (Dr) Robert," in *A Biographical Dictionary of Eminent Scotsmen*, Robert Chambers, vol. 4 (Glasgow: Blackie & Son, 1835), pp. 239-243; and "Simson," in *Lives of Men of Letters and Science, Who Flourished in the Time of George III*, Henry Brougham, vol. 1 (London: Charles Knight and Co., 1845), pp. 467-516. See also Helena M. Pycior, *Symbols, Impossible Numbers, and Geometric Entanglements: British Algebra Through the Commentaries on Newton's Universal Arithmetick* (Cambridge: Cambridge University Press, 1997), pp. 243-250.

analytic geometry, logarithms, mechanics, and geometrical optics in the course. Meanwhile, though, he began a lifelong effort to restore Greek mathematics, fixing his attention first on porisms. These were examples of Greek analysis which had been created to handle geometrical statements which fell between problems and theorems, such as classification of the properties of a geometrical locus. Written evidence of porisms, however, had only survived in a short account by Pappus, so Simson had to reconstruct source material into what he believed to be original form.[4] Simson explained some such propositions and the porisms they contained in a 1723 paper communicated to the Royal Society of London by Jurin.[5]

When Simson assumed the chair of mathematics, his most famous student was thirteen years old and already in his third year of study at Glasgow. Like Simson's later well-known students, such as Matthew Stewart, John Robison, and William Trail, Colin Maclaurin (1698-1746) was immersed in Simson's belief that classical geometry was the most certain branch of mathematics. He went on to become in all likelihood the greatest British mathematician of the eighteenth century. His courses at the University of Edinburgh, which included the useful subjects of surveying, gauging, navigation, and practical astronomy, were well-attended, and he helped to enlarge the Medical Society of Edinburgh into the Philosophical Society of Edinburgh.[6] His best-known publication was the 1742 *Treatise on Fluxions*, which showed in part that the calculus could be placed on the

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[4] On porisms, see Wilbur Richard Knorr, *The Ancient Tradition of Geometrical Problems* (New York: Dover Publishers, Inc., 1986), pp. 339-381; and Michael S. Mahoney, "Another Look at Greek Geometrical Analysis," *Archives for History of Exact Sciences* 5 (1968-1969): 381-348, on pp. 343-348.

[5] Robert Simson, "Two General Propositions of Pappus, in Which Many of Euclid's Porisms are Included," *Philosophical Transactions* 32 (1723): 330.

[6] Duncan K. Wilson, *The History of Mathematical Teaching in Scotland: To the End of the Eighteenth Century* (London: University of London Press, Ltd., 1935), p. 57.

same logical footing as ancient mathematics, or, in the equivalent for eighteenth-century mathematicians, as Euclidean geometry. Although Maclaurin did not favor Newtonian fluxions over the differential and integral calculus, the problem with his success in demonstrating the foundations of the calculus in this fashion was that other mathematicians tended to interpret the *Treatise* as saying that all mathematics should be done in the style and with the tools of ancient geometry.[7] Eighteenth-century British mathematicians convinced themselves that following those practices would prove them to be the inheritors of Newton's genius, which was not entirely unreasonable with the work of mathematicians such as Lagrange and Cauchy still in the future.

Simson, in the meantime, continued to work with Greek mathematics. He both published a purely geometrical treatment of conic sections and exchanged letters with Maclaurin on the subject.[8] The work exemplified Simson's determination to keep algebra and geometry completely separated.[9] Next, his restoration of *De locis planis* by Apollonius was published in 1749, while Simson's further research into porisms and restoration of another treatise by Apollonius appeared posthumously,

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[7] For re-evaluations of Maclaurin's role in British mathematics, see Niccoló Guicciardini, *The Development of Newtonian Calculus in Britain 1700-1800* (Cambridge: Cambridge University Press, 1989), pp. 25-51; and Erik Lars Sageng, "Colin MacLaurin and the Foundations of the Method of Fluxions" (Ph.D. diss., Princeton University, 1989). An older biography of Maclaurin is Herbert Westren Turnbull, *Bi-Centenary of the Death of Colin Maclaurin (1698-1746)* (Aberdeen: The University Press, 1951).

[8] See Ian Tweddle, "Some Results on Conic Sections in the Correspondence Between Colin MacLaurin and Robert Simson," *Archive for History of Exact Sciences* 41 (1991): 1-14. The title of Simson's book was *Sectionum Conicarum Libri V* (Edinburgh, 1735). On the long-term relationship between Simson and Maclaurin, see also P. J. Wallis, "The Maclaurin 'Circle': The Evidence of Subscription Lists," *Bibliotheck* 11 (1982): 38-54.

[9] Raymond Clare Archibald, "Robert Simson and Cantor's *Vorlesungen,*" *Scripta Mathematica* 2 (1933-1934): 73-75.

together with treatments of logarithms and limits in the ancient style.[10] The edition

of *De locis planis* did not sell well.[11] Perhaps Simson began to realize that Latin had

largely lost its place as the only language of mathematics; perhaps he became

concerned that available versions of the most important book of ancient

mathematics were dated as well as not carefully prepared.[12] In any event, he

delivered versions of the Euclidean Books I through VI, XI, and XII in both Latin and

English to the Foulis Press in 1756.[13]

For his *The Elements of Euclid*, Simson stated his purpose as the removal of all

errors introduced by previous editors in order to "restore the principal Books of the

Elements to their original accuracy."[14] He started from Commandinus's translation

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[10] *Apollonii Pergaei Locorum Planorum Libri II, restituti a R. Simson* (Glasgow, 1749); *Opera Quaedam Reliqua R. Simson*, ed. James Clow (Glasgow, 1776). Simson discussed the preparation and publication of *Locorum Planorum* in his correspondence as well. See "Mathematical Correspondence: Robert Simson, Matthew Stewart, James Stirling," *Proceedings of the Edinburgh Mathematical Society* 21 (1902-1903): 2-39.

[11] John Burnett, "Robert Simson's Euclid and the Foulis Press," *Bibliotheck* 11 (1983): 136-148, on p. 137; T. S. Davies, "Geometry and Geometers," *Philosophical Magazine*, 3d ser., 33 (1849): 201-206, 531-542, on p. 201; and Robert Simson to John Nourse, 29 June 1767, in Davies, "Geometry andGeometers," p. 204.

[12] According to Thomas L. Heath, trans. and intro., *The Thirteen Books of the Elements*, 2d ed., vol. 1 (New York: Dover Publications, Inc., 1956), pp. 106, 110-111, the most recent Latin edition of Euclid's *Elements* was one prepared in 1660 by a French scholar, Claude François Milliet Dechales, while versions in English since Isaac Barrow, *Euclide's Elements: The Whole Fifteen Books Compendiously Demonstrated* (London, 1660), included a 1685 translation of Dechales's edition by Reeve Williams, Edmund Scharburgh's 1705 *The English Euclide*, John Keill's 1708 translation from Commandinus, and William Whiston's 1714 translation into English from Latin of André Tacquet's 1654 *Elementa geometriae*. On Heath's treatment of Simson, see George A. Gibson, "Some Criticisms of Robert Simson by Sir T. L. Heath," *Proceedings of the Edinburgh Mathematical Society* 44 (1925-1926): 39-46.

[13] Burnett, "Simson's Euclid" (cit. n. 11), p. 138. The Latin version of *The Elements of Euclid* also languished at booksellers. See Robert Simson to John Nourse, 29 June 1767, in Davies, "Geometry and Geometers" (cit. n. 11), p. 204.

[14] Robert Simson, *The Elements of Euclid, Viz. the First Six Books, Together With the Eleventh and Twelfth* (Glasgow: Robert and Andrew Foulis, 1756), p. v. Some discussion of whether Simson's criticisms toward previous editors were valid may be found in Wilbur Richard Knorr, "The Wrong Text of Euclid: On Heiberg's Text and Its Alternatives," *Centaurus* 38 (1996): 208-276.

to reconstruct the proofs according to the Greek analytic method Simson had learned by reading Pappus.[15] The first print runs were small (803 copies in English, 543 in Latin), befitting the narrow audience of mathematicians, classicists, and university students for a book Simson himself perceived as mathematical scholarship.[16] Scottish universities needed to train young men to reason properly and to reach sound conclusions. Mathematics professors believed that Euclid's insistence on proceeding in proper order and refusal to admit hypothetical constructions — as preserved by Simson — fit this agenda perfectly.

The content of *The Elements of Euclid* followed what had become standard in the eighteenth century, for the eight Euclidean books chosen by Simson were the same ones which had been included in textbooks such as Dechales's and Tacquet's. These covered the fundamental principles of geometry, relationships between rectangles erected on lines, the geometry of circles, inscribed and circumscribed planar figures, proportionals, proportions within geometrical figures, solid geometry, and the relationships between different solids. And, while Barrow had been so keen to include every possible proposition of Euclid that he included two nonauthentic books and he employed symbols to make the *Elements* easier to understand, Simson was selective and committed to propagating the thorough detail of ancient methods.[17] Finally, Simson stressed that the importance of his labor went beyond restoring the believed accuracy of the original text. Geometry was so foundational as a mathematical discipline that any educated person needed to understand the principles as Euclid had: "[T]hese Elements are the foundation of a

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[15] Heath, *Elements* (cit. n. 12), vol. 1, p. 104; Burnett, "Simson's Euclid" (cit. n. 11), p. 142.

[16] Burnett, "Simson's Euclid" (cit. n. 11), pp. 138, 142.

[17] Barrow, *Euclide's Elements* (cit. n. 12).

science by which the investigation and discovery of useful truths, at least in mathematical learning, is promoted as far as the limited powers of the mind allow; and which likewise is of the greatest use in the arts both of peace and war, to many of which geometry is absolutely necessary."[18]

Indeed, Simson's London bookseller, John Nourse, saw the potential for employing the work as a school textbook, lowering the price and issuing it in octavo in 1762 with Simson's permission.[19] By 1780, *The Elements of Euclid* had been translated into Portuguese, Spanish, French, and German, and the book was well on its way to its twenty-sixth British edition.[20] Nourse and John Balfour, an Edinburgh printer, had realized that English schoolteachers had rediscovered the value of Euclidean geometry as a tool of logic in the eighteenth century. Previously emphasizing only the practical benefits of geometry and teaching the propositions from Books I through III without proof, they began to use the subject as well to establish a foundation for higher mathematics in those instances when students progressed beyond the most elementary material.[21] Gradually, teachers in the new academies furthermore developed an appreciation for the potential of following the logical structure of the *Elements* with all pupils so that they gained the useful lifelong skill of reasoning. By the late eighteenth century, Euclidean geometry was

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[18] Simson, *The Elements of Euclid* (cit. n. 14), pp. v-vi.

[19] Major changes to the text included Simson's reclassification of two theorems in Book I as problems, addition of the axiom, "Two straight lines cannot enclose a space," addition of one theorem to Book VI, and reorganization of Book XII from 13 theorems to 16 theorems and two problems. See, for example, Robert Simson, *The Elements of Euclid. Viz. the First Six Books, Together With the Eleventh and Twelfth* (Philadelphia: DeSilver and DeSilver, 1825), an American reprint of the 1762 second edition. Simson's translation of Euclid's *Data*, another work of geometrical analysis, was also appended to the second edition of *The Elements of Euclid*.

[20] Burnett, "Simson's Euclid" (cit. n. 11), pp. 144-145.

[21] Wilson, *Mathematical Teaching in Scotland* (cit. n. 6), p. 53.

ensconced in the cultural place it would hold throughout most of the nineteenth

century in England, its essential role for training up gentlemen at all levels of

education and not just at the universities. Simson's *The Elements of Euclid* appeared

to be the best medium through which to expose youths to the subject.[22]

Despite a sociable personality remarked upon by all his biographers, Simson

spent his career before his retirement in 1761 and death in 1768 focused on his own

writing — mostly unpublished — and on personal contact with students. Never a

member of any eighteenth-century Scottish scientific societies, he became separated

as well from the intellectual movement of the Scottish Enlightenment.[23] The new

literati, whose ideas were spread most widely from Edinburgh, are remembered

most for elucidating a philosophical realism which distinguished sensation and

perception, founding utilitarianism, and creating moral philosophy as the sociology

of man's behavior.[24] In the second half of the eighteenth century, the loose group of

Common Sense philosophers placed primary value on science and university

[22] See Joan Richards's account of the end of this era in "Euclid and the English Schoolchild," in *Mathematical Visions: The Pursuit of Geometry in Victorian England*, Joan L. Richards (Boston: Academic Press, Inc., 1988), pp. 161-198.

[23] For a general overview of the Scottish Enlightenment, see Hugh Trevor-Roper, "The Scottish Enlightenment," *Studies on Voltaire and the Eighteenth Century* 58 (1967): 1635-1658; Nicholas Phillipson, "The Scottish Enlightenment," in *The Enlightenment in National Context*, ed. Roy Porter and Mikulás Teich (Cambridge: Cambridge University Press, 1981), pp. 19-40; Anand C. Chitnis, *The Scottish Enlightenment & Early Victorian Society* (London: Croom Helm, 1986); David Daiches, Peter Jones, and Jean Jones, ed., *A Hotbed of Genius: The Scottish Enlightenment 1730-1790* (Edinburgh: University Press, 1986); and Stuart Brown, ed., *British Philosophy and the Age of Enlightenment* (London and New York: Routledge, 1996). On scientific societies in Scotland, see John R. R. Christie, "The Origins and Development of the Socttish Scientific Community, 1680-1760," *History of Science* 12 (1974): 122-141; Steven Shapin, "Property, Patronage, and the Politics of Science: The Founding of the Royal Society of Edinburgh," *British Journal for the History of Science* 7 (1974): 1-41; Roger C. Emerson, "The Philosophical Society of Edinburgh 1768-1783," *British Journal for the History of Science* 18 (1985): 255-303; and Roger C. Emerson, "The Scottish Enlightenment and the End of the Philosophical Society of Edinburgh," *British Journal for the History of Science* 21 (1988): 33-66.

[24] For an account of one major figure, see Norman Daniels, *Thomas Reid's Inquiry: The Geometry of Visibles and the Case for Realism* (New York: Bert Franklin & Co., 1974).

education.[25] While they advocated the study of new subjects within liberal education and teaching students practical skills, they believed that a vital feature of mathematics training was geometry — for many of the same reasons voiced by Simson. Geometrical proofs were rigorous, and taking students through each step revealed the reasoning process to them. Newer considerations helped the bias toward teaching geometry become even more firmly ingrained in Scotland: mathematics was considered a necessary prerequisite for eighteenth-century mathematized natural philosophy, and mathematical logic, which was epitomized by the reasoning in geometry, was viewed as a model of true knowledge.[26]

John Playfair and *Elements of Geometry*

No Scottish geometry textbook rivaled Simson's *The Elements of Euclid* until John Playfair published *Elements of Geometry* in 1795. Playfair was born on March 10, 1748.[27] His father was a minister who educated John at home until he entered the University of St. Andrews at age fourteen. There, he showed enough ability to be asked to fill in for his ill natural philosophy professor. He stood for his first chair of mathematics at age eighteen, but he first took over the parishes served by his father

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[25] On the development of the importance of science and universities, see Roger C. Emerson, "Science and the Origins and Concerns of the Scottish Enlightenment," *History of Science* 26 (1988): 333-366; and Ronald G. Cant, "Origins of the Enlightenment in Scotland: The Universities," in *The Origins and Nature of the Scottish Enlightenment*, ed. R. H. Campbell and A. Skinner (Edinburgh, 1982), pp. 42-64.

[26] On the interaction of epistemology and geometry in Common Sense philosophy, see Richard Olson, "Scottish Philosophy and Mathematics 1750-1830," *Journal of the History of Ideas* 32 (1971): 29-44; and Richard Olson, *Scottish Philosophy and British Physics, 1750-1880: A Study in the Foundations of the Victorian Scientific Style* (Princeton: Princeton University Press, 1975).

[27] Biographical information is taken from: John Challinor, "Playfair, John," in *DSB* (cit. n. 3), vol. 11, pp. 34-36; "Playfair, John," in Chambers, *Eminent Scotsmen* (cit. n. 3), vol. 4, pp. 106-112; "Playfair, John," in *Encyclopaedia Britannica* (cit. n. 3), vol. 19, p. 217; [James G. Playfair, ed., and Francis Jeffrey], "Biographical Memoir," in *Works of John Playfair*, vol. 1 (Edinburgh: Archibald Constable & Co., 1822), pp. ix-cv; and B. B. Woodward, "Playfair, John," in *DNB* (cit. n. 3), vol. 45, pp. 413-414.

in 1773 and then tutored the Ferguson family of Raith before he successfully entered

the employ of the University of Edinburgh in 1785 as Joint Professor of Mathematics

with Adam Ferguson (no relation to Playfair's charges), who was no longer healthy

enough to teach the classes. Playfair transferred over to the chair of natural

philosophy in 1805 after John Robison's death. At that time, he helped overcome

Moderate opposition to John Leslie's subsequent appointment to the professorship

of mathematics. Playfair's biographers all described him as a teacher who directed

students to the simplest methods of inquiry and who as well taught them to relish

the truth.

Playfair published his first article in 1778 in the *Philosophical Transactions of the*

Royal Society of London, and he was an active participant in the Edinburgh

intellectual community of the late Scottish Enlightenment, later becoming one of the

first and more prolific contributors on a wide range of subjects to the *Edinburgh*

Review.[28] Indeed, Playfair was an expository writer by and large, which earned him

accolades during his lifetime for a clear, mellow, and rich style and which is also one

characteristic making his writings useful for study. They reveal the pedagogical

impact of the ideas then current, and they show Playfair's attempts to shape the

thinking of non-specialists. Additionally, Playfair was one of the original members

of the Royal Society of Edinburgh, served as its general secretary for many years,

and published several articles in the Society's *Transactions*. His topics in this

publication varied from studies of geometry or of meteorological instruments to

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[28] Many of these articles dealt with geological publications, but roughly one-third of Playfair's more than sixty reviews examined works in mathematics and the exact sciences. The intellectual and political setting for Playfair and his *Edinburgh Review* colleagues is established by John Leonard Clive, *Scotch Reviewers: The Edinburgh Review, 1802-1815* (Cambridge: Harvard University Press, 1957); G. N. Cantor, "Henry Brougham and the Scottish Methodological Tradition," *Studies in History and Philosophy of Science* 2 (1971): 69-89; and Jack B. Morrell, "The University of Edinburgh in the Late Eighteenth Century: Its Scientific Eminence and Academic Structure," *Isis* 62 (1971): 158-171.

biographies and reports on Continental studies of Indian mathematics. Playfair was elected to the Royal Society of London in 1807.

Playfair did not author many books. He wrote two textbooks, the 1795 *Elements of Geometry* and the 1814 *Outlines of Natural Philosophy*. In 1797, his friend, James Hutton, died, and Playfair began work on the biographical memoir which grew as well into the popularization, *Illustrations of the Huttonian Theory of the Earth*, which was published in 1802. This work included a clear and direct exposition of Hutton's conclusions that heat had formed the sedimentary rocks and pushed up the floors of the oceans, that the world had no beginning or end that could be computed within a human time frame, and that nature therefore was designed to continue indefinitely.[29] One of Playfair's final publications was his 1816 contribution to the supplement to the fourth edition of the *Encyclopaedia Britannica*, "Dissertation on the Progress of Mathematical and Physical Science Since the Revival of Letters in Europe."[30] Playfair passed away after a period of declining health in 1819 without finishing a second edition of *Illustrations*, the work upon which most of his scientific reputation now rests.

Why would Playfair want to revise the labor of Simson, a highly revered teacher? One physical characteristic which made Simson's book a good publishing opportunity for Playfair was that the quality of the woodcut diagrams had deteriorated in editions of *The Elements of Euclid* printed by Balfour after Simson's death until the illustrations were nearly unreadable in 1793, the last pre-*Elements of*

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[29] See V. A. Eyles, "Hutton, James," in *DSB* (cit. n. 3), vol. 6, pp. 577-589.

[30] Richard Yeo explains the purpose of dissertations in the *Encyclopaedia Britannica*, as historical frameworks for the map of the sciences. See Richard Yeo, "Reading Encyclopaedias: Science and the Organization of Knowledge in British Dictionaries of Arts and Sciences, 1730-1850," *Isis* 82 (1991): 24-49.

Geometry printing.[31] On an scholarly level, Playfair described Simson as the last and most successful of the modern mathematicians who attempted to remove the blemishes in Euclid's *Elements* which had been introduced by ancient and medieval editors.[32] Yet, Playfair continued: "[A]fter all this was accomplished, something still remained to be done . . . . [S]ome alterations might be made upon [the *Elements*], that would accommodate them better to a state of the mathematical sciences, much more improved and extended than at any former period. This accordingly is the object of the edition now offered to the public. . . ."[33] For, he believed that the influence of Simson and Maclaurin upon British mathematicians to utilize so-called "synthetic" methods had resulted in those mathematicians not learning of or even ignoring the discoveries made by Continental mathematicians in the eighteenth century. Playfair also thought that some techniques of ancient geometry were unnecessarily unwieldy for modern students. Thus, he saw a need for a geometry text which was suitable for his own day.[34]

In addition, although Playfair was aged forty-seven and had been a professor at Edinburgh for eight years[35] when he wrote the *Elements*, it is plausible that his reputation needed to be bolstered by a traditional, widely-accepted publication. Since his first article in 1778, Playfair had published only four additional papers, all

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[31] Burnett, "Simson's Euclid" (cit. n. 11), p. 139.

[32] John Playfair, *Elements of Geometry; Containing the First Six Books of Euclid, with Two Books on the Geometry of Solids. To Which are Added, Elements of Plane and Spherical Trigonometry* (Edinburgh: Bell & Bradfute, 1795), pp. iii-iv.

[33] Playfair, *Elements* (cit. n. 32), p. v.

[34] Playfair, *Elements* (cit. n. 32), pp. iii-v.

[35] Although Playfair was elected Joint Professor in 1785, he did not actually stop tutoring the Ferguson brothers and assume his new duties until 1787.

in the *Transactions of the Royal Society of Edinburgh.*[36] John Robison had not yet

become ill enough to retire and open the way for Playfair's election in 1798 as

general secretary of the Society and editor of the *Transactions.*[37] Furthermore,

although Playfair was already troubled by the dearth of British mathematical

achievements in the eighteenth century in comparison to the Continent, it was not

yet politically expedient for Playfair to openly advocate French mathematics over

the British canon in writing. Although historians such as Jack B. Morrell have

explored Playfair's support for the French Revolution — Playfair himself wrote in

Robison's biography that the Revolution was necessary to bring about an extension

of democracy — and have called Playfair's mathematics chair "politically neutral,"[38] it

still would have been risky for a Whig like Playfair to advocate any aspect of French

culture during this period at the height of anti-republican zeal in Britain. Thus,

Playfair wrote in the preface to the *Elements* only generally about the current

advanced state of the mathematical sciences.[39] He avoided giving credit to any

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[36] John Playfair, "Biographical Account of the Late Matthew Stewart, D. D.," *Transactions of the Royal Society of Edinburgh* 1 (1788); John Playfair, "On the Causes Which Affect the Accuracy of Barometrical Measurements," *Transactions of the Royal Society of Edinburgh* 1 (1788); John Playfair, "Remarks on the Astronomy of the Brahmins," *Transactions of the Royal Society of Edinburgh* 2 (1790); and John Playfair, "On the Origin and Investigation of Porisms," *Transactions of the Royal Society of Edinburgh* 3 (1794). All four papers are reprinted in [Playfair], *Works* (cit. n. 27), vol. 3, pp. 33-251, and vol. 4, pp. 3-30.

[37] Neil Campbell and R. Martin Smellie, *The Royal Society of Edinburgh (1783-1983): The First Two Hundred Years,* (Edinburgh: Royal Society of Edinburgh, 1983), pp. 40-56; John Playfair, "Biographical Account of the Late John Robison, LL.D.," in [Playfair], *Works* (cit. n. 27), vol. 4, pp. 121-178, on p. 148. Meanwhile, Playfair was living — and supporting his mother and siblings — only on University of Edinburgh student fees. Adam Ferguson drew the salary of the chair of mathematics until his death in 1816. The possibility of a regular salary is therefore another reason why Playfair would have seen the chair of natural philosophy as a promotion when it became available upon Robison's death. For Ferguson's biography, see Francis Espinasse, "Ferguson, Adam," *DNB* (cit. n. 3), vol. 6, pp. 1200-1204.

[38] Jack B. Morrell, "Professors Robison and Playfair, and the *Theophobia Gallica*: Natural Philosophy, Religion and Politics in Edinburgh, 1789-1815," *Notes and Records of the Royal Society of London* 26 (1971): 43-63, on pp. 44-45, 59; Playfair, "Robison" (cit. n. 37), pp. 167-168.

[39] Playfair, *Elements* (cit. n. 32), p. v.

specific mathematicians or nations for this improvement, waiting to denigrate

British mathematical practice and publicly voice enthusiasm for French natural

philosophers until his own position in Edinburgh was even stronger.[40]

Finally, *Elements* may be viewed as an book-length example of Playfair's

natural inclination to clarify and popularize mathematics and natural philosophy.

Playfair utilized the same writing techniques which would earn accolades for the

later *Illustrations*, "a model of purity of diction, simplicity of style, and clearness of

explanation."[41] In both cases, he was trying to make the original works more

readable and useful.[42] Yet, there were more differences than similarities between

Playfair's two major popularizations. Both Simson and his textbook were famous

and highly regarded. In contrast, even though he devoted all of his time to natural

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[40] As noted in Chapter One, thanks to a series of articles in the *Edinburgh Review* beginning in 1804, historians generally name Playfair as one of the first mathematicians to articulate the concept of a "British decline" in mathematics, implying that his countrymen needed to master the contemporary research of natural philosophers like Laplace in order to catch up to the amazing eighteenth-century successes in applied mathematics. The articles cited most often are [John Playfair], review of *Euclidis Elementorum Libri Priores XII* and *Euclidis Datorum Liber*; by Samuel Horsley, *Edinburgh Review* 4 (1804): 257-272; and [John Playfair], review of *Traité de Méchanique Céleste*; by Pierre-Simon Laplace, *Edin. Rev.* 11 (1808): 249-284. The reader may also wish to consult [John Playfair], review of *Analytical Institutions*; by Donna Maria Gaetana Agnesi, trans. by John Colston, *Edin. Rev.* 3 (1804): 401-410; [John Playfair], review of *Discours sur les Progrès des Sciences, Lettres et Arts, depuis 1789 jusquà ce jour (1808)*, *Edin. Rev.* 15 (1809): 1-22; [John Playfair], review of *The System of the World*; by Pierre-Simon Laplace, trans. by J. Pond, *Edin. Rev.* 15 (1810): 396-417; and [John Playfair], review of *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry*; by John Leslie, *Edin. Rev.* 20 (1812): 79-100. Identifications of *Edinburgh Review* authors are based on Walter E. Houghton, ed., *The Wellesley Index to Victorian Periodicals: 1824-1900*, vol. 1 (Toronto: University of Toronto Press, 1966-1987), pp. 416-546. This early notion of British decline was criticized by Thomas Young in his review of *Théorie de l'Action Capillaire*; by Pierre-Simon Laplace, *Quarterly Review* 1 (1809): 107-112, on p. 108. Identifications of *Quarterly Review* authors are taken from Hill Shine and Helen Chadwick Shine, *The Quarterly Review Under Gifford: Identification of Contributors, 1809-1824* (Chapel Hill: The University of North Carolina Press, 1949).

[41] Woodward, "Playfair" in *DNB* (cit. n. 27), p. 414.

[42] On Playfair's reasons for systematizing Hutton's theory of the earth, see [Playfair], *Works* (cit. n. 27), pp. xix-xx; Morrell, "Robison and Playfair" (cit. n. 38), p. 52; George W. White, intro., *Illustrations of the Huttonian Theory of the Earth*, John Playfair (Edinburgh, 1802; reprint, New York: Dover Publishers, Inc., 1964); and "Playfair" in Chambers, *Eminent Scotsmen* (cit. n. 27), pp. 109-110.

philosophy after 1768, Hutton was not widely recognized outside the Edinburgh scientific elite, and several of those who did know him believed his theories were vague and his religion was suspect. Thus, Playfair's first tasks in *Illustrations* were to publicize and to defend his friend. Further, Euclidean geometry was already a complete system, while Playfair did much to develop geology into a new science with *Illustrations*. This meant that he needed to provide considerably more additional explanation — although Playfair rewrote the notes for *Elements*, that effort does not compare to the nearly four hundred pages he produced to add to Hutton's original and rather obtuse 140 pages. Playfair even laid part of the blame for the lack of an immediate impact of his friend's theory of geology on Hutton's manner of writing:

> Several causes probably contributed to produce this indifference. The world was tired out with unsuccessful attempts to form geological theories. . . . [Hutton's theory] was proposed too briefly, and with too little detail of facts, for a system which involved so much that was new, and opposite to the opinions generally received. The descriptions which it contains of the phenomena of geology, suppose in the reader too great a knowledge of the things described. The reasoning is sometimes embarrassed by the care taken to render it strictly logical; and the transitions, from the author's peculiar notions of arrangement, are often unexpected and abrupt.[43]

Turning to the content of *Elements of Geometry*, Playfair followed Simson's structure rather closely, repeating ninety percent of Simson's 241 proofs. He used the same books Simson had chosen from Euclid (I through VI, XI, and XII), although

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[43] John Playfair, "Biographical Account of the Late James Hutton, M.D.," in [Playfair], *Works* (cit. n. 27), vol. 4 , pp. 33-118, on pp. 64-65.

Playfair renumbered Books XI and XII as Books VII and VIII.[44] Modifications made

by Playfair included removing one axiom and recasting another, combining some of

Simson's definitions in his Books I and VII, adding fewer than five propositions to

each of Books II and VI, eliminating nearly ten propositions from each of Books V

and VII, and reorganizing Book VIII.[45] Playfair made changes to his textbook twice,

in 1804 and 1813. In these editions, he prepared almost no revisions to Books I

through VI — three theorems added in 1804 were removed in 1813, while a few

theorems were reclassified as problems. For the second edition of 1804, however,

Playfair replaced Books VII and VIII with a three-book supplement, which was then

left unchanged in the next version. In the supplement, he retained Euclid's method

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[44] Even so, Playfair's *Elements* is not often connected with Simson's *Elements of Euclid* in the literature. For example, R. C. Archibald did not list Playfair among the authors of English editions based on Simson's work. See Raymond Clare Archibald, "Simson and Cantor" (cit. n. 9), p. 74.

[45] Playfair removed the axiom, "Two straight lines cannot enclose a space," which Simson had added to the second edition of *The Elements of Euclid* and which Playfair argued was a corollary to his definition of a straight line; John Playfair, *Elements of Geometry, Containing the First Six Books of Euclid, With a Supplement on the Quadrature of theCircle and the Geometry of Solids* (Philadelphia: F. Nichols, 1806), p. 283. The change by Playfair most familiar to today's readers, however, was his substitution of "Playfair's Axiom," "Two straight lines, which intersect one another, cannot be both parallel to the same straight line" (p. 7), for the twelfth axiom, "If a straight line meet two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced, shall at length meet upon that side on which are the angles which are less than two right angles;" Simson, *The Elements of Euclid* (cit. n. 19), p. 12.

Playfair refused to accept this statement as an axiom because its converse was not self-evident but rather required demonstration (and in fact was Proposition I.17); Playfair, *Elements*, pp. 289-293; George A. Gibson, "Sketch of the History of Mathematics in Scotland to the End of the 18th Century," *Proceedings of the Edinburgh Mathematical Society*, 2d ser., 1 (1927-1929): 1-18, 71-93, on p. 88. Even Simson attempted to prove the twelfth axiom; Simson, *The Elements of Euclid* (cit. n. 19), pp. 300-305. Playfair chose his axiom as more obvious; although he introduced it as a "new" axiom, it was already centuries old when it became associated with his name. In addition, Playfair attempted to prove that the angles of a triangle are equal to two right angles in the 1813 third edition of *Elements of Geometry*, but he claimed to show the result held for both plane and spherical triangles; Florian Cajori, *A History of Elementary Mathematics with Hints on Methods of Teaching*, rev. and enl. ed. (New York: The Macmillan Company, 1925), p. 271.

Playfair's interest in the theory of parallels will not be pursued further in this project; again, the validity of the theory of parallels was not a significant concern in American colleges, where young and inexperienced tutors presented the material in Simson's and Playfair's geometry textbooks.

for investigating the intersection of planes, substituting in two propositions Playfair

wrote himself and one he took from Adrien-Marie Legendre's 1794 *Éléments de*

Géométrie.[46] Playfair also added material on the quadrature of the circle and rewrote

the geometry of solids. Interestingly, although *Elements* was printed a total of five

times with one thousand copies in each printing during his lifetime, Playfair only

taught from his textbook during its first edition.[47] By the time the second edition

appeared, Playfair was at the end of his career of teaching mathematics. Leslie, his

successor, taught his own material to Edinburgh students and published his own

Elements of Geometry in 1809.

Analysis and Synthesis as Mathematical Styles

The word "analysis" separated Great Britain and the Continent both

mathematically and culturally throughout the eighteenth century.[48] Historians'

usual story is that the separation between British and Continental mathematics was

initially caused by the Newton-Leibniz priority controversy, exacerbated (perhaps)

by Maclaurin's attempt to give the calculus the rigor of ancient Greek geometry in

the 1742 *Treatise of Fluxions*, and further widened by British distrust of French

Enlightenment and Revolutionary ideas, which leaned heavily on language

employing the word, "analysis."[49] Some writers of the French Enlightenment

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[46] The first printing of Playfair's *Elements* in the United States was of this 1804 second edition; Playfair, *Elements* (cit. n. 45), p. vii.

[47] [Playfair], *Works* (cit. n. 27), vol. 1, p. xviii.

[48] The standard source on the eighteenth century and the development of a "British geometrical method" is Guicciardini, *Newtonian Calculus* (cit. n. 7). Also helpful is Judith V. Grabiner, "Was Newton's Calculus a Dead End? The Continental Influence of Maclaurin's Treatise of Fluxions," *American Mathematical Monthly* 104 (1997): 393–410.

[49] Two sources on quantitative thinking, natural philosophy, and the Enlightenment are Thomas L. Hankins, *Science and the Enlightenment* (Cambridge: Cambridge University Press, 1985); and Tore Frängsmyr, John L. Heilbron, and Robin E. Rider, ed., *The Quantifying Spirit in the 18th Century* (Berkeley: University of California Press, 1990).

parlayed their fascination with the problem-solving power of algebra into attempts

to use mathematica to model all human thinking and interaction.[50] While any

number of intellectuals considered analysis-as-philosophy to be inherently limited,

response was especially vehement in England. For instance, Edmund Burke

dismissed rationalism as empty speculation unsuited for government or natural

philosophy.[51] In general, English thinkers argued that algebra was unproven, while

synthetic geometry had stood the tests of centuries as a model for proper reasoning

in addition to as a form for mathematical argument.

Although the philosophes' interpretations ultimately departed from

mathematics, they had initially responded to a variation of the understanding of

analysis and synthesis which associated analysis with algebra and synthesis with

geometry. The coexistence between algebra and geometry common in Descartes's

day and during the development of analytical geometry had given way to a rather

firm philosophical barrier between the disciplines among European mathematicians.

Early modern mathematicians had freed algebra from dependence on geometry as

its foundational justification, but the gulf between the disciplines was later

exacerbated by the national conflicts which arose over the different notations and

fundamental units between fluxions and Continental differential and integral

calculus, in addition to concerns over imaginary and negative numbers which were

generally unique to British mathematicians.[52] As mathematicians throughout Europe

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[50] For instance, see *Philosophical Writings of Etienne Bonnot, Abbé de Condillac*, trans. Franklin Philip and Harlan Lane (Hillsdale, NJ: Laurence Erlbaum Associates, 1982); and Isabel F. Knight, *The Geometric Spirit: The Abbé de Condillac and the French Enlightenment* (New Haven and London: Yale University Press, 1968).

[51] Edmund Burke, *Reflections on the Revolution in France*, ed. and intro. L. G. Mitchell (London, 1790; reprint, Oxford and New York: Oxford University Press, 1993), pp. 58, 61.

[52] On the early modern and eighteenth-century roots of British symbolic algebra, see Pycior, *Symbols, Impossible Numbers* (cit. n. 3).

were well aware that the dramatic discoveries of the eighteenth century had largely

come in algebra and the algebraic style of calculus and on the Continent, perhaps it

is not surprising that geometry — and the seemingly British tendency to be attached

to it — had gained a negative connotation in many quarters.[53] Still, while

commentators often perceived mathematics to be a choice between Continental

algebraic techniques or the British geometrical style, there was not a strict national

dichotomy between allegiance to this form of analysis and synthesis — algebra per se

was not anathema in Great Britain, nor was geometry wholly taboo in French

mathematics.[54] Instead, around the turn of the nineteenth century, mathematicians

asked under what conditions algebraic or geometrical methods were appropriate,

although admittedly to varying degrees at different times and in different places.

Indeed, it may already be evident that British mathematicians did not

respond in one voice with plans to correct the so-called "British decline." While

inside and outside perceptions of English mathematicians tended to be negative,

Edinburgh was appreciated as a center for exact science by French scientists in the

[53] Historians' attention with respect to concerns about British separation and "decline" has usually focused on the story of the Cambridge Analytical Society and the adoption of differential notation in England. See, for example, Philip C. Enros, "Cambridge University and the Adoption of Analytics in Early Nineteenth-Century England," in *Social History of Nineteenth Century Mathematics*, eds. Herbert Mehrtens, Henk Bos, and Ivo Schneider (Boston: Birkhäuser, 1981), pp. 135-148; Philip C. Enros, "The Analytical Society (1812-1813): Precursor of the Renewal of Cambridge Mathematics," *Historia Mathematica* 10 (1983): 24-47; and Harvey W. Becher, "William Whewell and Cambridge Mathematics," *Historical Studies in the Physical Sciences* 11 (1980): 1-48. See also Harvey W. Becher, "Radicals, Whigs and Conservatives: The Middle and Lower Classes in the Analytical Revolution at Cambridge in the Age of Aristocracy," *British Journal for the History of Science* 28 (1995): 405-426. A first examination of the Scottish perspective is Maria Panteki, "William Wallace and the Introduction of Continental Calculus to Britain: A Letter to George Peacock," *Historia Mathematica* 14 (1987): 119-132; followed by Alex D. D. Craik, "Calculus and Analysis in Early 19th-Century Britain: The Work of William Wallace," *Historia Mathematica* 26 (1999): 239-267.

[54] For example, Lorraine Daston brought to light the revival of French research in synthetic geometry in Lorraine J. Daston, "The Physicalist Tradition in Early Nineteenth Century French Geometry," *Studies in History and Philosophy of Science* 17 (1986): 269-295.

first quarter of the nineteenth century.[55] Yet, Scottish mathematicians both followed

developments in the differential and integral calculus and remained faithful to the

geometric spirit of Common Sense philosophy. They rejected algebra as a process

too easy to fulfill the philosophical benefits of mathematics.[56] On the other hand,

English mathematicians were more likely to associate French mathematics with

revolutionary France and to disavow the abstract reasoning supporting movements

toward formal rigor in higher mathematics, with the Cambridge Analytical Society

even replacing the limits in Lacroix's calculus textbook with Lagrange's power

series.[57] They were willing to study the foundations of algebra and thus took the

trend of research in England in the direction of symbolic algebra.[58] Still, in most

cases, English and Scottish mathematicians both preferred fluxional notation in the

calculus as a concrete representation of motion.[59]

English and Scottish mathematicians also worked within different

educational structures which assigned somewhat different roles to geometry in

mathematics education. In English universities, the students usually found their

individual meetings with tutors more valuable than professorial lectures for

memorizing geometrical propositions for written examinations, most notably the

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[55] Jack B. Morrell, "Science and Scottish University Reform: Edinburgh in 1826," *British Journal for the History of Science* 6 (1972): 39-56.

[56] Panteki, "William Wallace" (cit. n. 53), p. 124; Guicciardini, *Newtonian Calculus* (cit. n. 7), p. 95.

[57] Becher, "Radicals, Whigs" (cit. n. 53); Guicciardini, *Newtonian Calculus* (cit. n. 7), pp. 135-137.

[58] Joan L. Richards, "God, Truth, and Mathematics in Nineteenth-Century England," in *The Invention of Physical Science: Intersections of Mathematics, Theology, and Natural Philosophy Since the Seventeenth Century, Essays in Honor of Erwin N. Hiebert*, ed. Mary Jo Nye, Joan L. Richards, and Roger H. Stuewer (Dordrecht: Kluwer Academic Publishers, 1992), pp. 51-78.

[59] Enros, "Analytical Society" (cit. n. 53).

Cambridge Mathematical Tripos. University leaders wanted graduates to have absorbed the lessons that geometry was the language of mathematics and that mathematical logic was the pattern for reasoning. They preferred teaching the clarity and rigor of geometry to showing how mathematics was discovered. English university graduates were expected to have become gentlemen whose study of mathematics fit them for elite society and for the pursuit of applied mathematics (for those graduates who continued on with mathematical studies). In Scotland, professors influenced by Common Sense philosophy taught all of the courses and all examinations were oral. Because professors were paid with student fees rather than a regular salary, the professors felt pressure to present the material students wanted to learn. These students stayed in the university up to four years studying ancient languages and modern natural and moral philosophy, but they rarely paid the fees to graduate.[60] When "English" or "Scottish" are used with reference to geometry or educational factors in this study, then, these are some of the differences which are being indicated, while "British" is used when no distinction can be made between English or Scottish influences or when the generalizations being made more or less held true throughout Great Britain.

Yet, the sides in the public debate over the concept of "British decline" were not necessarily divided by whether the writers came from Scotland or from England, as participants proved willing to cover the differences between themselves in order to focus on drawing the contrast between mathematical practice in Great Britain and that on the Continent, chiefly in France. For example, although both camps tended to exaggerate the British commitment to geometry and that of French

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[60] David B. Wilson, "The Educational Matrix: Physics Education at Early-Victorian Cambridge, Edinburgh, and Glasgow Universities," in *Wranglers and Physicists: Studies on Cambridge Physics in the Nineteenth Century*, ed. Peter M. Harman (Manchester: Manchester University Press, 1985), pp. 12-48.

mathematicians to algebra,[61] on one hand were review writers such as English Thomas Young and Scottish John Leslie, who argued that a geometrical approach was not only the true way to do mathematics but also the most successful.[62] Both men believed that Continental mathematicians' publications were too abstruse to be understood by their readers, easing the way for them to claim to reject these works out of hand. They also argued that French writers' heavy reliance on algebraic methods had led them into error. As Leslie said, "Much as we admire the lofty flight and commanding skill of the Continental mathematicians, we are not blind to their defects and errors. They have long overrated the real value of the art of analysis, and have in many cases applied it to objects which it is not capable of attaining. . . . It would surely be wise to moderate the pretensions of analysis, and avoid the glaring abuse of symbols."[63] Similarly, while Young noted the "paucity of the continental publications" available in Great Britain and admitted that algebraic calculations were "indispensable" in certain cases, his final point in this review was that "the farther the . . . geometrical representation could be carried, the more simple, elegant, and satisfactory was the solution. . . ."[64] Like most British mathematicians of the early nineteenth century, Young and Leslie held the view that algebraic symbols lacked rigor and could only be appealed to with great care, while the proven propositions of geometry sufficed without further development or clarification.

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[61] For instance, Grabiner, "Newton's Calculus" (cit. n. 48), has made this point about exaggeration with respect to Playfair's motives.

[62] They agreed on this even though it was likely Young who later ridiculed Leslie in reviewing his 1809 *Elements of Geometry*: Review of *Elements of Geometry*, by John Leslie, *Quarterly Review* 4 (1810): 25–42.

[63] [John Leslie], review of *Memoires de Physique et Chimie, de la Société d'Arcueil, Edinburgh Review* 15 (1810): on p. 425.

[64] [Young], review of *Théorie de l'Action Capillaire* (cit. n. 40), p. 107, 111.

They were satisfied by the style of mathematics which had become traditional in Great Britain.

Playfair positioned himself along with such scholars as John Toplis and Robert Woodhouse in the camp which supported investigation of Continental accomplishments and increased willingness to appeal to algebraic techniques.[65] Members of this group said they emphasized the differences between British and French mathematics in order to raise the place of mathematics in British life back to its former standard of glory, achieved during Isaac Newton's lifetime. They wanted British intellectuals to direct their attention to mathematics and to recognize the fertile results gained through the differential and integral calculus. Thus, they criticized adherents to the geometrical style as closed-minded and detriments to the advancement of British mathematics. For instance, Toplis (who went on to translate part of Laplace's *Mécanique céleste*, among other activities) wrote, "It is remarkable, that amongst the very few men who still pursue mathematical studies in this country, a considerable part, instead of being dazzled and delighted by the wonderful and matchless powers of modern analysis, still obstinately attach themselves to geometry."[66] Playfair's reviews of Laplace's works, including

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[65] But while Woodhouse and Playfair were among the first professors to recognize the so-called "British decline" in mathematics, the two men do not seem to have shared their concerns with each other; in fact, either James Ivory or Henry Brougham defended Playfair's *Philosophical Transactions* paper against what they perceived as an unjust attack by Woodhouse in: Review of *On the Necessary Truth of Certain Conclusions Obtained by Means of Imaginary Expressions*; by Robert Woodhouse, *Edinburgh Review* 1 (1802-03): 407-412. Woodhouse and Playfair's circle apparently did reach common respect by the end of Playfair's life, as Woodhouse was praised for doing more than any other person to raise the state of exact science at Cambridge in: Review of *An Elementary Treatise on Astronomy*; by Robert Woodhouse, *Edinburgh Review* 31 (1819): 375-394.

[66] John Toplis, "On the Decline of Mathematical Studies, and the Sciences Dependent on Them," *Philosophical Magazine* 20 (1805): 25-31, on p. 28.

Mécanique céleste, both criticized the contemporary state of British mathematics and advocated an algebraic style of mathematics:[67]

> An attachment to the synthetical methods of the old geometers, in preference to those that are purely analytical, has often been assigned as the cause of this inferiority of the English mathematicians since the time of Newton. This cause is hinted at by several foreign writers, and we must say that we think it has had an inconsiderable effect.[68]

Another way in which Playfair dealt with the understanding of analysis and synthesis as mathematical styles, predating his reviews but not as often considered by historians, lay in the choices he made in preparing the second book in *Elements*, by replacing words in proofs with algebraic symbols.[69] As Playfair noted in his preface:

> In the Second Book, also, some algebraic signs have been introduced, for the sake of representing more readily the addition and subtraction of the rectangles on which the demonstrations depend. The use of such symbolical writing, in translating from an original, where no symbols are used, cannot, I think, be regarded as an unwarrantable liberty; for, if by that means the translation is not made into English, it is made

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[67] [Playfair], review of *Méchanique Céleste* (cit. n. 40); and [Playfair], review of *The System of the World* (cit. n. 40).

[68] [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 282.

[69] It should be noted that eighteenth- and nineteenth-century attempts to introduce algebra into Euclid's *Elements* need to be separated from the debate among modern historians over whether the Greeks practiced some manner of "geometric algebra." For a summation of that historiographical controversy, see Ivor Grattan-Guinness, "Numbers, Magnitudes, Ratios, and Proportions in Euclid's *Elements*: How Did He Handle Them?" *Historia Mathematica* 23 (1996): 355-375, on pp. 358-360.

into that universal language so much sought after in all the sciences,

but destined, it would seem, only for the mathematical.[70]

Playfair saw a number of advantages in making this change. Addition and

subtraction symbols helped clarify the complicated arguments in this notorious

book on the relationships between lines and figures.[71] In addition to facilitating

communication and learning, though, Playfair meant to bring attention to the goals

generally associated with the French Enlightenment. The algebraic style was rapidly

encompassing all of mathematics as the "universal language" of expression, even in

what Playfair considered to be a backwards Great Britain. Why not employ it in

areas traditionally placed within elementary geometry in British education but

unwieldy for young men to comprehend with the traditional approach?

Playfair appealed to symbols without altering the substance of the proofs, as

is demonstrated by a comparison between a typical proposition from Simson's *The

Elements of Euclid* and Playfair's *Elements*, Theorem II.6, "If a straight line be bisected,

and produced to any point; the rectangle contained by the whole line thus produced,

and the part of it produced, together with the square of half the line bisected, is

equal to the square of the straight line which is made up of the half and the part

produced."[72] (See Figure 2.1 for reproductions of both theorems and their proofs.)

The same figure accompanied the theorem in both works. Playfair quoted Simson

essentially verbatim in the proof until he reached the conclusion. Then, where

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[70] Playfair, *Elements* (cit. n. 45), p. v.

[71] Playfair was not the first English-language writer to introduce algebraic signs into Euclid's *Elements*. For instance, Reeve Williams retained the symbols and numbers when he translated, from French, Claude François Miliet Dechales, *The Elements of Euclid, Explained and Demonstrated in a New and Most Easie Method*, trans. Reeve Williams (London, 1685). This edition, however, had long since been rejected by British readers who favored Simson's more classical treatment.

[72] Simson, *The Elements of Euclid* (cit. n. 19), p. 54; Playfair, *Elements* (cit. n. 45), p. 54.

a)

If a straight line be bisected, and produced to any point; the rectangle contained by the whole line thus produced, and the part of it produced, together with the square of half the line bisected, is equal to the square of the straight line which is made up of the half and the part produced.

Let the straight line AB be bisected in C, and produced to the point D; the rectangle AD, DB, together with the square of CB, is equal to the square of CD.

Upon CD describe (46. 1.) the square of CEFD, join DE, and through B draw (31. 1.) BHG parallel to CE or DF, and through H draw KLM parallel to AD or EF, and also through A draw AK parallel to CL or DM: and because AC is equal to CB, the rectangle AL is equal (43. 1.) to CH; but CH is equal (36. 1.) to HF; therefore also AL is equal to HF: to each of these add CM; therefore the whole AM is equal to the gnomon CMG: and DM is the rectangle contained by AD, DB, for DM is equal (Cor. 4. 2.) to DB: therefore the gnomon CMG is equal to the rectangle AD, DB, add to each of these LG, which is equal to the square of CB, therefore the rectangle AD, DB, together with the square of CB, is equal to the gnomon CMG and the figure LG: but the gnomon CMG and LG make up the whole figure CEFD, which is the square of CD; therefore the rectangle AD, DB, together with the square of CB, is equal to the square of CD. Wherefore, if a straight line, &c. Q. E. D.
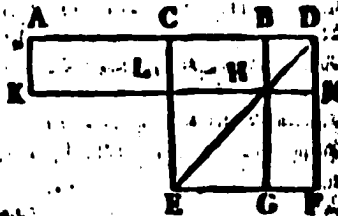
b)

IF a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced and the part of it produced, together with the square of half the line bisected, are equal to the square of the straight line which is made up of the half and the part produced.

Let the straight line AB be bisected in C, and produced to the point D; the rectangle AD.DB, together with the square of CB, are equal to the square of CD.

Upon CD describe the square CEFD, join DE, and through B draw BHG parallel to CE or DF, and through H draw KLM parallel to AD or EF, and also through A draw AK parallel to CL or DM. Because AC is equal to CB, the rectangle AL is equal to CH; but CH is equal to HF; therefore also AL is equal to HF. To each of these add CM; therefore the whole AM is equal to the gnomon CMG. Now AM =AD.DM=AD.DB, because DM=DB. Therefore gnomon CMG=AD.DB, and CMG+LG=AD.DB+CB². But CMG+LG=CF=CD²; therefore AD.DB+CB²=CD². Therefore, if a straight line, &c. Q. E. D.
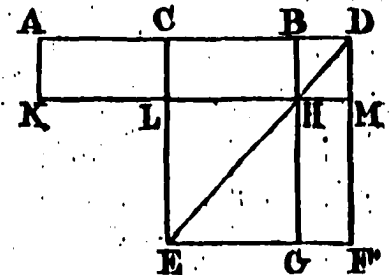
Figure 2.1. Comparison of Proposition II.6. a) From Robert Simson, *The Elements of Euclid* (Philadelphia, 1825), p. 54. b) From John Playfair, *Elements of Geometry* (Philadelphia, 1806), p. 54.

Simson laboriously explained, for instance, "DM is the rectangle contained by AD, DB, for DM is equal to DB . . . therefore the rectangle AD, DB, together with the square of CB, is equal to the square of CD," Playfair briefly noted, "Now AM = AD.DM = AD.DB, because DM = DB . . . therefore AD.DB + CB^2 = CD^2."[73] He simply substituted addition and equals signs for word descriptions of the relationships between the sides of a rectangle. There were a few propositions where he also added an algebraic alternative proof, set off by the word "otherwise." (See Figure 2.2 for a reproduction of one such proposition.) Yet, even with these formulas, Playfair was not advocating the introduction of the kind of analysis then popularly associated with Joseph Louis Lagrange, where there were no diagrams at all, into elementary geometry.[74] Rather, he had only devised an ad hoc arithmetic/algebra for conveying the principles of geometry.

In fact, even though he believed British mathematicians needed to realize that algebra was foundationally as sound as geometry, Playfair played down the significance of employing algebraic symbols in a geometry textbook. He originally thought his treatment of Book II was so unremarkable that he did not mention the

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[73] Simson, *The Elements of Euclid* (cit. n. 19), p. 54; Playfair, *Elements* (cit. n. 45), p. 54.

[74] See Joseph-Louis Lagrange, *Mécanique analytique*, 2 vols., Paris, 1788; and Auguste Boissonnade and Victor N. Vagliente, ed. and trans., *Analytical Mechanics*, Joseph-Louis Lagrange (Dordrecht and Boston: Kluwer Academic Publishers, 1997). Playfair refused to commit himself to a position on whether diagrams were helpful: "Whether the rejection of figures be in all respects an improvement, and whether it may not be in some degree hurtful to the powers of the imagination, we will not take upon us to decide. It is certain, however, that the perfection of Algebra tends to the banishment of diagrams, and of all reference to them. La Grange, in his treatise of *Analytical Mechanics*, has no reference to figures, notwithstanding the great number of mechanical problems which he resolves." [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 254.

IF a straight line be divided into any two parts, the squares of the whole line, and of one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square of the other part.

Let the straight line AB be divided into any two parts in the point C; the squares of AB, BC are equal to twice the rectangle AB.BC, together with the square of AC, or $AB^2 + BC^2 = 2AB.BC + AC^2$.

Upon AB describe the square ADEB, and construct the figure as in the preceding propositions. Because $AG = GE$, $AG + CK = GE + CK$, that is, $AK = CE$, and therefore $AK + CE = 2AK$. But $AK + CE$ = gnomon $AKF + CK$; therefore $AKF + CK = 2AK = 2AB.BK = 2AB.BC$, because $BK = BC$; therefore $AKF + CK + HF = 2AB.BC + HF$; and because $AKF + HF = AE = AB^2$, $AB^2 + CK = 2AB.BC + HF$, that is (since $CK = CB^2$, and $HF = AC^2$), $AB^2 + CB^2 = 2AB.BC + AC^2$. Wherefore, if a straight line, &c. Q. E. D.

a 46. 1
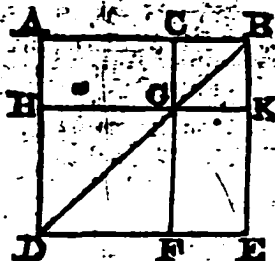
b Cor. 4. 2

x Otherwise. x

"Because $AB^2 = AC^2 + BC^2 + 2AC.BC$, if BC^2 be added to both, $AB^2 + BC^2 = AC^2 + 2BC^2 + 2AC.BC$. But $BC^2 + AC.BC = AB.BC$, and therefore $2BC^2 + 2AC.BC = 2AB.BC$; therefore $AB^2 + BC^2 = AC^2 + 2AB.BC$."

a 4. 2

b 3. 2

A————C————B

Figure 2.2. Example of John Playfair's alternative proofs: Proposition II.7. From John Playfair, *Elements of Geometry* (Philadelphia, 1806), p. 55.

changes he made in either the preface or the endnotes of the 1795 first edition.[75]

True, Playfair later advocated French analysis in *Edinburgh Review* mainly to correct "our inattention to the higher mathematics."[76] Still, he was also concerned with the fundamental disciplines of mathematics and with grounding students in their essential principles.[77] To Playfair, algebra and geometry were equally valid, and they could be intermingled without loss of generality. As he argued when he used algebraic language in Book V's theory of proportion, symbols did not alter the structure of the proofs: "It is plain, therefore, that the concise language of Algebra is directly calculated to remedy this inconvenience; and such a one I have, accordingly, endeavoured to introduce, in the simplest form, and without changing at all the nature of the reasoning, or departing in any thing from the rigour of geometrical demonstration."[78]

Yet, there was a philosophical change at stake in Britain at the turn of the nineteenth century, as was evidenced by the debate over the relative merits of algebra and geometry between those represented by Young and Leslie and those including Toplis, Woodhouse, and Playfair. It had been only seventeen years earlier that Playfair defended the existence of not only imaginary numbers, but the

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[75] The alterations to Books I through VI he listed included: revised definitions in Book I; remarks omitted from Book III; algebraic language, an additional definition, and revised propositions in the theory of proportion in Book V; and propositions replaced and added in Book VI. In the notes to Book II, he mentioned only a simpler proof of proposition 7 and two useful theorems he added, whereas in 1804, he wrote: "The demonstrations of this book are no otherwise changed than by introducing into them some characters similar to those of algebra." Playfair, *Elements* (cit. n. 32), pp. v-viii, 373; Playfair, *Elements* (cit. n. 45), p. 293.

[76] [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 281.

[77] See, for example, Playfair's glowing comments about the mathematics textbooks written in the wake of the French Revolution: [Playfair], review of *The System of the World* (cit. n. 40), pp. 396-397.

[78] Playfair, *Elements* (cit. n. 32), p. vi.

negatives as well: "In algebra, on the other hand [from geometry], the doctrine of negative quantities and its consequences have often perplexed the analyst, and involved him in the most intricate disputations."[79] At that time, Playfair resolved any philosophical difficulties caused by using negative numbers by reasoning from analogy that these numbers were meaningless but had utility: "Supported on so sure a foundation, the arithmetic of impossible quantities will always remain a useful instrument in the discovery of truth, and may be of service when a more rigid analysis can hardly be applied."[80] At least as late as 1811, though, other British mathematicians continued to suspect that symbols and algebra could not be a part of valid mathematical practice. For instance, Leslie cautioned those algebraists who would expose students too soon to the "loose and artificial operation of the modern analysis" that "we are persuaded that a young man will reap more essential and lasting advantage from an acquaintance with geometrical reasoning, than from a knowledge of the elements of algebra."[81] Geometry was inherently superior to algebra in Leslie's view. Thus, even though Playfair did not believe at first that his treatment of Book II was unusual and his work overall does strongly resemble Simson's version, his introduction of symbols would have caused his contemporaries to react by emphasizing the differences between algebra as an analytical method and geometry as the synthetical process. They were not yet

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[79] John Playfair, "On the Arithmetic of Impossible Quantities," *Philosophical Transactions* 68 (1778): 318-343, on p. 318.

[80] Playfair, "Impossible Quantities" (cit. n. 79), p. 342. Helena Pycior touched on Playfair's ideas while she explored the history of symbolic algebra in Helena M. Pycior, "George Peacock and the British Origins of Symbolical Algebra," *Historia Mathematica* 8 (1981): 23-45, on p. 30; and Helena M. Pycior, "Internalism, Externalism, and Beyond: 19th-Century British Algebra," *Historia Mathematica* 11 (1984): 424-441, on pp. 430-433.

[81] [John Leslie], review of *De l'Arithmetique des Grecs;* by Jean-Baptiste-Joseph Delambre, *Edinburgh Review* 18 (1811): 185-213, on pp. 186-187.

willing to accept both disciplines as equally appropriate, so Playfair's quiet

acceptance of algebraic symbols in a rigorous proof appeared too early to effect real

change — either with respect to British views on the relative merits of algebra and

geometry or with respect to British interest in the analytical style of Continental

mathematics.

Analysis and Synthesis as a Method of Proof

As well and not surprisingly given this setting, historians have thoroughly

documented the fact that the foundational standards of mathematics remained in

flux by the turn of the nineteenth century.[82] Although most attention was paid to the

foundations of the calculus, mathematics professors also sometimes debated the

ways in which proofs should be written. Specifically, in dealing with Baconian

empiricism, hypothetico-deduction, and other methods in the intellectual air, British

mathematicians from Thomas Hobbes and John Wallis to Simson and Maclaurin had

questioned the proper direction of proof, considering whether it was more

convincing to proceed forwards from cause to effect or backwards from effect to

cause.[83] Following his predecessors, Playfair brought up in the "Dissertation" that,

"truths can be very well conveyed in the synthetical way, [but] the methods of

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[82] One excellent starting place for reading on this topic is Joan L. Richards, "Rigor and Clarity: Foundations of Mathematics in France and England, 1800-1840," *Science in Context* 4 (1991): 297-319. Again, see as well Grabiner, "Newton's Calculus" (cit. n. 48).

[83] On British mathematicians and seventeenth-century philosophy of mathematics, see Chikara Sasaki, "The Acceptance of the Theory of Proportion in the Sixteenth and Seventeenth Centuries: Barrow's Reaction to the Analytical Mathematics," *Historia Scientiarum* 29 (1985): 83-116; Helena M. Pycior, "Mathematics and Philosophy: Wallis, Hobbes, Barrow, and Berkeley," *Journal of the History of Ideas* 48 (1987): 265-286; Richard A. Talaska, "Analytic and Synthetic Method According to Hobbes," *Journal of the History of Philosophy* 26 (1988): 207-237; Douglas M. Jesseph, "Philosophical Theory and Mathematical Practice in the Seventeenth Century," *Studies in History and Philosophy of Science* 20 (1989): 215-244; and Douglas M. Jesseph, "Of Analytics and Indivisibles: Hobbes on the Methods of Modern Mathematics," *Revue d'Histoire des Sciences* 46 (1993): 153-193. Helpful with the eighteenth century are Pycior, *Symbols, Impossible Numbers* (cit. n. 3), especially pp. 242-275 for Scottish ideas; and Sageng, "Colin MacLaurin" (cit. n. 7).

investigating truth are not communicated by it, nor the powers of invention directed

to their proper objects."[84] Rather than evoking the Kantian view of mathematics as a

synthetic, *a priori* enterprise, Playfair's interpretation deliberately echoed Pappus'

ancient meanings of the terms — in analysis, the mathematician began with a

statement known to be true and made deductions until he or she reached the desired

conclusion.[85] For the reverse, Playfair said: "By synthetical demonstrations I do not

mean reasonings where the algebraic language is not used, but reasonings, whatever

language be employed, where the solution of the proposed question is first laid

down, and afterwards demonstrated to be true."[86] Thus, in geometry, the analytic

part of the proof showed how the figure was originally constructed. The synthesis

inspected the completed figure, ascertaining that the conclusions reached were

valid.

Like Newton and unlike most eighteenth-century British mathematicians who

saw synthesis and analysis as hierarchical, Playfair believed that both analysis and

synthesis were necessary for a complete proof.[87] For example, to solve a problem

[84] John Playfair, *Dissertation Second: Exhibiting a General View of the Progress of Mathematical and Physical Science, Since the Revival of Letters in Europe*, Part II (Edinburgh, 1816; reprint, New York: Arno Press, 1975), p. 35.

[85] Immanuel Kant was tied to intuition much more closely than British mathematicians, who tended toward realism. He also set analytical and synthetical propositions in opposition in order to categorize human knowledge. Greater eloquence on Kant's philosophy of mathematics is provided by Willem de Jong, "Kant's Theory of Geometrical Reasoning and the Analytic-Synthetic Distinction. On Hintikka's Interpretation of Kant's Philosophy of Mathematics," *Studies in History and Philosophy of Science* 28 (1997): 141-166; and Michael Friedman, "Kant's Theory of Geometry," *The Philosophical Review* 94 (1985): 455-506. Jean-Michel Salanskis, "Analysis, Hermeneutics, Mathematics," in *Analysis and Synthesis in Mathematics: History and Philosophy*, ed. Michael Otte and Marco Panza (Dordrecht: Kluwer Academic Publishers, 1996), pp. 227-241, distinguishes between Pappus and Kant. For the text by Pappus, see *Book 7 of the Collection*, English and Greek, ed. with trans. and commentary by Alexander Jones, Parts 1 & 2 (New York: Springer, 1986).

[86] Playfair, *Dissertation* (cit. n. 84), part II, p. 35.

[87] This was how near-contemporaries of Playfair commonly described Newton's method. See Colin Maclaurin, *An Account of Sir Isaac Newton's Philosophical Discoveries*, 2d ed. (London, 1750), pp. 3-24; and John Toplis, "Analytical and Synthetical Reasoning" (cit. n. 66).

such as Proposition I.1 from *Elements of Geometry*, "To describe an equilateral triangle upon a given finite straight line,"[88] the geometer might begin by supposing such a triangle could be and was drawn. Then, one side of the triangle would be the radius of a circle, as would the side on the line. And, the other side and the side on the line would be radii of another circle. The completed analysis showed that, if one constructed these circles, one could construct the triangle. Thus, the first step in the formal proof in *Elements of Geometry* restated the proposition, focusing on the given conditions:

> Let AB be the given straight line; it is required to describe an
> equilateral triangle upon it.

Next was to set out the synthesis by beginning with construction of the triangle:

> From the centre A, at the distance AB, describe the circle BCD, and
> from the centre B, at the distance BA, describe the circle ACE; and from
> the point C, in which the circles cut one another, draw the straight
> lines CA, CB, to the points A, B: ABC shall be an equilateral triangle.

Then, to verify the proof, Playfair, repeating the reasoning found in Euclid's *Elements* and Simson's textbook, completed the synthesis with a demonstration that the construction worked, closing with another restatement of the construction problem and the statement of Quod Erat Faciendum, "Which was required to be done" or "constructed":

> Because the point A is the centre of the circle BCD, AC is equal to AB;
> and because the point B is the centre of the circle ACE, BC is equal to
> BA: but it has been proved that CA is equal to AB; therefore CA, CB

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[88] Playfair, *Elements* (cit. n. 32), p. 8.

are each of them equal to AB; but things which are equal to the same are equal to one another; therefore CA is equal to CB; wherefore CA, AB, BC are equal to one another; and the triangle ABC is therefore equilateral, and it is described upon the given straight line AB. Which was required to be done.

However, beyond those propositions proved by assuming that the converse of the statement was true and then reasoning to a contradiction, any hint of analysis as a method of proof was the exception, not the rule, in Euclid's Elements. Since ancient times, mathematicians have viewed geometry as "a science, of which the elements have always been synthetically delivered,"[89] in order to put the necessary constructions in front of readers as part of a logical process of demonstration.[90] Indeed, Playfair took the synthetical nature of the Elements for granted and did not use the words "analysis" or "synthesis" in the sense of direction of reasoning at any point in his textbook.[91] Furthermore, Playfair perserved the synthetic character of the proofs in Simson's textbook and did not reconstruct or introduce any analyses.

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[89] [Perhaps John Playfair or Henry Brougham], review of *The Principles of Fluxions, Designed for the Use of Students in the Universities*; by William Dealtry, *Edinburgh Review* 27 (1816): 87-98, on p. 87.

[90] On Euclid's method of proof, see, for example, Knorr, *Ancient Tradition* (cit. n. 4), pp. 1-14, 101-149; Ian Mueller, *Philosophy of Mathematics and Deductive Structure in Euclid's "Elements,"* (Cambridge and London: The MIT Press, 1981); David H. Fowler, "Investigating Euclid's *Elements*," review of *Philosophy of Mathematics and Deductive Structure in Euclid's "Elements;"* by Ian Mueller, *British Journal for the Philosophy of Science* 34 (1983): 57-70; and Ivor Bulmer-Thomas, "Euclid: Life and Works," in *DSB* (cit. n. 3), vol. 4, pp. 414-437, on pp. 415-425.

[91] Although there is no definitive proof Playfair wrote anything on the subject, it fits his character that he would have been concerned about relying on the synthetic method in inappropriate situations. For instance, Playfair may have been the *Edinburgh Review* author who warned that trigonometry should be developed analytically even though it was generally taught at the same time as geometry in the Scottish college curriculum (Playfair's *Elements of Plane and Spherical Trigonometry* was bound with the first edition of *Elements*) and/or the *Edinburgh Review* author who criticized a Portuguese textbook writer for teaching algebra with synthetical reasoning: Review of *A Treatise on Plane and Spherical Trigonometry*; by Robert Woodhouse, *Edinburgh Review* 17 (1810): 122-135, on p. 127; and Review of *Principes Mathematiques de feu Joseph-Anastase da Cunha*; trans. by J. M. D'Abreu, *Edinburgh Review* 20 (1812): 425-433.

That Playfair was nevertheless concerned about analysis and synthesis as method of proof is revealed by his interest in the best way to organize the material of Euclidean geometry. The most significant alternate presentation in Playfair's day was Legendre's *Éléments*. Its 1794 publication was too late to influence Playfair during the preparation of his *Elements*, but he did incorporate some of Legendre's propositions into his 1804 second edition.[92] When *Éléments* was later mentioned in Playfair's reviews, he lauded it and especially Legendre's treatment of parallel lines.[93] In his final evaluation of which works best demonstrated the "natural order" of geometrical truths (which meant an ideal deductive arrangement to Playfair), however, Playfair assigned *Éléments* to a secondary role in grasping the chain of underlying geometrical truths: "[We] have no doubt at all, that one who had studied Geometry in Euclid, might [then] read the Elements of Simpson, of Bossut, and still more of Le Gendre, with great advantage. The truths of Geometry might thus be expected to obtain in his mind this natural order — or an order founded on necessary connexion, and independent of all arbitrary or accidental association."[94] In other words, Euclid's propositions were already in "natural order," and Legendre's book only further illustrated their deductive connections to each other.

Also part of Playfair's concern for the method of proof was his willingness to accept what were, to his countrymen, unorthodox proof techniques. Most notably,

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[92] See, for example, Playfair, *Elements* (cit. n. 45), p. 307. It is certain only that Playfair owned the 1806 and 1817 editions of *Éléments; Catalogue of the Library of the Late John Playfair, Esq.* (Edinburgh: James Ballantyne and Co., 1820), p. 53. However, as Dennis Dean assumes for the most influential and useful geological books Playfair owned, Playfair's relatives must have held back the earlier — and likely heavily annotated — editions of *Éléments* from the auction of his books: Dennis R. Dean, "John Playfair and His Books," *Annals of Science* 40 (1983): 179-187, on p. 185n.

[93] [Playfair], review of *Discours sur les Progrès* (cit. n. 40), p. 3; and [Playfair], review of *Elements of Geometry*; by Leslie (cit. n. 40), pp. 89-90.

[94] Playfair, review of *Elements of Geometry*; by Leslie (cit. n. 40), p. 80. Playfair's definition of "natural order" also appears in Playfair, *Elements* (cit. n. 45), p. 289.

he separated himself from Euclid by recognizing the utility of hypothetical

constructions. In order to revise Simson's Book XII on solids including the round

bodies of cones, cylinders, and spheres, Playfair wrote that he broke the rule of

Euclid to never suppose "any thing to be done, as any line to be drawn, or any

figure to be constructed, the manner of doing which he has not previously

explained."[95] As stated earlier, this was an essential feature of the Euclidean system

of rigor. Playfair argued, though, that it was not always possible to demonstrate

geometrical objects which obviously exist, while he could use those objects to prove

interesting results. For example, by avoiding the "artificial difficulties" caused by

prohibiting hypothetical constructions, he presented a simpler form of the method

of exhaustion.[96] Yet, Playfair let his use of other possibly questionable proof

techniques pass almost without comment, such as superposition and *reductio ad*

absurdum.[97] Playfair did continue Simson's practice of avoiding reference to

particular diagrams. Propositions were stated in general form without assigning

names to the geometrical objects in the proposition, reflecting the labels in the

accompanying diagram.

However, for Scottish mathematicians, the most important aspect of analysis

and synthesis as method of proof was voiced in a third response to the separation

between British and French mathematics. In his biography of Simson, Henry

Brougham recorded the common Scottish belief that the French had equated

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[95] Playfair, *Elements* (cit. n. 32), p. viii. See also Playfair, *Elements* (cit. n. 45), p. 286.

[96] Playfair, *Elements* (cit. n. 32), pp. ix-x.

[97] Playfair, *Elements* (cit. n. 45), pp. 283, 293, 299. *Reductio ad absurdum* is generally considered to be an analytic proof rather than synthetic even though reasoning to a contradiction is commonly used in Euclidean geometry. Thus, some geometry reformers tried to perfect the synthetic reasoning of Euclidean geometry by rewriting proofs based on *reductio ad absurdum* into direct form. See, for example, Eric G. Forbes, "Descartes and the Birth of Analytical Geometry," *Historia Mathematica* 4 (1977): 141-151; and Heath, *Elements* (cit. n. 12).

analysis with algebra and thus ignored the ancient writers who employed analysis.[98] What he was referring to was the hypothesis that the constructions and theorems in works such as Euclid's *Elements* were originally found through the method of analysis described in this section, and that ancient Greek mathematicians then removed the analytical part of the demonstrations, since they knew synthesis was sufficient to show the truth of a theorem or problem.[99] This explained why the *Elements* had been transmitted through the ages as a synthetical textbook. Although some British writers appear to have resented the Greeks for "hiding" their analytical research,[100] others enthusiastically followed Newton, Edmund Halley, and Simson by attempting to restore long-lost analytical portions of these proofs.[101] Perhaps most prominent among this number was Matthew Stewart, former holder of the chair of mathematics at the University of Edinburgh and father of Dugald Stewart, Playfair's colleague.

Playfair involved himself in the pursuit of this discipline called "geometrical analysis" by often fondly mentioning the subject. From drawing an analogy between the determination of problems in geometrical analysis and the role of imaginary expressions in the algebraic calculus in his very first paper, Playfair ultimately described geometrical analysis as "the most intricate and paradoxical subject in the

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[98] Brougham, "Simson" (cit. n. 3).

[99] Burnett, "Simson's Euclid" (cit. n. 11). In the past thirty years, there has been another renaissance of interest in this "geometrical analysis" as it was practiced in ancient Greece. Different interpretations are presented by Mahoney, "Another Look" (cit. n. 4); Jaakko Hintikka and Unto Remes, *The Method of Analysis: Its Geometrical Origins and Its General Significance* (Dordrecht: D. Reidel Publishing Company, 1974); Knorr, *Ancient Tradition* (cit. n. 4); and Ali Behboud, "Greek Geometrical Analysis," *Centaurus* 37 (1994): 52-86.

[100] Review of *A Mathematical and Philosophical Dictionary*, by Charles Hutton, *Monthly Review* 25 (1798): 364-383, on p. 376.

[101] Guicciardini, *Newtonian Calculus* (cit. n. 7).

history of the ancient mathematics" and "one of the most ingenious and beautiful contrivances in the mathematicks [sic]."[102] In fact, despite his denigration of the so-called "British decline" in mathematics but probably in part because he had a lifelong interest in the history of science, Playfair treated the restoration of ancient mathematics as the significant accomplishment of his Scottish predecessors. Thus, even though Playfair pointed out that Matthew Stewart's insistence on geometrical reasoning had sometimes led him into error when solving complicated research problems of his own day such as the parallax of the sun, Playfair at the same time valued the propositions Stewart discovered by practicing ancient analysis.[103] Playfair himself was excited about the possibility of understanding how Greek mathematical problems had been originally solved.

To this end, Playfair applied geometrical analysis to the rediscovery of porisms. These statements between problems and theorems were constructed through an analysis done on a problem in order to determine whether the problem had a porism attached, with the synthesis following easily at that point.[104] Playfair positioned his research as a continuation of Simson's efforts to determine what the lost book on porisms, attributed to Euclid by Pappus, had contained.[105] Playfair wanted to find the process through which ancient geometers were led to porisms and additionally to classify these statements within the larger context of

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[102] Playfair, "Impossible Quantities" (cit. n. 79), p. 320; Playfair, "Matthew Stewart" (cit. n. 36), p. 5; Playfair, Dissertation (cit. n. 84), part I, p. 9.

[103] Playfair, "Matthew Stewart" (cit. n. 36), pp. 16-18, 23-25.

[104] See Playfair's definition of "porism" in Playfair, "Porisms" (cit. n. 36), pp. 189-190, 202. Another way to consider "porism" is as a geometrical proposition with too few independent conditions to reach a determinate number of solutions.

[105] Playfair, "Porisms" (cit. n. 36), pp. 181-185.

mathematics and reasoning.[106] He presented seven examples to demonstrate that porisms were particular cases with indeterminate solution of geometrical problems which were generally true.[107] He showed that these were relevant to modern mathematicians because they could be connected with problems of maxima and minima, and Playfair promised to publish a sequel in which he would employ algebra in the investigation of porisms as part of higher geometry.[108] This paper, though, was never written.[109]

Still, Playfair's work on porisms and geometrical analysis in 1794 would have been fresh on his mind when he prepared the *Elements*. It is therefore not surprising that he raised the subject in the preface to the textbook. In addition, Playfair had to justify his omission of Euclid's *Data*, another of the Greek analytical works and a reconstruction by Simson usually published with *The Elements of Euclid* after 1762. Playfair explained that *Data* was not for beginners in mathematics, the audience for *Elements*, and he again indicated his desire to reserve his comments on the discipline "for a work on that [geometrical] analysis, which I have long meditated."[110] By refusing to admit geometrical analysis as an elementary subject, Playfair differed with contemporaries whom he later reviewed. For example, the whole purpose of the textbook by the Italian, Lorenzo Mascheroni, to solve problems in plane geometry with compass only rather than with ruler and compass, was not

[106] Playfair, "Porisms" (cit. n. 36), p. 186.

[107] Playfair, "Porisms" (cit. n. 36), p. 227.

[108] Playfair, "Porisms" (cit. n. 36), pp. 211, 248, 252.

[109] [Playfair], *Works* (cit. n. 27), p. xvii.

[110] Playfair, *Elements* (cit. n. 32), pp. xi-xii.

elementary in Playfair's estimation.[111] Playfair further wished, if Mascheroni had to present this exercise to students, that he at least had given the propositions in analytical (arithmetical) form so that readers could see how to investigate the properties of the intersection of two circles.[112] However, by the time John Leslie included an entire section on geometrical analysis in his 1809 *Elements of Geometry*, Playfair had decided instead that the topic was "a great acquisition to elementary Geometry."[113] He apparently thought that Leslie's careful enunciation of both the analysis and the synthesis clearly illustrated the two parts of proof to students. Perhaps Playfair had also concluded that textbooks were the only conduit through which others would learn the "beautiful and interesting branch" of geometrical analysis.[114] Finally, he was aware by this time that his own advocacy of geometrical analysis as a field of research within modern mathematics was going unheeded.[115]

Analysis and Synthesis as Educational Techniques

Underlying Playfair's first two understandings of analysis and synthesis was the fact that the terms were central features in education and textbooks at the turn of the nineteenth century. Besides being associated with algebra and the deductive reasoning of decomposition, analysis was often described as a process of discovery.

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[111] [John Playfair], review of *Geometrie du Compas, &c;* by Lorenzo Mascheroni, *Edinburgh Review* 9 (1806): 161-168, on p. 162. Playfair repeated this point in his review of *Elements of Geometry;* by Leslie (cit. n. 40), p. 97.

[112] [Playfair], review of *Geometrie du Compas* (cit. n. 111), p. 166.

[113] [Playfair], review of *Elements of Geometry;* by Leslie (cit. n. 40), p. 98.

[114] [Playfair], review of *Elements of Geometry;* by Leslie (cit. n. 40), p. 98.

[115] For instance, Playfair vainly urged French mathematicians to cultivate the subject in his review of *Discours sur les Progrès* (cit. n. 40), p. 5. At the same time, he sighed that the Italian Society was the only body outside Britain which was interested in ancient analysis. On the Italian contest over this form of analysis and synthesis, see Massimo Mazzotti, "The Geometers of God: Mathematics and Reaction in the Kingdom of Naples," *Isis* 89 (1998): 674-701.

For instance, John Leslie (1766-1832) ended his definition of analysis in the *Edinburgh Encyclopaedia* with, "Analysis, therefore, presents the medium of invention; while synthesis naturally directs the course of instruction."[116] In this usage, analysis and synthesis encompassed more than the merits of differential notation against fluxions or even the philosophical certainty of geometry relative to algebra. Rather, an analytical approach was a whole system of thinking and communicating closely related to analysis as the direction of a proof, while the synthetical treatment of science presented the known principles of a discipline as an organized system.

As a complete body of knowledge, Euclidean geometry was the paradigm of this form of synthesis to British mathematics educators. Generally, the chief end of mathematical training in English universities and one of the goals in Scotland as well was to equip the students, as generally-educated people and future ministers, to learn to think logically and to reason to wise decisions.[117] As Leslie described his philosophy of mathematics education in 1811:

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[116] [John Leslie], "Analysis," in *Edinburgh Encyclopaedia*, ed. David Brewster, 2d ed., vol. 1 (Edinburgh, 1830), pp. 718-723, on p. 719. The identification is given in Craik, "Calculus and Analysis" (cit. n. 53), p. 244.

[117] It takes some effort to unearth histories of British mathematics education for the period before non-Euclidean geometry upset the apple cart in which Euclid's *Elements* was entrenched, as is documented by Joan Richards in *Mathematical Visions* (cit. n. 22). An introduction to the Association for the Improvement of Geometrical Teaching, formed during that era, is W. H. Brock, "Geometry and the Universities: Euclid and His Modern Rivals, 1860-1901," *History of Education* 4 (1975): 21-35. As helpful for course content as for the lives of the mathematics teachers he profiles is G. Howson, *A History of Mathematics Education in England* (Cambridge: Cambridge University Press, 1982). Sources which cover British geometry teaching at least in part tend to be older but include some useful information, such as Florian Cajori, "Attempts Made During the Eighteenth and Nineteenth Centuries to Reform the Teaching of Geometry," *American Mathematical Monthly* 17 (1910): 181-201; George A. Gibson, "History of Mathematics in Scotland" (cit. n. 45); and Wilson, *Mathematical Teaching in Scotland* (cit. n. 6). Within their larger studies, Peter M. Harman, "Newton to Maxwell: The *Principia* and British Physics," *Notes and Records of the Royal Society of London* 42 (1988): 75-96; and Peter Slee, "The Oxford Idea of a Liberal Education 1800-1860: The Invention of Tradition and the Manufacture of Practice," *History of Universities* 7 (1988): 61-87; compare and contrast the Scottish setting with that in English universities, while David B. Wilson, "Educational Matrix" (cit. n. 60); takes that aspect of the story beyond the time period of this chapter.

The study of Mathematics, when rightly conducted, ought, we presume, to aim at two capital objects. It should not only lead to an intimate acquaintance with those relations of figure and quantity, which are so highly instructive, and confer such immediate and important advantages in the business of life, and the prosecution of the physical sciences; but it should also train the mind to the invaluable habits of patient attention, accurate arrangement, nice discrimination, and close reasoning. This latter advantage, in the view to general education, is perhaps the most essential.[118]

British mathematicians saw the synthetical aspect of geometry proofs as the best medium for developing this mental discipline because synthesis laid out the steps of reasoning to the readers. Students did not need to discover results on their own when, in many cases, teachers in universities and schools were satisfied to see pupils memorize the truths presented in class and were convinced that trying to understand the analysis was far too complicated for novices.

Although Playfair clearly respected the central role of Euclidean geometry in mathematics education, his opinions about mathematics education filtered through the lens of analysis and synthesis into something rather different from the above generally-accepted view.[119] Throughout his writings, he argued for what he called the study of analytical reasoning, a method which appears to have operated on a variety of levels. First of all, Playfair used analysis and synthesis to urge that Euclidean geometry be taught within proper bounds and in a manner which fostered true mastery of the material. For instance, in reviewing Samuel Horsley's

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[118] [Leslie], review of *De l'Arithmetique des Grècs* (cit. n. 81), p. 186.

[119] See, for example, [Playfair], review of *Elements of Geometry*, by Leslie (cit. n. 40), p. 79.

Latin edition of Euclid's *Elements*, Playfair criticized the editor's desire to add Books

VII through X to those usually taught in grammar schools (I through VI, with XI and

XII added in universities but which Horsley classified as arts rather than as

sciences).[120] Horsley wanted to teach the essential principles of geometry first and

then, to demonstrate the rules of arithmetic and algebra, present these additional

books. Playfair responded by once more justifying algebra in its own right and

continuing, "One very great disadvantage that would necessarily arise from forcing

the student of mathematics to read the Seventh, &c. of the Elements, is, that it would

detain him long in the study of synthetical reasonings, when he ought to be

applying his mind to those that are analytical, and that lead to understanding the

methods of investigation."[121] He hoped that those teaching in universities would

recognize that exposing boys to Horsley's textbook would only further retard the

progress of science in Britain by showing them only the geometrical style of

mathematics.[122] Playfair preferred a balanced education in mathematics which

included geometry and algebra where each subject was appropriate. Because he

advocated both algebra and geometry in mathematics education and research,

Playfair railed against those who believed there was a choice between either algebra

or geometry throughout his career. Of the entire translation by Horsley, Playfair

tepidly commented, "little occurs to be censured, and not much to be praised."[123]

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[120] [Playfair], review of *Euclidis Elementorum* (cit. n. 40), p. 260.

[121] [Playfair], review of *Euclidis Elementorum* (cit. n. 40), pp. 260-261. In England, the *Monthly Review* also faulted Horsley for rejecting algebra as incompetent; Review of *Euclidis Datorum Liber*, by Samuel Horsley, *Monthly Review* 47 (1805): 57-61.

[122] [Playfair], review of *Euclidis Elementorum* (cit. n. 40), pp. 261-262.

[123] [Playfair], review of *Euclidis Elementorum* (cit. n. 40), p. 269.

Playfair's aim for his own textbook of Euclidean geometry reflected as well the Common Sense buzzword of "utility": "The edition now offered to the public . . . is intended not so much to give the writings of Euclid the form which they originally had, as that which may at present render them most useful."[124] Whereas someone like Simson saw geometry as an end in itself, Playfair's philosophy of mathematics maintained that geometry led to additional mathematics. Taking full advantage of his direct contact with students through the Scottish professorial system to encourage them to become doers of mathematics, Playfair in addition cautioned against wearing students out needlessly with unwieldy constructions or with undue insistence on the form of geometrical reasoning at the expense of all the other benefits of geometry. An example of Playfair's implementation of his concerns is his treatment of the theory of proportions, Book V. The traditional geometrical method was extremely difficult for neophytes in mathematics because they were required to visualize several lengths of lines and the relationships between those lines before they could understand any proposition. As with Book II, therefore, Playfair introduced concise symbols in order to streamline the process of comprehending proportion and to enable students to proceed on to the many important applications of the topic. (Figure 2.3 compares the different versions of Proposition V.5 from Simson's and Playfair's textbooks.) Once again, Playfair argued, the desired level of rigor in reasoning necessary for students and typical of Euclidean geometry was preserved despite the introduction of algebraic language.[125] Representing proportion as number was merely a pragmatic step which brought no loss of generality with it. In fact, Playfair thought he had actually made proportion more

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[124] Playfair, *Elements* (cit. n. 32), p. v.

[125] Playfair, *Elements* (cit. n. 32), pp. vi-vii; Playfair, *Elements* (cit. n. 45), p. 299.

a)

If one magnitude be the same multiple of another, which a magnitude taken from the first is of a magnitude taken. from the other; the remainder shall be the same multiple of the remainder, that the whole is of the whole.

Let the magnitude AB be the same multiple of CD, that AE taken from the first, is of CF taken from the other; the remainder EB shall be the same multiple of the remainder FD, that the whole AB is of the whole CD.

Take AG the same multiple of FD, that AE is of CF: therefore AE is (1. 5.) the same multiple of CF, that EG is of CD; but AE, by the hypothesis, is the same multiple of CF, that AB is of CD, therefore EG is the same multiple of CD that AB is of CD; wherefore EG is equal to AB (1. Ax. 5.). Take from them the common magnitude AE; the remainder AG is equal to the remainder EB. Wherefore, since AE is the same multiple of CF, that AG is of FD, and that AG is equal to EB; therefore AE is the same multiple of CF, that EB is of FD: but

AE is the same multiple of CF, that AB is of CD; therefore EB is the same multiple of FD, that AB is of CD. Therefore, if any magnitude, &c. Q. E. D.
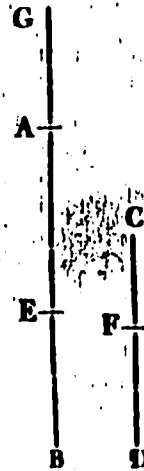
b)

IF one magnitude be the same multiple of another which a magnitude taken from the first is of a magnitude taken from the other, the remainder is the same multiple of the remainder that the whole is of the whole.

Let mA and mB be any equimultiples of the two magnitudes A and B, of which A is greater than B; $mA - mB$ is the same multiple of A—B that mA is of A, that is, $mA - mB = m.\overline{A - B}$.

Let D be the excess of A above B, then A—B=D; therefore by adding B to both, A=D+B; therefore* $mA = mD + mB$; take mB from both, then $mA - mB = mD$. But D=A—B, therefore $mA - mB = m.\overline{A - B}$. Therefore, &c. Q. E. D.

a 1. 5.

Figure 2.3. Comparison of Proposition V.5. a) From Robert Simson, *The Elements of Euclid* (Philadelphia, 1825), pp. 125-126. b) From John Playfair, *Elements of Geometry* (Philadelphia, 1806), p. 138.

general by extending it to relationships between numbers.[126] Yet, although Playfair also pared the thirty-four theorems given by Simson down to a list of twenty-five, he accepted the fundamentals of Book V as the best possible explanation of the theory of proportion. When Leslie discarded this body of theorems entirely, Playfair responded: "So far is Euclid's definition of proportion, and his methods of demonstration, from meriting the censure which are here so inconsiderately thrown out [by Leslie], that we believe no one can explain the properties of proportionals, universally and directly, more easily than he has done; nor do we believe any definition can be given, on which it is so easy to reason correctly, as that which he has employed."[127]

By advocating an analytical, useful style of learning, Playfair's ideas clashed with those he saw practiced in other educational centers, most notably Oxford and Cambridge. In his first review of Laplace's *Mécanique céleste*, the best known of Playfair's statements on British mathematical inferiority, Playfair assigned a major share of the blame for the decline to "the two great centres from which knowledge is supposed to radiate over all the rest of the island."[128] Playfair quickly dismissed Oxford as a place where "the mathematical sciences have never flourished; and the scholar has no means of advancing beyond the mere elements of geometry," saving his strongest critique for the so-called synthetical method forced on students at the mathematically-oriented Cambridge.[129] Playfair painted a bleak picture of the thirst

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[126] Playfair, *Elements* (cit. n. 45), p. 299.

[127] [Playfair], review of *Elements of Geometry* by Leslie (cit. n. 40), p. 93.

[128] [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 283.

[129] [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 283. Even though Playfair himself was more concerned about student cramming in preparation for the Cambridge Mathematical Tripos, his one sentence on Oxford was what drew a vehement response from English readers. After Oxford was mentioned critically in the *Edinburgh Review* several times in addition to Playfair's remark, Edward Copleston's *A Reply to the Calumnies of the Edinburgh Review Against Oxford* was published

for knowledge not being awakened in Cambridge students' minds: "He must study [pure and mixed mathematics], not to learn the spirit of geometry . . . but to know them as a child does his catechism, by heart, so as to answer readily to certain interrogations. In all this, the invention finds no exercise; the student is confined in narrow limits; his curiosity is not roused; the spirit of discovery is not awakened."[130] To Playfair, a synthetical approach to teaching, when practiced by itself, meant that students encountered mathematics as a finished body of theorems fit merely for memorization. He wanted synthesis combined with analysis so that students would grasp the structure of mathematics and retain what they learned. Further, he believed that each subject within the broad curriculum common in Scotland, from classical literature to modern chemistry and medicine, was important in its own right. Mathematics should not be cast as the handmaiden to liberal education or even to philosophy, as he believed it was at Cambridge.

Even though Playfair's entire presentation was appealing to Scottish and English mathematics teachers alike, readers did not necessarily see the different understandings of analysis and synthesis described above in its text, and they certainly did not value the same aspects that Playfair did. *Elements of Geometry* was printed at least thirteen times in Great Britain before 1875 because teachers considered the book to be a more accessible version of the best available model of rigor and reasoning, Euclid's *Elements*. Those who used the textbook accepted

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anonymously in 1810. Playfair, R. Payne Knight, and Sydney Smith reviewed the booklet in *Edinburgh Review* 16 (1810): 158-187; after which *A Second Reply to the Edinburgh Review* appeared and John Davison discussed all three publications in *Quarterly Review* 4 (1810): 177-206. However, the part of the dispute which dealt with the place of mathematics set aside analysis and synthesis to quibble over the definition of "elementary mathematics." Peter Slee examined the dispute and the definition of English university education from the Oxford perspective and identified Copleston in Slee, "Oxford Idea" (cit. n. 117), pp. 64-67, 85.

[130] [Playfair], review of *Méchanique Céleste* (cit. n. 40), p. 283.

Playfair's algebraic language in Books II and V as an evil necessary to make the subject easier for students to learn. What was important about the work to its users, however, was that it contained the synthetic proofs central to educational systems which were not interested in developing creative mathematicians, regardless of any support Playfair showed for analytical teaching.

One alternative for British teachers and tutors to using an edition of Euclid was Leslie's *Elements of Geometry*, which was printed four times between 1809 and 1820.[131] Like Legendre, Leslie wanted to improve upon the arrangement of the propositions, and he also added explanations at the beginning of each book to "facilitate [the students'] progress, by rendering more continuous the chain of demonstration."[132] His explanations, though, often proved to be just as philosophical and detailed as his opening essay on the nature of geometry, perhaps in some ways comparable to Legendre's dense yet more profound endnotes.[133] In the work's six books, Leslie treated the properties of plane figures, equivalence of figures, properties of the circle, quadrature of the circle, proportion, and the division of lines. Overall, Leslie retained 121 of the 173 propositions in the first six books of Simson's *The Elements of Euclid* and added another eighty-three of his own.[134] Leslie made the most changes to the theory of proportion of Book V, removing two-thirds of the usual (Euclidean) propositions, taking one from Book VII and two from Book

[131] For biographies of Leslie, see "Leslie (Sir) John," in *Eminent Scotsmen*, Chambers (cit. n. 3), vol. 3, pp. 407–417; and John Leslie, *Treatises on Various Subjects of Natural and Chemical Philosophy* (Edinburgh: Adam and Charles Black, 1838), pp. 3-46.

[132] John Leslie, *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry* (Edinburgh: James Ballantyne and Co., 1809), pp. vi-vii.

[133] Leslie, *Elements of Geometry* (cit. n. 132), pp. 1-2. See also, for example, Leslie's Book V, pp. 45-173.

[134] Leslie, *Elements of Geometry* (cit. n. 132), pp. xiii-xvi. This count does not include the twelve additional theorems in Simson, *The Elements of Euclid* (cit. n. 14), Books V and VI.

X of Euclid's *Elements*, and writing sixteen new theorems with proofs for a total body of twenty-eight theorems; thus, it was not surprising when Playfair critiqued Leslie for discarding the standard treatment.[135]

In several ways, *Elements of Geometry* reflected Leslie's understandings of the various uses of "analysis" and "synthesis." As was previously noted, Leslie included geometrical analysis within elementary geometry. As well as illustrating the ancient method of two-directional proof, geometrical analysis also fulfilled the educational purpose of training the mind in reasoning in his view. Leslie believed that students must learn to follow proper rigor through such examples as his section on geometrical analysis and another appendix taken from Mascheroni's geometry of compasses. In the preface to *Elements of Geometry*, Leslie gave the same negative view of algebra as a mathematical style that he espoused in *Edinburgh Review*: "It is [a] matter of deep regret, that Algebra or the Modern Analysis, from the mechanical facility of its operations, has contributed especially on the Continent, to vitiate the taste and destroy the proper relish for the strictness and purity so conspicuous in the ancient method of demonstration."[136] Yet, following Playfair, Leslie appealed to algebraic notation in presenting the properties of proportion. He justified this by arguing that geometry had been brought under the dominion of arithmetic in this area — arithmetic was separate from and unlike algebra because it was foundationally sound — and that this language clarified the demonstrations so students learned these particular relationships more quickly.[137] In other words, Leslie's symbols were acceptable because they were arithmetical rather than

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[135] [Playfair], review of *Elements of Geometry* (cit. n. 40), p. 93.

[136] Leslie, *Elements of Geometry* (cit. n. 132), p. viii-ix.

[137] Leslie, *Elements of Geometry* (cit. n. 132), p. 145.

algebraic. Still, unlike Playfair, Leslie believed he was sacrificing mathematical certainty to help his students. Despite this, it is not clear how useful Leslie's work was to learners. The order of theorems in Book V was not from most simple to most complex and thus confused readers, who could not see the connection with practical applications either, and the proofs themselves were not more plain than those in other geometry textbooks. (See Figure 2.4 for a comparison of theorems from Playfair's *Elements* and Leslie's *Elements of Geometry*.) In general, *Elements of Geometry* was too philosophical and too much of a departure from Euclid's *Elements* to be attractive to many British teachers, tutors, or professors.

David Brewster and Thomas Carlyle

Playfair's successor at the center of Edinburgh intellectual culture, David Brewster (1781-1868) also was a member of the second generation of British scientists who talked about the "British decline." That conversation was re-energized around 1830 by former members of the Cambridge Analytical Society, such as Charles Babbage and William Whewell, who had since become socially prominent and gained influence over the Cambridge curriculum. Additionally, these men were increasingly eager to establish societies of professional scientists in Great Britain. When written statements on British decline reappeared with Babbage's 1830 *Reflections on the Decline of Science*, then, the central points were quite different from those raised earlier in the century. While Playfair sought to raise British awareness of interesting mathematical and scientific work done on the Continent and argued that the best scientists should teach, Babbage and Whewell, along with Brewster, wanted public funding for scientific research and asked that the best scientists be

a)   PROP. XIV. THEOR.

IF the first have to the second the same ratio which the third has to the fourth, and if the first be greater than the third, the second will be greater than the fourth; if equal, equal; and if less, less.

If A : B : : C : D, and if A>C, B>D; if A=C, B=D; and if A<C, B<D.

First, let A>C, then A : B>C : B; but A : B : C : D, *a. 5.* therefore C : D>C : B, therefore B>D. *b 13. 5.*

In the same manner it can be proved that if A=C, B=D; *c 10. 5.* and if A<C, B<D. Therefore, &c. Q. E. D.

b)   PROP. IV. THEOR.

In proportional quantities, according as the first term is greater, equal, or less than the second, the third term is greater, equal, or less than the fourth.

Let A : B : : C : D; then if A>B, C>D; if A=B, C =D; and if A<B, C<D.

For, if A be greater than B, A : B is a ratio of majority; whence C : D, being the same with it, is likewise a ratio of majority, and consequently C is greater than D.

If A be equal to B, A : B must be a ratio of equality, and hence C : D is also a ratio of equality, or C is equal to D.

But, if A be less than B, A : B is a ratio of minority, and so is, therefore, C : D, or C is less than D.

Figure 2.4. Comparison of similar theorems. a) From John Playfair, *Elements of Geometry* (Philadelphia, 1806), p. 145. b) From John Leslie, *Elements of Geometry* (Edinburgh, 1809), p. 155.

freed from teaching.[138] As one example, Brewster helped found the British Association for the Advancement of Science as a voice for British science.[139]

As a scientific journalist for much of his adult life, Brewster had opportunity to spread his views and to provide employment to other Scottish intellectuals.[140] After studying under Dugald Stewart, Playfair, and Robison at the University of Edinburgh, Brewster parlayed his contributions to the *Edinburgh Magazine* into its editorship in 1802. That journal folded the next year, but Brewster later watched over the *Edinburgh Encyclopaedia* through its twenty-two years of preparation between 1808 and 1830. In the 1820s, his attempt to launch the *Edinburgh Philosophical Journal* degenerated amid contentious relationships with his co-editor and his publisher. Brewster then took charge of the *Edinburgh Journal of Science*, which was financially sponsored by the publisher of the *Edinburgh Encyclopaedia* and which merged with *Philosophical Magazine* in 1832. Through all of these editorial roles, Brewster needed to hire article authors for a variety of topics.

For instance, even before he began complaining about the so-called "British decline," Brewster publicized Continental mathematics and science through his contracted authors. One such writer, William Wallace (1768-1843), in 1815 not only penned a well-known treatise explaining the differential and integral calculus in

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[138] See Charles Babbage, *Reflections on the Decline of Science in England and on Some of Its Causes* (London, 1830; reprint, New York: Augustus M. Kelley, 1970); David Brewster, review of *Reflexions on the Decline of Science in England*, by Charles Babbage, *Quarterly Review* 13 (1830): 305-342; and Review of *Thoughts on the Study of Mathematics as a Part of Liberal Education*, by William Whewell, *Edinburgh Review* 52 (1836): 218-252. A historical account is Jack B. Morrell, "Individualism and the Structure of British Science in 1830," *Historical Studies in the Physical Sciences* 3 (1971): 183-204.

[139] The history of the British Association is Jack B. Morrell and Arnold Thackray, *Gentlemen of Science: Early Years of the British Association for the Advancement of Science* (Oxford, 1981). On Brewster's many facets, see A. D. Morrison-Low and J. R. R. Christie, ed., *'Martyr of Science': Sir David Brewster 1781-1868* (Edinburgh: Royal Scottish Museum, 1984).

[140] On Brewster's career in scientific journalism, see W. H. Brock, "Brewster as a Scientific Journalist," in Morrison-Low and Christie, *'Martyr of Science'* (cit. n. 139), pp. 37-42.

"Fluxions" for the *Edinburgh Encyclopaedia* but also made a partial translation of

Legendre's *Éléments* available to British readers in the reference work's "Geometry"

article.[141] In "Geometry," Wallace began with a thirteen-page history of geometry,

including the teaching of Euclid's *Elements*.[142] Wallace concentrated on ancient

geometers and geometrical analysis for most of this section, but when he reached the

seventeenth century a few paragraphs from the end, he remarked, "The history of

geometry becomes now interwoven with that of the modern analysis, and is chiefly

interesting by the extent to which the science has been carried by that powerful

instrument of invention."[143] Wallace was willing to intermingle algebra and

geometry, for he saw analytical geometry as a necessary precursor to the calculus,

which then revolutionized the study of nature. Still, while Wallace referred to the

"curious circumstance" that Maclaurin had relied on geometry in his defense of

fluxions, he did want students of higher mathematics to gain a thorough knowledge

of geometry as part of their mathematical training. He proposed that geometry

students peruse Legendre's *Éléments* together with any of the standard Scottish

works: Simson's *The Elements of Euclid*, Playfair's *Elements*, or Leslie's *Elements of

Geometry*.[144] Until that time, Legendre's *Éléments* had been referred to only in

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[141] Wallace's contributions were long overshadowed by the emphasis on Cambridge students and mathematicians, but Panteki, "William Wallace" (cit. n. 53), and Craik, "Calculus and Analysis" (cit. n. 53), provide studies of Wallace, who taught at the Royal Military College and succeeded John Leslie in the Edinburgh chair of mathematics. The evidence for Wallace's authorship of "Geometry" is in a letter by Wallace quoted by Craik: "I was just in the middle of an article preparing for Dr. Brewster and about to begin another. . . . I mean the article *Fluxions* . . . I do not recollect any other besides *Geometry* for which I am engaged to him (I mean of any length)." William Wallace to Macvey Napier, 3 April 1815, British Library, London, Macvey Napier Correspondence: Add. MSS. 34, 611, f. 189; quoted in Craik, "Calculus and Analysis" (cit. n. 53), p. 245.

[142] [William Wallace], "Geometry," in *Edinburgh Encyclopaedia* (cit. n. 116), vol. 10, pp. 185-240, on p. 188.

[143] [Wallace], "Geometry" (cit. n. 142), p. 196.

[144] [Wallace], "Geometry," (cit. n. 142), pp. 197-198.

passing by English or Scottish writers. For example, Playfair and Leslie both pointed out interesting but isolated propositions in their geometry textbooks — Leslie did not critique Legendre's understanding of the theory of parallels until the third edition of *Elements of Geometry*, published in 1817.[145] In contrast, Wallace relied heavily on Legendre's work in the forty-page summary of the elements of geometry which comprised the remainder of the article.

Although Wallace translated roughly two-thirds of the first six books of Legendre's *Éléments* for "Geometry," Brewster later assigned the entire textbook to one of his young protégés, Thomas Carlyle (1795-1881). Carlyle had mainly been drifting ever since his 1809 graduation from the University of Edinburgh when he translated an article by Jakob Berzelius and a German work on crystallography and reviewed a book on magnetism for Brewster in 1819.[146] Brewster then assigned

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[145] See, for example, Playfair, *Elements* (cit. n. 45), pp. 307-308, 311; [Playfair], review of *Discours sur les Progrès* (cit. n. 40), p. 3; [Playfair], review of *Elements of Geometry*, by Leslie (cit. n. 40), pp. 89-90; and Leslie, *Elements of Geometry* (cit. n. 132), pp. 490, 492. Legendre's response to Leslie's objections may be found in [Thomas Carlyle, trans.], *Elements of Geometry and Trigonometry; With Notes. Translated From the French of A. M. Legendre*, ed. David Brewster (Edinburgh: Oliver & Boyd, 1824), pp. 227-230. The debate between the two will not receive much attention in this study because the Americans teaching geometry voiced little, if any, concern for the status of the parallel postulate. For example, John Farrar omitted most of Legendre's discussion of the subject, telling readers to go look at the critique in Leslie's *Elements of Geometry*. Later, Charles Davies included Carlyle's translation of Legendre's endnotes on the theory of parallels in the first version of *Elements of Geometry and Trigonometry* but then took it out without comment the first time he significantly revised the textbook in 1834. [John Farrar, trans.], *Elements of Geometry*, by Adrien-Marie Legendre (Cambridge, MA: Cummings & Hilliard, 1819); [Charles Davies, ed.], *Elements of Geometry and Trigonometry; With Notes. Translated from the French of A. M. Legendre*, trans. [Thomas Carlyle] and David Brewster, 3d ed., rev. (New York, 1828); [Charles Davies, ed.], *Elements of Geometry and Trigonometry, Translated from the French of A. M. Legendre by [Thomas Carlyle and] David Brewster*, rev. ed. (Philadelphia: A. S. Barnes & Co., [1834]).

[146] Carlisle Moore, "Carlyle, Mathematics and 'Mathesis,'" in *Carlyle Past and Present: A Collection of New Essays*, ed. K. J. Fielding and Rodger L. Tarr (New York: Barnes and Noble, 1976), pp. 61-95, on p. 75. Moore examined Carlyle's considerable attention to mathematics in his early life and argued that the habits he acquired then continued throughout his literary career. He also gave an account of Carlyle's best-known mathematical achievement, the solution of a quadratic equation which appeared in the third edition of Leslie's *Elements of Geometry*. Another source on Carlyle's early years is James Anthony Froude, *Thomas Carlyle: A History of the First Forty Years of His Life, 1795-1835* (London: Longmans, Green, and Co., 1882).

Carlyle more than twenty articles in the *Edinburgh Encyclopaedia*, his first steady

employment as a writer. While he was preparing these, Brewster offered him more

work in November 1821, a translation of Legendre's *Éléments*. Carlyle wrote to his

father that he did not expect the task to be difficult, but at this point Carlyle was

already becoming involved in activities more closely related to the literary career for

which he was later known and thus he procrastinated for a month before beginning

the translation.[147] By the following April, Carlyle complained that he found no

pleasure in the work and enlisted his brother's help, finally finishing in the last days

of July 1822.[148] Carlyle earned £50 for his and his brother's efforts, a sum he

considered paltry later in life even though Brewster had originally offered the

equivalent of only £15, 15s.[149]

Elements of Geometry and Trigonometry was almost entirely a literal translation

of the 1817 eleventh edition of Legendre's *Éléments*, with Carlyle retaining even

Legendre's notes and the decimal division of the circle in the trigonometry

section.[150] By the time of this edition, Legendre had replaced the book's

advertisement three times and revised the endnotes, which most notably contained

his discussion of the theory of parallels, but he made only minor changes to the

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[147] See Thomas Carlyle to James Carlyle, 17 November 1821, in Charles Richard Sanders, ed., *The Collected Letters of Thomas and Jane Welsh Carlyle*, Duke-Edinburgh ed., vol. 1 (Durham: Duke University Press, 1970), pp. 395-397; and Thomas Carlyle to John Carlyle, 11 December 1821, in Sanders, *Collected Letters*, vol. 1, pp. 407-409.

[148] Thomas Carlyle to John Carlyle, 7 April 1822 and 25 July 1822, in Sanders, *Collected Letters* (cit. n. 147), vol. 2, pp. 79-82, 151-154. John owned a copy of Legendre's book and had familiarized Carlyle with it when he began the project. See Thomas Carlyle to James Carlyle, 17 November 1821, in Sanders, *Collected Letters*, vol. 1, pp. 395-397.

[149] See, for example, Moore, "Carlyle" (cit. n. 146), p. 79, and Charles Richard Sanders's notes to the letter from Thomas Carlyle to Alexander Carlyle, 21 November 1821, in Sanders, *Collected Letters* (cit. n. 147), vol. 1, pp. 397-400. On the first offer of 15 guineas, see Thomas Carlyle to James Carlyle, 17 November 1821, in Sanders, *Collected Letters*, vol. 1, pp. 395-397.

[150] Adrien-Marie Legendre, *Éléments de Géométrie avec des Notes*, 11th ed. (Paris, 1817).

body of propositions over the years. Brewster added a brief preface in which he noted that Legendre had approved the idea of a translation and sent three items to add to the endnotes which were to appear in the twelfth edition of *Éléments*, published in 1823: Queret's demonstration of the solidity of the pyramid, Legendre's reply to Leslie's critique of his theory of parallel lines, and Jean-Frédéric-Théodore Maurice's defense of Legendre's theory.[151] Brewster also had the engraved plates for *Éléments* replaced with diagrams prepared from woodcuts. These appeared with the corresponding propositions, as was common of the various British versions of Euclid's *Elements*, rather than at the end of the text, which was generally a feature of French textbooks, including *Éléments*. Interestingly, Brewster's artist added shading to the figures in the books on solid geometry, which was unusual for either British or French works but which was probably made easier by the use of woodcuts rather than metal plates. (See Figure 2.5 for an example.) It is not clear, though, whether these illustrations were meant to hold any philosophical significance.

The major aspect of analysis and synthesis in *Elements of Geometry and Trigonometry* was the influence of mathematical styles. By choosing to have Legendre's *Éléments* translated, Brewster demonstrated the desire for a British audience to become well-read in contemporary mathematical developments, including gaining familiarity with the manner in which French students were exposed to elementary geometry and the prominent book read by them or their teachers. Although Carlyle preserved the synthetic form of Legendre's *Éléments* in terms of the method of proof, he brought attention to Legendre's approach to algebra within those proofs. Carlyle translated Legendre's statements using the term

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[151] [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 145), pp. v-vi. Maurice (1775-1851) was a Swiss mathematician, astronomer, and administrator who later wrote a memoir on the life and writings of Legendre; "Maurice (le baron Jean-Frédéric-Théodore)" in *Bibliographie Universelle: Ancienne et Moderne*, ed. J. Fr. Michaud, new ed., vol. 27 (Paris, 1854), pp. 345-346.

PROPOSITION VIII. THEOREM.

If the line AB is parallel to a straight line CD drawn in the plane MN, it will be parallel to that plane.

For if the line AB, which lies in the plane ABCD, could meet the plane MN, this could only be in some point of the line CD, the common intersection of the two planes: but AB cannot meet CD, since they are parallel; hence it will not meet the plane MN; hence (*Def.* 2.) it is parallel to that plane.
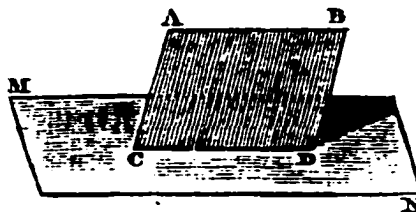
PROPOSITION IX. THEOREM.

Two planes MN, PQ perpendicular to the same straight line AB, are parallel to each other.

For, if they can meet anywhere, let O be one of their common points, and join OA, OB; the line AB which is perpendicular to the plane MN, is perpendicular to the straight line OA drawn through its foot in that plane; for the same reason AB is perpendicular to BO; therefore OA and OB are two perpendiculars let fall, from the same point O, upon the same straight line; which is impossible: therefore the planes MN, PQ, cannot meet each other; therefore they are parallel.
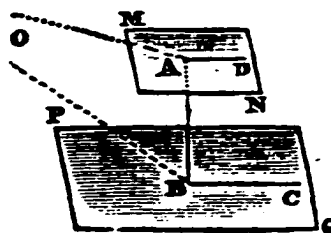
Figure 2.5. Examples of Brewster's shaded, woodcut diagrams for solid geometry. From [Thomas Carlyle, trans.], *Elements of Geometry and Trigonometry* (Edinburgh, 1824), pp. 115-116.

"analytical," which was found in the endnotes as a synonym for "algebraic": "In all this, we have supposed that surfaces are measured by the product of two lines, and solids by the product of three; a truth which it is easy to demonstrate by Analysis, in like manner."[152] Carlyle was not necessarily sympathetic himself to the substitution of modern algebraic techniques for geometric methods—Carlisle Moore argued that Leslie was the overriding mathematical influence on Carlyle—but he did prepare an introduction on proportion for the translation, apparently in the belief that British students needed an explanation of the algebraic approach to proportion since they generally studied the subject only while mastering Euclid's *Elements*.[153] Like Leslie, Carlyle considered the theory of proportion to belong properly to arithmetic, rather than to algebra or geometry.[154] He employed the same sort of notation used by Playfair and Leslie, but he condensed the essential knowledge to only three theorems: that the product of the extremes equalled that of the means, that the ratio of two magnitudes was not changed when the magnitudes were increased by other magnitudes in the same ratio, and that the products of corresponding terms were proportional.[155]

Elements of Geometry and Trigonometry was never a successful textbook in Great Britain. There are a number of possible reasons why mathematics teachers failed to use it. For example, publication was delayed by more than Carlyle's slow translation work, which was partially offset by the printers' preparation of sheets as

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[152] [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 145), pp. 226-227. See also pp. 228, 229, 235n, 246, 277, 304, 360ff.

[153] Moore, "Carlyle" (cit. n. 146), p. 66, 79; [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 145), p. ix. Although he claimed in later life to have composed the essay in half a day, the dates when Carlyle composed it are not clear.

[154] [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 145), p. ix.

[155] [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 145), pp. xiii-xvi.

the translation progressed. Although Carlyle corrected the final proofs in November 1822, Brewster then became embroiled in a disagreement with the printers, Oliver & Boyd in Edinburgh.[156] A full printing of *Elements of Geometry and Trigonometry* did not appear until 1824 — bringing the Scottish experience with Legendre's *Éléments* full circle, Brewster dedicated the textbook to William Wallace. The problems getting the textbook into print may have helped doom its influence in Scotland and England, for Oliver & Boyd ended up losing £351 14s. 16d. on the venture.[157] Additionally, Carlyle wrote that Brewster intended the translation to be a publication of an "embryo society for the encouragement of the arts," perhaps along with Robison's *Essays on Mechanical Philosophy* and Euler's *Letters to a German Princess*, also prepared in 1822 and 1823 and the only other books edited by Brewster besides revisions of three textbooks on astronomy and natural philosophy by James Ferguson.[158] Yet, the society was apparently such an embryonic idea that it is not mentioned in biographical articles on Brewster and it had no noticeable impact on contemporaries.[159] Perhaps most importantly, British readers viewed Legendre's

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[156] Thomas Carlyle to Alexander Galloway, 6 November 1822 and 4 December 1822, in Sanders, *Collected Letters* (cit. n. 147), vol. 2, pp. 193-196, 214-216. Carlyle scholar Rodger L. Tarr suggested that Brewster found an unacceptable number of typographical errors in a proof copy carrying the date of 1822; bolstering this theory, almost no copies exist of this or the second issue, printed in 1823; Rodger L. Tarr, *Thomas Carlyle: A Descriptive Bibliography* (Pittsburgh: University of Pittsburgh Press, 1989), pp. 2-5. Brewster was also enmeshed in a legal dispute with the Constable publishing house over the ownership and printing rights for the *Edinburgh Philosophical Journal* during this same time period, October 1822-1824.

[157] Tarr, *Thomas Carlyle* (cit. n. 156), p. 4.

[158] Thomas Carlyle to James Johnston, 15 December 1821, in Sanders, *Collected Letters* (cit. n. 147), vol. 1, pp. 410-412. The textbooks revised by Brewster were: James Ferguson, *Lectures on Select Subjects in Mechanics, Hydrostatics, Pneumatics and Optics* (Edinburgh, 1805); James Ferguson, *Astronomy Explained Upon Sir Isaac Newton's Principles*, 2 vols. (Edinburgh, 1811); and James Ferguson, *Essays and Treatises on Astronomy, Drawing in Perspective, Electricity &c.* (Edinburgh, 1823).

[159] For instance, see Morrison-Low and Christie, '*Martyr of Science*' (cit. n. 139); Robert Hunt, "Brewster, Sir David," in *DNB* (cit. n. 3), vol. 2, pp. 1207-1211; and Edgar W. Morse, "Brewster, David," in *DSB* (cit. n. 3), vol. 2, pp. 451-454.

Éléments as advanced mathematical research, a purpose for which it was outdated by 1824, since the only significant changes made to the textbook by Legendre over the years were updated reports on his thinking with respect to the theory of parallels.[160] In English schools, especially, Playfair's *Elements* was seen as the faithful rendering of Euclid's *Elements* for modern, young eyes and thus was the proper textbook throughout the nineteenth century.

Conclusion

In summary, Scottish authors between 1750 and 1825 produced a rather broad array of potential geometry textbooks which formed a whole of influence upon college professors who appreciated the educational ethos of Scotland, such as the Americans. Yet, professors in American colleges ultimately emphasized certain pieces from the whole over the rest. Simson's vision of a corrected Euclid was one of the first choices of the nine colleges in existence before the American Revolution and remained in use in certain areas well into the nineteenth century, but professors and boards of overseers in many more institutions appreciated that Playfair had sought to popularize Euclid.[161] They replaced older versions of Euclid's *Elements* with Playfair's *Elements*, resulting in the enormously successful publishing history of this book in the United States which was noted at the beginning of this chapter. In general, Leslie's *Elements of Geometry* appealed to different sensibilities than the various British and American textbooks which influenced American teaching. For instance, Leslie's book appears to be the only geometry textbook prepared in

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[160] Augustus De Morgan was said to be the only British mathematician to propose the adoption of the translation as a textbook; he preferred Legendre's explanation of solid geometry to those given in versions of Euclid's *Elements*; Cajori, "Attempts" (cit. n. 117).

[161] Although Florian Cajori, in *The Teaching and History of Mathematics in the United States* (Washington, DC: Government Printing Office, 1890), did not name any institutions which used *The Elements of Euclid* after the 1810s, the textbook was reprinted in the United States from 1803 to 1876.

Scotland which was translated into French and read by the synthetic geometers such as Monge and Chasles.[162] In contrast, and despite Leslie's own brief sojourn as a tutor in Virginia from 1788 to 1790, *Elements of Geometry* was never printed in the United States nor taught in the colleges.[163] Finally, Carlyle completed a literal translation into English of Legendre's highly respected textbook. Carlyle left mathematics permanently shortly after he finished the task, and he apparently never knew of his work's long life — and alteration — in the United States, to be discussed in Chapter Five.[164]

In addition to providing the course material in geometry for American colleges, all of these "pieces" or geometry textbooks from Scotland helped make French mathematics known in the United States, initiate discussions about mathematical practice and educational philosophy, and transmit diverse notions about analysis and synthesis. As the textbook distributed most widely, Playfair's *Elements* especially exemplified the prevalent understandings. He was a proponent of the mathematical style represented by the differential and integral calculus associated with French mathematics. Playfair distinguished himself from other Scottish professors by accepting algebra and geometry as equally valid and willingly intermingling the disciplines. He wanted to update Euclid's *Elements* to enable students to progress on to higher mathematics and practical applications, but he

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[162] Olson, "Scottish Philosophy" (cit. n. 26). Gaspard Monge and Michel Chasles found *Elements of Geometry* (cit. n. 132) helpful with pursuing their research programs, but French readers were probably also interested in the dispute between Leslie and Legendre over the theory of parallels.

[163] Chambers, "Leslie" (cit. n. 131), p. 408; *NUC* (cit. n. 2), vol. 328, pp. 253-255. *Elements of Geometry* was apparently short-lived in Edinburgh as well — Thomas Carlyle noted that Leslie's successor in the chair of mathematics, William Wallace, reintroduced Playfair's *Elements*; Thomas Carlyle to Robert Mitchell, 18 November 1819, in Sanders, *Collected Letters* (cit. n. 147), vol. 1, pp. 207-210.

[164] Tarr, *Thomas Carlyle* (cit. n. 156), p. 9.

continued to view Euclidean geometry as a central part of mathematics education and supported efforts to do research with geometrical analysis. Playfair accepted hypothetical constructions as a method of proof but did not take a position on the validity of reference to particular diagrams. Finally, he favored training students according to the concept of analysis as invention, by learning actively. The study turns next to the story of Jeremiah Day in order to explore the themes raised by these varied Scottish geometry textbooks in the American context.

CHAPTER THREE

TO FIND THE AREA OF A PARALLELOGRAM: JEREMIAH DAY, CREATING AN AMERICAN GENTLEMAN THROUGH MATHEMATICS

Higher education in the nineteenth-century United States was a small world. For example, Jeremiah Day (1773-1867) and John Farrar updated each other on their mathematics textbook projects, as will be explored in greater detail shortly. The junior member of the threesome, Charles Davies, apparently crossed paths with either of the other two only once, in a brief and businesslike 1840 note to Day, where the rising king of American mathematics textbooks reported that the portrait of Jared Mansfield requested nine months earlier by the prominent president of Yale was on its way to the Trumbull Gallery.[1] The letter indicated a mathematical connection as well as Day's desire to increase the collection on display in a building constructed during his presidency to house the works of Colonel John Trumbull — Mansfield had survived expulsion from Yale in 1777 to receive the degree of A.M. from Yale in 1787, to write the first American book of original mathematical research in 1801, *Essays, Mathematical and Physical*, and to become Davies's father-in-law in 1825.[2]

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[1] Charles Davies to Jeremiah Day, 10 January 1840, Day Family Papers, MS 175, Series III, Box 50, Manuscripts and Archives, Yale University Library, New Haven.

[2] Kovarik, Alois F., "Mansfield, Jared," in *Dictionary of American Biography* (hereinafter cited *DAB*), ed. Allen Johnson, vol. 6, part 2 (New York: Charles Scribner's Sons, 1927-1936), pp. 256-257. On Day and the Trumbull Gallery, see "Day, Jeremiah," in *The National Cyclopedia of American Biography* (hereinafter cited *NCAB*), vol. 1 (New York and Clifton, NJ: James T. White and Company, 1898-1984), pp. 169-170.

Indeed, despite their differences in age, the careers of Day and Davies as well as Farrar as American college mathematics educators paralleled in a number of ways. All three were largely self-trained and yet produced the textbook series which shaped college mathematics and geometry education in the nineteenth century. Members of a "bridge generation" of American mathematicians and scientists involved in republic- and university-building, Day, Farrar, and Davies actively received European ideas which arrived mainly via Scotland. They relied upon their understandings of these ideas, including the various usages of "analysis" and "synthesis," to popularize French mathematics in their own ways. Additionally, as the study shows, they sought to put geometry into a form their students could comprehend and find useful. One facet of this was that Day, Farrar, and Davies all advocated liberal education for its ability to discipline the mind but were not unduly committed to the formal learning process. They considered science and mathematics — including applications of mathematics — to be part of liberal education. Day and Farrar even anticipated curriculum reform by suggesting that students be given a limited choice of courses.

Day also spent much of his career drawing parallels or, as he put it, seeking balances to undergird mathematics teaching at Yale. He felt that the ideal college course was both paternal and thorough. In his most famous publication, a report on the state of education at Yale in 1828, he further argued that college education should balance literature and science, abstract and practical knowledge. He was eager to accommodate modern knowledge to traditional modes of teaching, especially the tutor system and its inherent reliance on memorization. Thus, he added new subjects to the Yale curriculum in mathematics while remaining one of the most influential adherents to Playfair's *Elements*. Through direct contact with students and textbooks such as the 1816 *A Practical Application of the Principles of*

Geometry to the Mensuration of Superficies and Solids, Day helped develop Yale

students into American gentlemen.

Crossing the Atlantic: Scotland and American Colleges

When historians have noted the multi-faceted relationship between Scotland

and colonial America, the majority have focused on higher education.[3] The story of

the college in America began in 1642, with the first graduates from Harvard. Despite

its constant search for financial support, Harvard soon became known for teaching

the classical liberal arts in Latin through recitations and disputations.[4] Students were

instructed through the tutor system, whereby a recent Harvard graduate guided one

class through material taken from textbooks in all subjects, including mathematics in

the third and final year. Natural philosophy was commonly taught in the

Newtonian fashion by the time William and Mary and Yale were founded near the

turn of the eighteenth century. The level of science and mathematics teaching waxed

and waned in the growing number of institutions over time, as college boards hired

professors of natural philosophy and mathematics and then tried to ensure that

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[3] The author identified four book-length works on Scotland and America in preparing this study; three of these, in addition to more than one chapter in the fourth, dealt entirely with the colleges: George S. Pryde, *The Scottish Universities and the Colleges of Colonial America* (Glasgow: Jackson, Son & Company, 1957); Richard J. Petersen, "Scottish Common Sense in America, 1768-1850: An Evaluation of Its Influence" (Ph.D. diss., American University, 1963); Douglas Sloan, *The Scottish Enlightenment and the American College Ideal* (Teachers College, Columbia University: Teachers College Press, 1971); and Andrew Hook, *Scotland and America: A Study of Cultural Relations 1750-1835* (Glasgow and London: Blackie and Son, 1975). See also T. J. Wertenbaker, *Early Scotch Contributions to the United States, Being a Lecture Delivered Within the University of Glasgow on 8th March, 1945* (Glasgow: Jackson, Son, & Company, 1945); Archie Turnbull, "Scotland and America, 1730-90," in *A Hotbed of Genius: The Scottish Enlightenment 1730-1790,* ed. David Daiches, Peter Jones, and Jean Jones (Edinburgh: University Press, 1986), pp. 137-152; J. H. Plumb, "Britain and America—The Cultural Tradition," *Notes and Records of the Royal Society of London* 31 (1977): 227-240; and Winifred Bryan Horner, "Nineteenth-Century Higher Education: The Scottish American Connection," and Deborah Brunton, "Edinburgh and Philadelphia: The Scottish Model of Medical Education," both in *Scottish Universities: Distinctiveness and Diversity,* ed. Jennifer J. Carter and Donald J. Withrington (Edinburgh: John Donald Publishers Ltd., 1992), pp. 34-39, 80-86.

[4] On seventeenth-century Harvard, see Samuel Eliot Morison, *Harvard College in the Seventeenth Century* (Cambridge: Harvard University Press, 1936).

these men stayed sober enough to deliver lectures complementing the recitation sessions.[5] The curriculum, comprised of classics, philosophy, and mathematics and physics, remained fixed in the eighteenth century, although colleges began to place mathematics earlier in the college curriculum. When the American Revolution broke out, the nine American colleges in existence by then had a well-established societal role: they taught by rote gentlemen who would be entering the clergy.

Of these colleges, the College of New Jersey (later Princeton) was most evident for a direct connection with Scotland circa 1776.[6] The only Presbyterian institution of higher education in the colonies, its president from 1768 to 1794 was John Witherspoon (1723-1794), who was educated at the University of Edinburgh and was an outspoken Evangelical leader. He introduced a moral philosophy course based on the writings of Thomas Reid and Francis Hutcheson, emphasizing the utilitarian realism of Common Sense philosophy. Witherspoon also gave philosophy the status of an academic discipline, advocating its study as a means for developing the character of young men into that of gentlemen. Although he did not alter the rest of the curriculum, he did re-energize course content.[7] In all of his activities at the

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[5] The first professor of mathematics and natural philosophy in the colonies was Tanaquil Lefevre, who began to lecture at William and Mary in 1711. See H. R. Phalen, "The First Professorship of Mathematics in the Colonies," *American Mathematical Monthly* 53 (1946): 579-582. Perhaps the archetypal example of American mathematics professors who struggled with intemperance was Isaac Greenwood, who is discussed in "History of Harvard Mathematics" in Chapter Four.

[6] On earlier colonial educators from Scotland, see Pryde, *Scottish Universities* (cit. n. 3), pp. 9-25.

[7] Sloan, *Scottish Enlightenment* (cit. n. 3), pp. 1-35, 103-145: Pryde, *Scottish Universities* (cit. n. 3), pp. 29-35; Wertenbaker, *Scotch Contributions* (cit. n. 3), pp. 18-20; Jurgen Herbst,"American Higher Education in the Age of the College," *History of Universities* 7 (1988): 37-59, on p. 41. Witherspoon also hired Walter Minto, a Scot who had made astronomical observations in Italy while tutoring the two sons of a member of Parliament and who had co-written a biography of John Napier with the Earl of Buchan, as professor of mathematics and natural philosophy in 1787. Minto's influence was limited to New Jersey, as he passed away in 1796 without publishing a manuscript textbook of mathematics he had written. See Luther P. Eisenhart, "Minto, Walter," in *DAB* (cit. n. 2), vol. 13, p. 32.

College of New Jersey, Witherspoon typified the characteristics of Scottish education listed by Douglas Sloan: appreciation of learning, orientation toward utility and service to society, devotion to national progress, and emphasis on the connection between education and the Church.[8] In addition to graduating American Presbyterian ministers, Witherspoon produced political leaders inculcated in his own republican principles — Witherspoon himself was active in the American Revolution from its beginning as a signer of the Declaration of Independence. Leaders educated in Princeton also went on to preside over colleges across the South.[9]

Yet, New Jersey did not hold a monopoly on commonalties with Scotland. For example, Eliphalet Nott at Union and Francis Wayland at Brown were other college presidents imbued in the Scottish Common Sense philosophy. In addition, the historian Archie Turnbull has counted eighty-one men from throughout the colonies who attended the medical school at the University of Edinburgh before 1770.[10] One of the most notable of these students, Benjamin Rush, recruited Witherspoon to come to New Jersey and also later signed the Declaration of Independence. American doctors established schools based on the Edinburgh model, including the University of Pennsylvania medical school.[11] They also brought back aspects of the Scottish professorial system. Although American professors were never directly dependent on student fees, they were inspired to raise their own level of scholarship by the examples they observed. College

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[8] Sloan, *Scottish Enlightenment* (cit. n. 3), pp. 1-35.

[9] Petersen, "Scottish Common Sense" (cit. n. 3), pp. 47-65.

[10] Turnbull, "Scotland and America" (cit. n. 3), p. 141.

[11] Brunton, "Edinburgh and Philadelphia" (cit. n. 3).

educators followed Scottish universities and English dissenting academies in building close relationships with groups of students, and some gave public lectures to inform their communities about natural philosophy.[12] George Pryde has even attributed the origin of the widely-copied Harvard practice of defending theses at graduation to Edinburgh.[13]

People in Scotland and the colonies each also battled the provincial image Londoners held of them. For instance, Scottish and American natural philosophers sought to participate on an equal level in the Royal Society of London. Frank Freeman has noted that Americans contributed papers to *Philosophical Transactions* between 1753 and 1775 which were at an average level of science for the journal and which were not more practical than typical papers, even though their submissions comprised merely 3.8% of the total number of articles published in the journal.[14] Meanwhile, people in both the colonies and Scotland turned their agreement on the virtues of education for the middle class into admiration for each other and denigrated Cambridge and Oxford in the eighteenth century as havens for the idle rich.[15]

The American Revolution was only a temporary impediment to the close relationship which had developed between American colleges and Scotland. In fact, Andrew Hook argued that Scotland's influence further increased after the Revolution.[16] Americans continued to depend on Great Britain for consumer goods

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[12] Hook, *Scotland and America* (cit. n. 3), pp. 17-46.

[13] Pryde, *Scottish Universities* (cit. n. 3), pp. 3-5.

[14] Frank R. Freeman, "American Colonial Scientists Who Published in the *Philosophical Transactions* of the Royal Society," *Notes and Records of the Royal Society of London* 39 (1985): 191-206.

[15] Horner, "Scottish-American Connection" (cit. n. 3).

[16] Hook, *Scotland and America* (cit. n. 3), p. 72.

and books and established a British-looking political structure and culture. They learned about European developments, including French rationalism, through Scottish sources.[17] Indeed, news of the French Revolution arrived in slightly muddled fashion through hostile British newspapers, although Americans were more like the Scots than the English by welcoming the early developments. Simon Newman has reported that Americans celebrated French military victories in public festivals through 1800, until Democratic Republican leaders and then ordinary Americans experienced the decline of French-American relations.[18] Meanwhile, American professors and natural philosophers visited Scotland in the early nineteenth century, cementing the Scottish roles as purveyor of European culture and as proponent of both educational reform and the teaching of French research in mathematics and natural philosophy.

All of these influences were largely implicit at Yale when Jeremiah Day began his studies there in 1789 at age sixteen, which was then the typical matriculation age. In the mid-eighteenth century, the college had been especially active in astronomy under presidents Thomas Clap and Ezra Stiles. Stiles and Nehemiah Strong taught Newtonian natural philosophy, with Stiles making a careful study of a copy of the *Principia* which Newton himself had given to Yale. Yale students did not scatter during the American Revolution, although financial problems and disputes with the Yale Corporation left Stiles as the sole professor in New Haven in 1781. He

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[17] Dirk J. Struik, *Yankee Science in the Making* (Boston: Little, Brown, and Company, 1948), p. 336.

[18] Simon P. Newman, *Parades and the Politics of the Street: Festive Culture in the Early American Republic* (Philadelphia: University of Pennsylvania Press, 1997), pp. 120-151. On the high levels of literacy in both the elite and artisan classes in Revolutionary America, see E. Jennifer Monaghan, "Literacy Instruction and Gender in Colonial New England," *American Quarterly* 40 (1988): 18-41; and David Paul Nord, "A Republican Literature: A Study of Magazine Reading and Readers in Late Eighteenth-Century New York," *American Quarterly* 40 (1988): 42-64.

persevered through student disorders to reassemble the library, make the senior examinations more difficult, organize a literary society, begin to teach history as a subject, and build Union Hall before he died suddenly in 1795.[19]

Jeremiah Day and Yale

Jeremiah Day was born on August 3, 1773, in New Preston, Connecticut, where his father was pastor of the local Congregational church.[20] The senior Day, who had taught school himself between his graduation from Yale in 1756 and assumption of his parish in 1763, hired Nathan Hale's brother, David, and then John Kingsley to tutor Jeremiah. Presumably the boy learned Latin, Greek, and the elements of mathematics in order to enter Yale. He began the curriculum there in 1789, but he left in 1791 with a pulmonary ailment. After teaching school for two years, he returned to Yale and earned a bachelor's degree in 1795.

Day was a student during the years Stiles brought what Brooks Mather Kelley called the "university spirit" to Yale.[21] Stiles died shortly before Day's graduation, however, and Timothy Dwight was elected to replace him. Dwight was socially and politically conservative, and he was not particularly oriented toward science. He had to deal with the violent uprisings of students then common in American colleges, but he responded differently than Stiles had, by reinforcing the paternalistic system which was already in place for regimenting every aspect of the

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[19] Frederick E. Brasch, "The Newtonian Epoch in the American Colonies (1680-1783)," *Proceedings of the American Antiquarian Society,* new ser., 49 (1939): 314-332; L. W. McKeehan, *Yale Science: The First Hundred Years, 1701-1801* (New York: Henry Schuman, 1947); Brooks Mather Kelley, *Yale: A History* (New Haven and London: Yale University Press, 1974), pp. 91-112.

[20] Two of the major biographies of Day are "President Woolsey's Address at the Funeral of President Day, Commemorative of His Life and Services," *New Englander* 26 (1867): 692-724; and Harris Elwood Starr, "Day, Jeremiah," in *DAB* (cit. n. 2), vol. 5, pp. 161-162. See also "Jeremiah Day," *American Journal of Science* 94 (1867): 291-292.

[21] Kelley, *Yale* (cit. n. 19), p. 91.

students' lives. Day was one of the few students to find favor with Dwight, for Day

succeeded the older man as principal of the Greenfield Hill Academy Dwight had

also founded. Day held this position only a short time before taking an appointment

as tutor at Williams College.[22] Then, in 1798, Day became tutor of the freshman class

at Yale, leading the students through Euclid's *Elements*, along with John Ward's

Young Mathematician's Guide and American Nicholas Pike's 1788 *New and Complete*

System of Arithmetic, both of which were replaced by Samuel Webber's *Mathematics*,

another compendium, after 1801.[23]

Afflicted with tuberculosis during these years, Day was still able to be

licensed to preach in 1800. Later that year, Dwight forced out the professor of

mathematics and natural philosophy, Josiah Meigs (1757-1822), with whom he had

clashed repeatedly over issues such as Meigs's sympathy for the French

Revolution.[24] Dwight offered the vacant post to Day despite his lack of experience in

mathematics, but the younger man was by that time seriously ill and departed for

Bermuda after accepting the professorship.[25] Although Day's friends are said to

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[22] Kelley, *Yale* (cit. n. 19), pp. 115-139; Charles Henry Smith, "History of Yale University, 1700-1898," in *Universities and Their Sons*, ed. Joshua L. Chamberlain, vol. 1 (Boston: R. Herndon Company, 1898), p. 260; Daniel C. Gilman, "Reminiscences of President Day," *The Congregationalist*, 19 September 1867, held in Pamphlets, Clippings on President Day [Yb13 D4 1], Manuscripts and Archives, Yale University, New Haven.

[23] The version of Euclid's *Elements* was probably Robert Simson's *The Elements of Euclid*, although this is not certain. See Florian Cajori, *The Teaching and History of Mathematics in the United States* (Washington, DC: Government Printing Office, 1890), p. 63; Nicholas Pike, *New and Complete System of Arithmetic* (Newburyport, MA, 1788); John Ward, *The Young Mathematician's Guide, Being a Plain & Easie Introduction to the Mathematicks . . . with an Appendix of Practical Geometry* (London: J. T. & J. Woodward, 1707). Ward's textbook covered arithmetic, algebra, conic sections, the arithmetic of infinites, gauging, compound interest, and logarithms. One of the most popular English textbooks of the eighteenth century, it was published twelve times by 1771; *The National Union Catalog: Pre-1956 Imprints* (hereinafter cited *NUC*), vol. 244, (London: Mansell, 1976), p. 245.

[24] Chandos Michael Brown, *Benjamin Silliman: A Life in the Young Republic* (Princeton: Princeton University Press, 1989), pp. 62-99; McKeehan, *Yale Science* (cit. n. 19), pp. 55-58.

[25] Meigs apparently harbored no ill feelings toward Day over his departure from Yale, for the pair exchanged a number of friendly letters with scientific hints and advice between 1816 and 1821.

have feared he would never return—his condition worsened when doctors bled his arm, and Day returned the following April to his father's home to die—Benjamin Silliman (1779-1864), with whom Day had been a student and then a tutor, remained upbeat in his letters to Day.[26] Day did survive, experiencing something of a "miracle cure" when a doctor prescribed iron in 1803, and Silliman worked out arrangements to give the natural philosophy lectures in chemistry first in order to give Day time to finish preparing his botany and natural history experiments.[27] Day assumed his duties later that year.

As professor of mathematics and natural philosophy, Day met with the juniors once per week and the seniors twice per week for natural philosophy lectures and demonstrations. Silliman warned him early on to keep his naturally extemporaneous style and not fall prey to "modus Professoris."[28] Day also supervised the tutors who taught geometry and algebra to the freshmen and sophomores. Even

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Meigs even sighed that the death of Dwight "left a chasm in the Social State;" Josiah Meigs to Jeremiah Day, 24 June 1817, Day Papers: Letters to Jeremiah Day, Beinecke Rare Book & Manuscript Library, Yale University, New Haven. Of course, Meigs moved directly from Yale to become both president and professor of mathematics, natural philosophy, and chemistry at the University of Georgia, an institution he built up from literally nothing. (See "Meigs, Josiah," in NCAB (cit. n. 2), vol. 9, pp. 178-179.) By 1821, his memory of Yale was that he had "resigned" his mathematical professorship in 1800; Josiah Meigs to Jeremiah Day, 2 July 1821, Day Papers: Letters to Jeremiah Day.

[26] Silliman earned the Yale A.B. in 1796 and became a tutor in 1799. For Silliman's hopeful messages to Day, see Benjamin Silliman to Jeremiah Day, 9 January 1801 and 20 June 1802, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[27] The medical care administered to Day, including his demeanor as "like one raised from the dead" after the iron treatment, is described in S. G. Hubbard, Medical History of the Late President Day (New Haven, 1868), Pamphlets, Clippings on President Day (cit. n. 22). For Day's and Silliman's plans for the natural philosophy course, see Benjamin Silliman to Jeremiah Day, 29 January 1803, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[28] Benjamin Silliman to Jeremiah Day, 8 March 1804, Day Papers: Letters to Jeremiah Day (cit. n. 25). Silliman spent part of 1803 and 1804 in Philadelphia, attending lectures at the University of Pennsylvania Medical School. For biographies of Silliman, see Charles H. Warren, "Silliman, Benjamin," in DAB (cit. n. 2), vol. 17, pp. 160-163; Brown, Benjamin Silliman (cit. n. 24); and Leonard G. Wilson, ed., Benjamin Silliman and His Circle: Studies on the Influence of Benjamin Silliman on Science in America Prepared in Honor of Elizabeth H. Thomson (New York: Science History Publications, 1979).

though Day's interest in science and mathematics was previously untapped, he seems to have been stimulated by his professorship to expand his own knowledge. Perhaps parlaying his friendship with the increasingly prominent Silliman into contacts with other American natural philosophers in addition to commercial ventures such as the mineral water enterprise the pair entered into with Noyes Darling and Stephen Twining in 1809, Day exchanged latitude measurements with Nathaniel Bowditch (1773-1838) in 1810. He then served as one of Bowditch's collectors of observations for the eclipse on September 17, 1811.[29] The eclipse and a comet which also appeared in 1811 inspired two of the only four scientific papers ever published by Day. (See Table 3.1 for a full list of Day's publications.) His initial submission to the Connecticut Academy of Arts and Sciences, one of several local scientific societies which flourished briefly in the early nineteenth century, was the Baconian "Of the Quantity of Rain Which Falls, on Different Days of the Moon."[30] Day then put aside his rudimentary data collection to write about celestial phenomena. His second paper was an essay listing four different theories about meteors and the objections to each one, with Day ultimately deciding in favor of Thomas Clap's theory, that meteors broke off from comets when they approached the earth, even though Day admitted that this theory was not supported by proof

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[29] Nathaniel Bowditch to Jeremiah Day, 14 March 1810, 20 August 1811, 20 March 1812, Day Papers: Letters to Jeremiah Day (cit. n. 25). Bowditch used measurements taken during the eclipses of 1806 and 1811 to improve the accuracy of the longitude computations in his *The New American Practical Navigator*, 4th ed. (New York, 1817); John C. Greene, *American Science in the Age of Jefferson* (Ames: Iowa State University Press, 1984), p. 146.

[30] Jeremiah Day, "A Statement of the Quantity of Rain Which Falls, on Different Days of the Moon," *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 125-127. The Connecticut Academy was founded in 1799.

Table 3.1. Publications by Jeremiah Day.

"A Statement of the Quantity of Rain Which Falls, On Different Days of the Moon." *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 125-127.

"A View Of the Theories Which Have Been Proposed, To Explain the Origin of Meteoric Stones." *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 163-174.

"Observations On the Comet of 1811." *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 341-352.

With James L. Kingsley. "Calculation Of the Longitude of Yale College, From the Solar Eclipse of September 17th, 1811." *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 353-361.

An Introduction to Algebra, Being the First Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges. New Haven: Howe & Deforest, 1814. 67 printings by 1850. 23,000 more copies sold between 1852 and 1869.

A Treatise of Plane Trigonometry. To Which is Prefixed, A Summary View of the Nature and Use of Logarithms. Being the Second Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges. New Haven: Oliver Steele for Howe & Deforest, 1815. Circa 9 printings.

A Practical Application of the Principles of Geometry to the Mensuration of Superficies and Solids. Being the Third Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges. New Haven: Oliver Steele, [1816]. Circa 8 printings.

The Mathematical Principles of Navigation and Surveying. Being the Fourth Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges. New Haven: Steele & Gray, 1817. Circa 6 printings. 10 printings of Parts 2-4 between 1831 and 1858.

A Sermon, Delivered in Boston, Sept. 17, 1823, Before the American Board of Commissioners for Foreign Missions. Boston: Crocker and Brewster, 1823.

With James Luce Kingsley. "Original Papers in Relation to a Course of Liberal Education." *American Journal of Science* 15 (1829): 297-351.

The Christian Preacher's Commission. New Haven: Hezekiah Howe, 1831.

Table 3.1. (continued)

Declaring the Whole Counsel of God . . . Delivered . . . October 30th, 1833. New Haven: Baldwin & Peck, 1833.

An Inquiry Respecting the Self-Determining Power of the Will. New Haven: Herrick & Noyes, 1838.

"Contending for the Faith." *American National Preacher* 13 (1839).

An Examination of President Edward's Inquiry On the Freedom of the Will. New Haven: Durrie & Peck, 1841.

Key to Day's Mathematics. New Haven: Noyes, 1845.

"The Inaugurating Address." In *Discourses and Addresses at the Ordination of the Rev. Theodore Dwight Woolsey.* . . . New Haven, 1846. Pp. 55-72.

Psalms and Hymns, for Christian Use and Worship. New Haven: Durrie & Peck, 1859.

and observation.[31] Day also published a report containing his observations of the comet of 1811 and calculations based on those observations in order to attempt to determine the period of the comet.[32] He even made sure in this paper to comfort

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[31] Jeremiah Day, "A View of the Theories Which Have Been Proposed, To Explain the Origin of Meteoric Stones," *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 163-174. Day's paper was anonymously reviewed in *General Repository and Review* 3 (1813): 140-164. See also Greene, *American Science* (cit. n. 29), pp. 150-151; and Dorrit Hoffleit, *Astronomy at Yale, 1701-1968* (New Haven: Connecticut Academy of Arts and Sciences, 1992), pp. 12-16.

[32] Jeremiah Day, "Observations On the Comet of 1811," *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 341-352. One of the most widely-viewed comets ever, it was found by Honori Flaugergues on March 25, 1811, and named for Jean-Louis Pons, who spotted it three weeks later. The enormous head, which appeared larger than the sun for many of the ten months it was visible, and tail of the comet resulted in a vast amount of popular mythology, including Russian peasants' association of the comet with Napoleon, as Leo Tolstoy incorporated into *War and Peace*. See, for example, David H. Levy, *The Quest for Comets: An Explosive Trail of Beauty and Danger* (New York and London: Plenum Press, 1994), pp. 53-54; Carl Sagan and Ann Druyan, *Comet* (New York: Random House, 1985), pp. 136, 372; and Peter Lancaster Brown, *Halley's Comet & the Principia* (Aldeburgh, Suffolk, England: Aries Press, 1986), pp. 153-154, 167.

readers that the comet could never hit the earth. Finally, Day joined with the Yale

Professor of Hebrew, Greek, and Latin, James Luce Kingsley (1778-1852), to use the

1811 eclipse to calculate Yale's longitude, a method considered more accurate than

determining longitude by the motion of the moon.[33]

Perhaps Day was encouraged to rethink his expectations for the classroom by

enthusiastic students, as well. His most notable protégé during this period may have

been Theodore Strong (1790-1869). An 1812 Yale graduate, Strong became one of the

most prolific contributors to the short-lived American mathematical journals of the

early nineteenth century and to Silliman's own *American Journal of Science*, which

began publication in 1819.[34] Strong submitted the first of these papers, in which he

solved six propositions from the article "Circle" in Abraham Rees's *Cyclopaidea* with

algebraic formulas, to Day, saying, "Since I know you to be a lover of truth and

scientific investigation and as I have good reason to believe that you are friendly to

me I will with your permission submit to your examination whatever propositions I

may have the ability to discover or to demonstrate in a different manner from what

others have done."[35] Although Strong learned no Continental mathematics from

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[33] Jeremiah Day and James L. Kingsley, "Calculation Of the Longitude of Yale College, from the Solar Eclipse of September 17th, 1811," *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 353-361. Kingsley was apparently not related to John Kingsley, Day's boyhood tutor.

[34] For biographies of Strong, see Edward R. Hogan, "Theodore Strong and Ante-Bellum American Mathematics," *Historia Mathematica* 8 (1981): 439-455; and Raymond Clare Archibald, "Strong, Theodore," in *DAB* (cit. n. 2), vol. 18, p. 152. On Silliman's efforts to establish a national readership for the *American Journal of Science*, see Simon Baatz, "'Squinting at Silliman': Scientific Periodicals in the Early American Republic, 1810-1833," *Isis* 82 (1991): 223-244.

[35] Theodore Strong to Jeremiah Day, 15 November 1813, Day Papers: Letters to Jeremiah Day (cit. n. 25). At the time, Strong was tutor in mathematics at the recently organized Hamilton College. After Day made corrections, the paper appeared as Theodore Strong, "Demonstrations of Stewart's Propositions of the Circle," *Memoirs of the Connecticut Academy of Arts and Sciences* 1 (1810-1816): 393-412. See Theodore Strong to Jeremiah Day, 26 August 1816, Day Papers: Letters to Jeremiah Day (cit. n. 25). Rees had revised Ephraim Chambers's well-known eighteenth-century encyclopedia into Abraham Rees, ed., *Cyclopaidea: or, An Universal Dictionary of Arts and Sciences*, Ephraim Chambers, rev. ed., 4 vols. (London, 1786).

Day as a Yale student, Day had purchased some of Lagrange's works by 1813.[36]

Strong and Day were soon each involved in the study of contemporary mathematics

and astronomy, with Strong reporting in 1816 that he had purchased one hundred

European books, including works by Delambre, Laplace, Lacroix, Lagrange, and

Gauss, as well as Legendre's 1793 *Mémoire sur les transcendantes elliptiques*, for

Hamilton College, where he had just been named Professor of Mathematics and

Natural Philosophy.[37]

While Strong went on to become known in his day for introducing

Continental mathematics in the United States and for practicing an erudite form of

mathematics, Day directed his dissatisfaction with the current state of Yale

mathematics toward the preparation of a series of mathematical textbooks.[38] *An*

Introduction to Algebra, A Treatise of Plane Trigonometry, A Practical Application of the

Principles of Geometry to the Mensuration of Superficies and Solids, and *The Mathematical*

Principles of Navigation and Surveying all appeared after 1813 but before Day's life

took a surprising turn in 1817.[39] Timothy Dwight died, and Henry Davis, the

president of Middlebury College who was elected to succeed him, declined the

position.[40] The Yale Corporation then turned to Day. Although he proved

reluctantly modest at first, the man no one thought would survive long enough to

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[36] Jeremiah Day, Account Book, 1812-1814, Day Family Papers (cit. n. 1), Box 19. Note that, thus, at least some books from Europe made it into the United States before the end of the War of 1812.

[37] Theodore Strong to Jeremiah Day, 26 August 1816, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[38] See Hogan, "Theodore Strong" (cit. n. 34); and Cajori, *Teaching and History* (cit. n. 23), p. 398. The earliest notes for *An Introduction to Algebra* in Day's papers are dated 27 July 1812; Day Family Papers (cit. n. 1), Boxes 23-24.

[39] "President Woolsey's Address" (cit. n. 20), p. 700.

[40] Brown, *Benjamin Silliman* (cit. n. 24), p. 302.

take up a professorship was installed as president of Yale and ordained as a minister on July 23. Isaac Lewis, who gave the sermon at Day's ordination, exhorted him to be a "Paul" to the "Timothys" studying under his direction. Day was to diligently study both the Bible and the "vast field of science," all while comporting himself with godliness, dignity, and perseverance. Lewis reminded his audience that taking responsibility for the education of youths would be work just as interesting, important, extensive, and arduous for Day or anyone as the labors of a pastor.[41]

While most of Day's time was spent engaged in a number of other, more mundane matters related to his presidential responsibilities, two of the most notable events of Day's presidency were his roles in the preparation of the Yale Report and in the Conic Sections Revolt.[42] In 1827, Noyes Darling, the participant in the mineral water venture who was by then a Senator and member of the Yale Corporation, took his former partners by surprise by moving to eliminate classical languages from the curriculum.[43] A committee was appointed in September to prepare a thorough study of the Yale course of instruction, which resulted in the famous "Yale Report," researched and written between April and August, 1828, by Day and Kingsley on behalf of the entire faculty.[44] The paper was read by the committee in August,

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[41] Isaac Lewis, *A Sermon . . . at the Ordination of the Rev. Jeremiah Day . . . July 23, 1817* (New Haven, 1817).

[42] See Day Family Papers (cit. n. 1), Boxes 50-52; and Day Papers: Letters to Jeremiah Day (cit. n. 25) for examples of correspondence written or received by Day while carrying out his duties. For example, he dealt with students trying to avoid the examinations held twice per year, collected tuition from tardy students and parents, raised funds for the expansion of Yale's buildings and faculty, and received resignations from the Yale Corporation. On Day's activities during his presidency, see also Kelley, *Yale* (cit. n. 19), pp. 140-170, and Smith, "History of Yale" (cit. n. 22), pp. 260-266.

[43] Kelley, *Yale* (cit. n. 19), pp. 160-170.

[44] Kelley, *Yale* (cit. n. 19), pp. 140-170.

accepted by the Yale Corporation in September, and subsequently published in *American Journal of Science* and as a separate pamphlet.[45]

Day's responsibility was the first part of the Report, a summary of the plan of education at Yale.[46] To Day, there were two keys to the character of Yale's course: paternalism and thoroughness. First, the students entered Yale young enough to require "that a substitute be provided for *parental superintendence*."[47] College governance should be organized as a family to guide the students' steps through "kind and persuasive influence" and to punish them only when absolutely necessary. The faculty even took meals with the students in college buildings to help ensure that discipline was learned in every aspect of daily life.[48] Because American students needed this parental control, Day argued that it would be ill-advised to copy German universities, where the students were older and already prepared to embark upon professional studies.[49] Second, the course of instruction had to be thorough in order to expand the powers of the mind and fill it with knowledge. As Day even wrote in capital letters, the purpose of an American college such as Yale was to "LAY THE FOUNDATION OF A SUPERIOR EDUCATION."[50] To lay this foundation, all the mental faculties had to be trained: reasoning, imagination, taste, decision-

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[45] "Original Papers in Relation to a Course of Liberal Education,"*American Journal of Science* 15 (1829): 297-351; also published as *Reports on the Course of Instruction in Yale College* (New Haven: Hezekiah Howe, 1830). In his capacity as editor of *American Journal of Science*, Silliman informed readers of the history of the Report on pp. 297-298.

[46] "Original Papers" (cit. n. 45), pp. 298-324.

[47] "Original Papers" (cit. n. 45), p. 303; emphasis in source. When Day was a student, the average age of entering students was sixteen. By the 1820s, freshmen were significantly younger, so it is not too surprising that Day would feel they needed parental authority.

[48] "Original Papers" (cit. n. 45), pp. 303, 306-307.

[49] "Original Papers" (cit. n. 45), pp. 315-316.

[50] "Original Papers" (cit. n. 45), p. 300; emphasis in source.

making, memorization, invention, and communication. To exercise and develop all these skills, the ideal college course contained a balance between literary and scientific subjects and between theory and practice.[51]

A number of lesser themes were introduced along with Day's emphases on paternalism and thoroughness. For example, he repeatedly stressed that a college education was merely preparatory — graduates of Yale had learned how to learn and could continue to teach themselves throughout life or enter a professional school to master medicine, theology, or the law.[52] The actual college courses were balanced between lectures and demonstrations by professors and daily recitations, where students were examined by tutors over their active mastery of a textbook.[53] Day stressed that the material taught at Yale changed constantly albeit gradually; he named chemistry, mineralogy, geology, and political economy as recent additions to the curriculum.[54] Day also made it clear that his statements about the state of education were to apply only to Yale and that other colleges were free to choose their own methods, although he was also firmly convinced that other institutions would serve their students and nation well by requiring a similarly thorough course.[55] Finally, Day advocated democratic accessibility to education but without any loss of the high standards of liberal education and not at the cost of converting the colleges into academies.[56] Even more than a reflection of the Scottish

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[51] "Original Papers" (cit. n. 45), pp. 301-302, 311-312.

[52] Among other mentions of this, see "Original Papers" (cit. n. 45), pp. 308-309, 313. Day improved Yale's medical school, which had been founded in 1813, established the divinity school, and partnered with a local law school run by three Yale graduates during his presidency.

[53] "Original Papers" (cit. n. 45), p. 304.

[54] "Original Papers" (cit. n. 45), p. 299.

[55] "Original Papers" (cit. n. 45), p. 321.

[56] "Original Papers" (cit. n. 45), pp. 317-318, 321-324.

commitment to democratic education Day had read about in the *Edinburgh Review*,

one of the sources he relied upon in writing the Report, his comments were an

expression of a patriotic desire to train the nation's businessmen to manage their

growing resources wisely.[57]

Yet, the Yale Report gained a reputation in many quarters as a reactionary

document.[58] Through its publication in the *American Journal of Science*, the Report

was disseminated widely enough (from the first issues published in 1819, the journal

sold more than 1000 copies per issue) that perhaps a misinterpretation of it was

inevitable and was certainly not helped by Day's own publicly increased

conservatism in his old age.[59] Still, as scholars who have recently reevaluated the

Report have found, it truly was a moderating influence that showed colleges how to

uphold both tradition and innovation.[60] Day made one of the first efforts to define

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[57] The other references Day named in his notes were the American periodicals, *Journal of Education* and *Christian Spectator*, and the Harvard electives advocate, George Ticknor; Jeremiah Day, Outline of Report on Course of Instruction, Miscellaneous Undated File, Day Family Papers (cit. n. 1), Box 29.

[58] See Hogan, "Theodore Strong" (cit. n. 34); and Herbst, "American Higher Education" (cit. n. 7). Even before the Report, it was apparently a common charge that Yale had not changed since it opened in 1701, for Day and Kingsley both took pains to respond to the contrary. One example is "Original Papers" (cit. n. 45), pp. 337-340. See also Jeremiah Day, "The Inaugural Address," in *Discourses and Addresses at the Ordination of the Rev. Theodore Dwight Woolsey, LL.D. to the Ministry of the Gospel, and His Inauguration as President of Yale College, October 21, 1846* (New Haven: B. L. Hamlen, 1846), pp. 55-72, on p. 69. For a historian of education who agreed that the Report contained no new ideas and prevented any change in the college curriculum until after the Civil War, see Frederick Rudolph, *The American College and University: A History* (New York: A. Knopf, 1962; reprint, intro. John R. Thelin, Athens and London: The University of Georgia Press, 1990), pp. 110-135; and Frederick Rudolph, *Curriculum: A History of the American Undergraduate Course of Study Since 1636* (San Francisco: Jossey-Bass Publishers, 1977), pp. 69-83.

[59] Stanley M. Guralnick, "Sources of Misconception on the Role of Science in the Nineteenth-Century American College," *Isis* 65 (1974): 352-366, on p. 353; Brown, *Benjamin Silliman* (cit. n. 24), pp. 300-310; Kelley, *Yale* (cit. n. 19), p. 170.

[60] In addition to Guralnick, "Sources of Misconception" (cit. n. 59); see Stanley M. Guralnick, *Science and the Antebellum American College* (Philadelphia: American Philosophical Society, 1975), pp. 18-46; Herbst, "American Higher Education" (cit. n. 7); and Lawrence A. Cremin, *American Education: The National Experience 1783-1876* (New York: Harper & Row, 1980), pp. 253-297.

an American philosophy of education and presided over a curriculum which incorporated more and more science alongside the classical languages.

The other most public event during Day's tenure was the Conic Sections Revolt of 1830.[61] Student resentment of the increased role in the administration that Day had delegated to the faculty of the college, combined with warm weather and challenging lessons, pushed the sophomores too far that summer. Under usual recitation practice, tutors required the students to each demonstrate one of the propositions assigned the day before, guided only by the accompanying diagram posted on the blackboard. The sophomores of 1830 submitted a petition on July 20 demanding that they be allowed to use their textbook for help as well. After their tutor informed them the next day that the faculty had refused their request, eight or nine students refused to recite from the diagram. The tutor dismissed the class in the face of this resistance, and the class petitioned the faculty again, asking that the lessons be shortened. Day noted, though, that the course of conic sections was not longer than in previous years as the students claimed; the problem seemed to be that spherical trigonometry had been studied before conic sections in 1830 instead of after it, which was the traditional practice at Yale. Thus, feeling that the students had no valid complaint, the faculty decided to ask the sophomores to promise "to recite Conic Sections, in the manner prescribed by our instructors."[62] Meanwhile, however, a group of the students submitted three more petitions reiterating their refusal to follow instructions, so the faculty decided to expel the forty-three sophomores who continued to resist. The remaining students bowed to Yale's parental authority and

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[61] Although Kelley, *Yale* (cit. n. 19), pp. 160-170; and Smith, "History of Yale" (cit. n. 22), pp. 260-266, both provide accounts of the rebellion, this narrative is also based upon a rough draft of Day's report on the matter, which is preserved on the reverse side of his notes on theological readings. Jeremiah Day, Theological Notes, 1837, Day Family Papers (cit. n. 1), Box 30.

[62] Jeremiah Day, Theological Notes, 1837, Day Family Papers (cit. n. 1), Box 30.

returned to memorizing their proofs. By the following January, though, the faculty still had not been able to craft a satisfactory document for the expelled students to present when they attempted to gain admittance to other colleges.[63] To Day's contemporaries, the event exemplified the constant danger that students would rebel against the college's parental authority. As well, it demonstrated the perpetual student frustration with the recitation mode of instruction and the difficulties faced by the generally young and inexperienced tutors expected to lead their charges through the material.[64] Finally, the Conic Sections Revolt was significant as an example of Day's beliefs about education being put into practice.

Day's mathematics series was completed during his presidency. The fifth and sixth parts of the course were published in one volume in 1824 as *An Elementary Treatise on Conic Sections, Spherical Geometry, and Spherical Trigonometry*. While Day was listed as the editor, he apparently contributed little, if anything, to the content of the text. In the first place, Day had no longer wanted to write on conic sections after preparing the first four books in the 1810s. He then waited unsuccessfully for Harvard professor John Farrar to publish a revision of the mathematics textbook by Samuel Webber and from which Day and his tutors continued to teach portions of the mathematics course, including conic sections. In 1817, Farrar wrote Day that printing of "Conic Sections etc." had not yet begun and would probably take more

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[63] Jeremiah Day to Unidentified Parent of Dismissed Student, 27 January 1831, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[64] The tutors' struggles were exacerbated by the fact that Yale tutors were assigned by class rather than by subject, requiring them to teach to both their strengths and weaknesses. That finally changed late in 1830 on the suggestion from one of the tutors, Frederick A. P. Barnard, who later became president of Columbia around the time that Charles Davies became an emeritus professor there. See William J. Chute, *Damn Yankee! The First Career of Frederick A. P. Barnard: Educator, Scientist, Idealist* (Port Washington, NY: National University Publishers, 1978), pp. 37-39.

than a year.[65] By March 1818, the bookseller Francis Nichols was warning Day, "If

you wait for Conic Sections, and mathematical philosophy, you will be disappointed

for some years. Mr. Farrar has not examined books on those subjects, and has not

fixed on any."[66] Indeed, Farrar never completed nor printed his revision. Secondly,

there are no rough drafts of these texts in Day's papers at Yale.[67] Finally and most

obviously, Matthew Rice Dutton was named as the primary author on the title page.

Day had hired Alexander Metcalf Fisher as adjunct professor of mathematics when

he became president of Yale, although Day continued to give the natural philosophy

lectures for two more years.[68] After Fisher died in a boating accident near Ireland in

1822, Dutton succeeded him as professor of mathematics and natural philosophy.[69]

Dutton unfortunately then passed away himself in 1825. His replacement was

Denison Olmsted.[70]

The later years of Day's tenure passed relatively peacefully. He weathered

financial problems in the middle of his term to establish a $100,000 fund raising

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[65] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 25). Farrar ultimately abandoned this project and translated a series of mathematical textbooks instead. See Chapter Four.

[66] Francis Nichols to Jeremiah Day, 23 March 1818, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[67] Admittedly, that may not mean they do not exist somewhere. See (cit. n. 89).

[68] P. F. Smith, "The Department of Mathematics," in A Century of Mathematics in America, ed. Peter Duren, vol. 2 (Providence, RI: American Mathematical Society, 1989), pp. 121-126; Brown, Benjamin Silliman (cit. n. 24), p. 300.

[69] Smith, "History of Yale" (cit. n. 22), pp. 260-266; Alexander Metcalf Fisher to Jeremiah Day, 31 March 1822, and General Spring to Jeremiah Day, 28 May 1822, Day Family Papers (cit. n. 1), Box 50.

[70] Jeremiah Day to Congregational Church, undated draft, Day Papers: Letters to Jeremiah Day (cit. n. 25).

campaign in the early 1830s.[71] Day also expanded the number of Yale's buildings to seventeen and increased the number of students to ninety per class.[72] He hired a total of thirty-seven new professors and created a variety of new departments. During these years, Day additionally published several theological treatises: *The Christian Preacher's Commission* in 1831, *An Inquiry Respecting the Self-Determining Power of the Will* in 1838, and *An Examination of President Edward's Inquiry on the Freedom of the Will* in 1841. Feeling that it was in the College's best interests, Day retired in 1846 at age seventy-three. He chose Theodore D. Woolsey, whom he had hired as Professor of Greek in 1831, as his successor and then delivered the address at Woolsey's inauguration. Day's friends and faculty were not ready to see him leave, however, and they convinced him to serve on the Yale Corporation until his death in 1867. Despite chronically poor health and one frightening episode in 1836 with his permanently irregular heartbeat, Day was never in serious danger of death again until the last year of his life.[73] Sadly, though, Day had rather poor luck with women, as nearly all those in his life died at a young age. His first wife, Martha Sherman, lived only one year after their marriage in 1805. Their son became a surveyor and may not have remained in close contact with Day.[74] Day married again in 1811. He and Olivia Jones had three daughters, all of whom died as young adults; Olivia also passed away before Day retired.[75] After he resigned the Yale presidency,

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[71] Silliman often solicited funds for Yale on his travels. See, for instance, Benjamin Silliman to Jeremiah Day, 31 May 1831 and 1 June 1831, Day Papers: Letters to Jeremiah Day (cit. n. 25). See also letters documenting the efforts of Wyllys Warner and Seth Bliss between 1830 and 1832, Day Family Papers (cit. n. 1), Box 51.

[72] Smith, "History of Yale" (cit. n. 22), pp. 260-266.

[73] Gilman, "Reminiscences" (cit. n. 22).

[74] See Jeremiah Day to Unidentified, 9 May 1853, Day Papers: Letters to Jeremiah Day (cit. n. 25); where Day said he was not involved in his son's mercantile relations.

[75] " President Woolsey's Address" (cit. n. 20).

the twice-widowed Day moved in with his son-in-law, Thomas Anthony Thacher, and Thacher's five children.

The Preparation of *Mensuration*

At Yale, as at other American colleges, the mathematics and natural philosophy course was expanded in the late eighteenth century. By about 1800, freshmen studied arithmetic and algebra; sophomores learned Euclidean geometry and trigonometry; juniors turned to natural philosophy, astronomy, and fluxions; and seniors delved more deeply into natural philosophy and astronomy. For as long as fifty years in the eighteenth century, Yale tutors presented the material from the compendium by the English mathematician, John Ward.[76] The usual practice in American colleges was for tutors to assign pages from the textbook to each group within a class year, perhaps explain the material to students, and then require each student to recite a portion of the assignment in the following day's meeting. When a compendium printed in the United States by an American author, Samuel Webber, became available in 1801, the Yale Corporation replaced Ward's textbook with Webber's, even though it was also taken from eighteenth-century English sources.[77]

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[76] Cajori, *Teaching and History* (cit. n. 23), pp. 32, 63. Available records and secondary sources are vague, but it is likely that versions of Euclid were taught alongside the compendium throughout the eighteenth century. By late in the century, the chosen textbook would have been Robert Simson's *The Elements of Euclid*.

[77] Samuel Webber, *Mathematics, Compiled from the Best Authors*, 2 vols. (Boston: Thomas & Andrews, 1801). Although no one has sorted out what appears to have been a hodgepodge of English textbook influences on Webber, Lao Genevra Simons raised and Helena Pycior repeated the claim that Webber was only a copying compiler in Lao Genevra Simons, *Introduction of Algebra into American Schools in the Eighteenth Century*, Bureau of Education Bulletin No. 18 (Washington, DC: Government Printing Office, 1924), pp. 75-76; Lao Genevra Simons, *Bibliography of Early American Textbooks on Algebra*, Scripta Mathematica Studies No. 1 (New York: Scripta Mathematica, 1936), pp. 6-10; and Helena M. Pycior, "British Synthetic Vs. French Analytic Styles of Algebra in the Early American Republic," in *The History of Modern Mathematics*, ed. David E. Rowe and John McCleary, vol. 1 (San Diego: Academic Press, Inc., 1989), pp. 125-154, on p. 126. For example, Webber used the chapter divisions from Charles Hutton's *A Course of Mathematics*, published in London between 1798 and 1801, and he selected propositions for the chapter on geometry from Hutton's section on the measurement of lines and angles in his 1767-1770 *A Treatise on Mensuration*.

Separate books by the English authors William Enfield and Samuel Vince were also put into use in the natural philosophy and astronomy and fluxions sections. In the meantime, Kingsley, who graduated from Yale in 1799 and served as a tutor before he was appointed to a professorship, introduced Playfair's *Elements of Geometry*. It is unclear how much institutional support he received for this move, since Dwight, Yale's conservative president, is said to have preferred a return to Ward's textbook for all mathematics subjects.[78]

Still, the concept of separate textbooks for separate topics within a larger field continued to take hold in the Yale curriculum. When he became the professor of mathematics and natural philosophy, Day seems to have taken responsibility for procuring the various textbooks. For example, to supply the sophomores, Day placed an order with Francis Nichols, a Philadelphia printer who published the first American edition of Playfair's *Elements* in 1806.[79] The first shipment to Yale was delayed "[b]y reason of the bad faith of an engraver," but Nichols offered to ship the Books I through VI at once in boards and then send the remaining portions when they were ready for a projected total price of $1.75 to $2.00 per volume. He apparently believed Day was not fully convinced of the importance of Playfair's textbook, for he added, "The 2d edition of Playfair's Euclid is far superior to

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[78] And probably Simson's *The Elements of Euclid*, as well. Kelley, *Yale* (cit. n. 19), pp. 115-139.

[79] It is not clear whether Nichols's papers survive anywhere to explain why he decided to go to the effort of setting up printing plates rather than continuing to import copies of Playfair's textbook. However, in addition to whatever incentive was provided by the non-existent international copyright laws, Nichols may have found it impossible to receive shipments from Scotland during an 1806 Congressional boycott of British goods under the Non-Importation Act. See John Mack Faragher, et al, *Out of Many: A History of the American People*, 2d ed. (Upper Saddle River, NJ: Prentice-Hall, 1997), p. 253. In any event, Nichols started the long tradition of Playfair's *Elements* as an American geometry textbook which was first introduced in Chapter One.

Simson's Euclid, & may be read by a student in less time & with less labour."[80] Like

British readers, Nichols appreciated the clarifications made by Playfair in 1804 and

was willing to substitute the more readily understood, albeit reorganized,

supplement for Simson's Books XI and XII. Nichols also shared the view that

mathematics education ought to be synthetical; that is, that all the material should be

presented to students in complete and perfect form, as Playfair's *Elements* was

considered to do. Day continued to purchase Playfair's *Elements* and other

mathematics and natural philosophy textbooks from Nichols through at least 1818.

But by the early 1810s, Webber's *Mathematics* was increasingly hard to find,

while Day's increasing expertise had made him discontented with the low quality of

the textbook's content. With American shipping in difficulty due to the Napoleonic

Wars and then the War of 1812, importing quantities of different textbooks from

Europe was not an option. Day decided the solution was to prepare his own

mathematical material, and he took the next logical step, given that individual books

for Euclidean geometry and fluxions were already in place in the Yale curriculum.

From the beginning of his project, Day had a mathematical series in mind — the first

American author to conceive of separate textbooks to undergird college

mathematics education.[81] In the advertisement to the first book, the 1814 *An

Introduction to Algebra*, Day informed readers that he expected the course to

comprise two or three volumes containing algebra, plane trigonometry, the

mensuration of superficies and solids, navigation and surveying, conic sections,

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[80] Francis Nichols to Jeremiah Day, 10 March 1806, Day Papers: Letters to Jeremiah Day (cit. n. 25). See also Francis Nichols to Jeremiah Day, 26 May 1807 and 3 October 1818, in the same location.

[81] By 1814, Day was also involved in an "Academic Convention" which evaluated the textbooks used in colleges and urged for more uniformity between different institutions. See John T. Kirkland to Jeremiah Day, 8 May 1814, Day Papers: Letters to Jeremiah Day (cit. n. 25).

spherical geometry and trigonometry, and fluxions.[82] Day's series would eventually be six parts in five volumes of one hundred to two hundred pages each, with no work on fluxions or the differential and integral calculus.

An Introduction to Algebra, printed by a local printer and distributed by a local bookseller, Howe & Deforest, was immediately popular. Although Strong was tepid at best in his praise for *An Introduction to Algebra*, writing to Day, "You desire me to give my opinion (without reserve) concerning your treatise on Algebra, but I wish to be excused, for it is by no means my desire to presume to criticise your words," other colleges and academies rapidly adopted Day's series.[83] Howe & Deforest conducted an active trade in the textbook, often sending one or two hundred copies of the algebra to other booksellers at a time.[84] The algebra was also picked up by another publisher, John Wiley, to become the first of many mathematics books to issue from the New York-based Wiley & Sons. Perhaps this version was the improved copy on "whiter" paper that Day sent to Frederick Hall, professor at Middlebury College.[85] In a time before stereotyping was adopted in the United States and when the type for most American books was therefore broken up after printing because the pieces were needed for other books, Day's algebra was so

[82] Advertisement to Jeremiah Day, *An Introduction to Algebra, Being the First Part of a Course of Mathematics, Adapted to theMethod of Instruction in the American Colleges* (New Haven: Howe & Deforest, 1814). For a discussion of the content of Day's printed algebra, see Pycior, "British Synthetic" (cit. n. 77), pp. 126-129.

[83] Theodore Strong to Jeremiah Day, 23 July 1815, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[84] Hezekiah Howe & Co. Letter Books, 1816-1818, Beinecke Rare Book & Manuscript Library, Yale University, New Haven.

[85] Jeremiah Day to Frederick Hall, 31 July 1815, Yale Miscellaneous Manuscripts, MS 1258, Box 6, Folder 216, Manuscripts and Archives, Yale University Library, New Haven.

valued that it appeared in sixty-seven editions by 1852.[86] Meanwhile, Day continued

directly on to writing the trigonometry textbook, which also sold well.[87] His success

as an author additionally helped result in his election to the American Academy of

Arts and Sciences in 1815.[88]

By the time Day next began to make notes for *Mensuration* on June 1, 1815, he

had established a standard writing process for his textbooks.[89] He would jot down

necessary problems as they occurred to him and then go back in later months to fill

in proofs and additional explanations, as he did with *Mensuration* from December

1815 to January 1816. He would then copy these sections into the next draft, crossing

out the older version. Day often said what he wanted in his first effort, though. Day

also listed the topics he wanted to address in the margins of his notebooks, marking

out items after he had written about them. Indeed, by January and February 1816,

[86] H. B. Phillips, Guilford L. Spencer, and Dick Wick Hall, "Numbers: Books on Mathematics," in *The First One Hundred and Fifty Years: A History of John Wiley and Sons, Incorporated 1807-1957* (New York: John Wiley & Sons, Inc., 1957), pp. 69-74, on pp. 69-70; Henry Walcott Boynton, *Annals of American Bookselling 1638-1850* (New York: John Wiley & Sons, 1932), pp. 138-151; *NUC* (cit. n. 23), vol. 135, pp. 404-409. In 1852, Day and Anthony D. Stanley published an alteration of *An Introduction to Algebra* which sold an additional 23,000 copies by 1869. In an apparently separate venture, James Bates Thomson prepared an abridgement which appeared in twelve editions between 1843 and 1849.

[87] Jeremiah Day, *A Treatise of Plane Trigonometry. To Which Is Prefixed, A Summary View of the Nature and Use of Logarithms. Being the Second Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges* (New Haven: Oliver Steele for Howe & Deforest, 1815).

[88] Josiah Quincy to Jeremiah Day, 9 September 1815, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[89] Jeremiah Day, Notes on Mensuration, Day Family Papers (cit. n. 1), Boxes 28-29. Please note that this manuscript is misfiled at Yale with the notes for *The Mathematical Principles of Navigation and Surveying*, the fourth part of Day's course, which was published in 1817. A few additional notes are tucked within a "Miscellaneous Undated" folder in Box 29. In addition, "1811" is incorrectly printed on the title page of Jeremiah Day, *A Practical Application of the Principles of Geometry to the Mensuration of Superficies and Solids. Being the Third Part of a Course of Mathematics, Adapted to the Method of Instruction in the American Colleges* (New Haven: Oliver Steele, [1816]); and is often given by bibliographers and historians as the book's year of publication despite the obvious logical inconsistency this date implies of the third part of the course appearing before the first part, *An Introduction to Algebra* (1814).

although the numbered sections appear out of order in his notes, the content of these sections was complete and each paragraph was printed unchanged from Day's manuscript.[90] As could be surmised from the brief eight-month preparation of *Mensuration*, Day generally wrote in a hurry, omitting words and sometimes employing a shorthand illegible to others.[91] At other times, there are different handwritings in Day's notes, indicating that the "weak eyes" mentioned by Farrar led Day to employ secretarial assistants.[92]

Whoever actually put the words on paper, however, Day drew upon a wide variety of sources in the preparation of all his textbooks, including *Mensuration*.[93] Over the course of his professorship, Day recorded reading several of the eighteenth-century compendia still in common use in colleges as one-size-fits-all lesson books, by authors including Charles Hutton, John Bonnycastle, and John Ward.[94] He referred to dictionaries by Rees and Hutton and to Scottish authors: Playfair, Black, Reid, and Maclaurin. Day also mastered subject textbooks. For example, while preparing *Mensuration*, Day consulted existing mensuration

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[90] The diagrams, however, were revised considerably before *Mensuration* was published, perhaps in order to meet the considerations of printing. Compare Jeremiah Day, Notes on Mensuration, Day Family Papers (cit. n. 1), Boxes 28-29, to Day, *Mensuration* (cit. n. 89).

[91] An anonymous archivist's note in another box of the Day Family Papers identifies the shorthand as John Byrom's system, which was published in 1767. See Jeremiah Day, Diary 1797-1801, Day Family Papers (cit. n. 1), Box 19. On Byrom and his system, which was considered elegant in appearance but impossible to write rapidly and therefore not favored by professional stenographers, see Leslie Stephen, "Byrom, John," in *Dictionary of National Biography* (hereinafter cited *DNB*), ed. George Smith, vol. 3 (London: Oxford University Press, 1885-1901), pp. 581-584.

[92] John Farrar to Jeremiah Day, 15 April 1817, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[93] These are listed in Day's manuscript notes (cit. n. 89) and in footnotes in the printed text (cit. n. 89). At the beginning of Sections 2, 4, and 5, Day additionally directed readers to some of these works for more complete demonstrations of the material presented in those sections.

[94] In addition to Day's textbooks, consult the references he listed in the notes for his natural philosophy lectures, Day Family Papers (cit. n. 1), Boxes 20-22.

textbooks by Hutton and Bonnycastle and a trigonometry textbook by Samuel Horsley, as well as an older practical geometry by Samuel Hawney and Bowditch's manual for navigation. For Euclidean geometry, Day read Playfair's *Elements* and Legendre's *Éléments*.

In *Mensuration*'s advertisement, printed unchanged from the draft Day wrote around February 20, 1816, Day stated his purpose as "little more, than an application of the principles of Geometry, to the numerical calculation of the superficial and solid contents of such figures as are treated of in the Elements of Euclid."[95] Indeed, the book's five sections dealt with the areas of figures bounded by right lines, the quadrature of the circle and its parts, solids bounded by plane surfaces, the three round bodies (cylinder, cone, and sphere), and isoperimetry. In each section, Day explained the process for finding various areas and volumes, often providing the reader with a calculating factor he arrived at by combining Euclidean relationships, such as the fact that a sphere is two-thirds of the circumscribing cylinder, with a numerical approximation for π.[96] He justified his arguments with propositions from Playfair's *Elements* and principles from his own *An Introduction to Algebra* and *Treatise of Plane Trigonometry*. Day also included "promiscuous problems," word problems applying the rules of measurement presented in the text. He ended the book with an appendix on measuring the conic sections and the gauging of casks.[97] A table of the segments of the circle and the diagrams were placed at the end of *Mensuration*. Day never revised the textbook.

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[95] Advertisement to Day, *Mensuration* (cit. n. 89). See also Jeremiah Day, Notebook C, Notes on Mensuration, Day Family Papers (cit. n. 1), Box 29.

[96] This relationship is in Proposition 21 of Supplement III in John Playfair, *Elements of Geometry, Containing the First Six Books of Euclid, With a Supplement on the Quadrature of the Circle and the Geometry of Solids* (Philadelphia: F. Nichols, 1806), p. 276.

[97] Ullage has been an underregarded concern of eighteenth- and nineteenth-century mathematicians. See Judith V. Grabiner, "A Mathematician Among the Molasses Barrels: Maclaurin's

As was clear from *Mensuration*'s advertisement, Day did not intend the book

to be a comprehensive source of pure geometry. Rather, he designed it as a

complement to Playfair's *Elements*, which in fact was the way it was used at Yale for

many years. *Mensuration*'s use reflected the trend in American colleges toward

adding more and more mathematical instruction to the curriculum in the early

nineteenth century. Courses in practical mathematics, especially, grew in

importance in order to meet the demands of faculty and students. For example, in

1821, Yale freshmen studied the arithmetic portion of the 1808 second edition of

Webber's *Mathematics* in the first term and *An Introduction to Algebra* in the second.[98]

Sophomores began to memorize and present Playfair's *Elements* before their tutors

during the second term, also adding Day's *A Treatise of Plane Trigonometry* and

Mensuration then. In the third term, they worked through Day's textbook on

navigation and surveying, the fourth part of the mathematical series, published in

1817, and learned conic sections and spherical geometry. Juniors finished the

required course in mathematics with spherical trigonometry.[99] They also began the

natural philosophy course in the first term. In the third term, they studied

astronomy and chose between fluxions, Greek, and Hebrew. In other words, Yale

students had added several mathematical topics since the turn of the century

(mensuration, navigation and surveying, conic sections, spherical geometry, and

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Unpublished Memoir on Volumes," *Proceedings of the Edinburgh Mathematical Society* 39 (1996): 193–240; and Judith V. Grabiner, "'Some Disputes of Consequence': Maclaurin Among the Molasses Barrels," *Social Studies of Science* 28 (1998): 139-168.

[98] *A Statement of the Course of Instruction, Expense, &c. in Yale College, New Haven, Connecticut* (New Haven, 1821), in Volume of Pamphlets, Dr. Jacob Porter Collection, Beinecke Rare Book & Manuscript Library, Yale University, New Haven.

[99] No textbooks are named for conic sections, spherical geometry, and spherical trigonometry in the pamphlet (cit. n. 98), but it appears most likely that tutors taught these subjects with the aid of the appropriate section of Samuel Webber's compendium.

spherical trigonometry), and they had also gained the privilege to choose whether they wanted to study fluxions.[100]

Although perhaps no textbook could sell as well as Day's *An Introduction to Algebra, Mensuration* was still also commercially successful. It was published alone as many as eight times between 1816 and 1851. In addition, the entire course of four volumes was published together twice, and the trigonometry, mensuration, and navigation and surveying were published in one volume under the popular designation of "Day's *Mathematics*" up to ten times between 1831 and 1858.[101] Furthermore, professors such as Josiah Meigs, who told Day that, "I rejoice at every exertion made to advance the Mathematical Sciences," made warm comments about receiving *Mensuration*.[102] Yet, while Playfair's *Elements*, which *Mensuration* was to accompany, was widely used in higher education in the United States, Day's course remained an uncommon choice in the American college curriculum.[103] Only four of the nineteen institutions in existence before the Civil War and profiled in Florian Cajori's *Teaching and History of Mathematics in the United States* taught from

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[100] There appears to be no study which states definitively when the notation of the differential and integral calculus displaced that for fluxions in the United States (George M. Rosenstein, "The Best Method. American Calculus Textbooks of the Nineteenth Century," in Duren, *Century of Mathematics* (cit. n. 68), vol. 3, pp. 77-109; concentrates on the late nineteenth century, after the change had been accomplished), but Lao Genevra Simons said fluxions were used through the first quarter of the nineteenth century, and Florian Cajori named Farrar's 1824 translation of Etienne Bézout's *First Principles of the Differential Calculus* as the first American work using Leibnizian notation. Lao Genevra Simons, "The Adoption of the Method of Fluxions in American Schools," *Scripta Mathematica* 4 (1936): 207-219, on pp. 217-219; Cajori, *Teaching and History* (cit. n. 23), p. 395. When reading American journal articles from the period, one gets the sense that the word "fluxions" was sometimes used as a generic label for any aspect of the subject, the way "calculus" is employed today.

[101] *NUC* (cit. n. 23), vol. 135, pp. 404-409.

[102] Josiah Meigs to Jeremiah Day, 11 July 1816, Day Papers: Letters to Jeremiah Day (cit. n. 25).

[103] For example, in August 1817, Francis Nichols reported to Day that *Mensuration* and *Navigation and Surveying* were already no longer available in New York. Francis Nichols to Jeremiah Day, 26 August 1817, Day Papers: Letters to Jeremiah Day (cit. n. 25).

Mensuration: Yale, the University of North Carolina around 1823, and Dartmouth and Transylvania University in Kentucky in the 1830s.[104] Day's works were ultimately most popular with academies — including those institutions which were colleges in name only — and the high schools which came into existence beginning in the 1830s.

Day and Educational Technique

Still, Day, like Farrar and Davies, did put forth mathematical ideas which influenced the American intellectual community, including notions about analysis and synthesis related to Euclidean geometry, even though none of the three made much reference to the terms directly. For example, Day only used the words themselves in print once. Early in the Yale Report, as he argued that liberal education should force the student to develop his own mind, Day wrote: "The analytic method must be combined with the synthetic. Analysis is most efficacious in directing the powers of invention; but is far too slow in its progress to teach, within a moderate space of time, the circle of the sciences."[105] Farrar discussed analysis as an educational method in a letter to Day: "If a boy is to learn Spherics to any purpose can he learn it on the whole more *easily* to say nothing of other recommendations than by analysis," while Davies did not use the terms until he wrote a work on the logic and utility of mathematics and a mathematical dictionary in the 1850s.[106] Yet, although the three professors were not spending great amounts

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[104] Cajori, *Teaching and History* (cit. n. 23).

[105] "Original Papers" (cit. n. 45), p. 302. He also struggled to define analytical teaching as it was meant by education reformers in his notes for the report; Jeremiah Day, Outline of Report on Course of Instruction, Miscellaneous Undated File, Day Family Papers (cit. n. 1), Box 29.

[106] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 25), emphasis in source; Charles Davies, *The Logic and Utility of Mathematics, With the Best Methods of Instruction Explained and Illustrated* (New York: A. S. Barnes & Co., 1850); Charles Davies and William G. Peck, *Mathematical Dictionary and Cyclopedia of Mathematical Science* (New York: A. S. Barnes & Burr, 1855).

of time explicitly discussing "analysis" and "synthesis," these terms were a prevalent part of contemporary mathematical discourse — for instance, three of the four reviews of Day's and Farrar's series used the words liberally. Further, just as these issues were seen in Playfair's *Elements* by the light of his other writings, the understandings of the terms were central features of geometry textbooks. Day, Farrar, and Davies wrestled with the relationship between algebra and geometry while they compared the mathematical styles of British and French mathematics, and, to a lesser extent, they raised questions about the method of proof in geometry. Overall, though, Americans were probably most concerned with educational techniques which maximized the educative value of mathematics within college liberal education.

Thus, the reasons for teaching mathematics Day voiced in the Yale Report were rooted in the manner in which Day discussed the role of mathematics education in his textbook series. Since Day's plan was that the entire series would be studied as a whole, he laid out his pedagogical motivations most fully in the first volume, *An Introduction to Algebra*. His central argument was that mathematics was key to liberal education because this subject developed mental discipline in the students: "The time and attention devoted to [mathematics], is for the purpose of forming *sound reasoners*, rather than expert mathematicians."[107] In the 1828 Yale Report, Day provided a long list of the benefits of mental discipline: "fixing the attention, directing the train of thought, analyzing a subject proposed for investigation; following, with accurate discrimination, the course of argument; balancing nicely the evidence presented to the judgment; awakening, elevating, and controlling the imagination; arranging, with skill, the treasures which memory

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[107] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. iv; emphasis in source.

gathers; rousing and guiding the powers of genius."[108] To help the students build up these skills, the instructor needed to clearly explain and demonstrate mathematical principles, finding a balance between that which was too obvious and that which was too obscure — thus avoiding the two faults Day found in existing British textbooks. Euclidean geometry, most especially, exemplified the clear, exact, logical thinking desired.

Since his chief reason for teaching mathematics was to convey mental discipline, Day generally believed that the status quo was an adequate mode of teaching. In other words, as professor and president of Yale, he retained instruction by tutors who required students to memorize and recite demonstrations from the textbook. Since the tutors were imparting only the elements of subjects, they were not required to be overly experienced. Day therefore crafted his textbooks for these unique needs of American colleges, where students had to gain much of their knowledge on their own while retaining more of what they memorized than only a few rules for practical use.[109] Similarly, Day limited the role of original mathematics in teaching. He stated that elementary books such as *An Introduction to Algebra* were to collect, arrange, and illustrate mathematics which were already known. While the few students with the leisure to continue in mathematics could be well guided by the genius and spirit of original authors, "[o]riginal discoveries are not for the benefit of *beginners*, though they may be of great importance to the advancement of science."[110] Instead of teaching students to discover mathematical principles on their own, as Playfair suggested was possible, Day argued that they should be provided

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[108] "Original Papers" (cit. n. 45), pp. 300-301.

[109] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. iii.

[110] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. vi; emphasis in source.

with full explanations of the rules. While content was an important part of liberal education, Day often stated that the guiding purpose of an American college was to lay the foundation for lifelong learning or advanced study in the professional schools in divinity, medicine, and law established during Day's tenure at Yale.[111]

Thus, Day's understanding of analysis as invention and synthesis as instruction was markedly different from Playfair's. Playfair, of course, apparently wanted the best professors to teach, as he did under the Scottish professorial system. Additionally, he could be interpreted as advocating the training of students in how to conduct mathematical and scientific research, and he was certainly against too much reliance on memorization. In contrast, although Day included the inventive powers among the mental faculties to be exercised in education, noting in the Yale Report, "However abundant may be the acquisitions of the student, if he has no talent at forming new combinations of thought, he will be dull and inefficient," he continued on to say that, "[t]he most gifted understanding cannot greatly enlarge the amount of science to which the wisdom of ages has contributed."[112] Day did not expect any of his charges at Yale to make a significant difference in theoretical mathematics. After leaving college, Yale graduates would live in the antebellum world of political and technological change. Therefore, Day said, the student needed a substantial body of knowledge gained from others to be truly prepared for "the business of life."[113] In other words, development of the inventive faculties was one part of mental discipline, which together with instruction were both vital parts of liberal education, liberal education's analysis and synthesis. Day may not have seen

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[111] "Original Papers" (cit. n. 45), p. 308.

[112] "Original Papers" (cit. n. 45), p. 302.

[113] "Original Papers" (cit. n. 45), p. 302.

mathematics as the handmaiden of liberal education, as Playfair feared, but there was an essential connection between the two for him. That was one reason why he stood on the side of memorization during the Conic Sections Revolt.

Yet, Day was not arguing that a liberal education could not also be scientific. In his discussion of his purpose for the textbook series, he wrote that he taught mathematics not just to mold students' thinking into clear and logical paths but also to connect mathematics to the physical sciences.[114] *Mensuration* clearly was a step toward the second purpose, since students read Playfair's *Elements* as a model of good reasoning, and Day also presided over Yale's adoption of a limited departure from the fixed course of liberal education around 1820. Day was always proud that the leaders of Yale gradually expanded the curriculum in mathematics and added the new scientific subjects of the early nineteenth century, such as chemistry and mineralogy. He reminded readers of the Yale Report that Yale's object was "to maintain such a proportion between the different branches of literature and science, as to form in the student a proper *balance* of character."[115] Even as late as his address at Woolsey's inauguration in 1846, Day emphasized the ongoing introduction of scientific courses at Yale as evidence that the college incorporated modern understandings of the physical world while preserving the solid foundation of the classics.[116] Still, as part of liberal education, these newer courses were not to get bogged down in minutiae which obscured the overall importance of the subject. In the preface to *Navigation and Surveying*, Day defended omitting some of the details utilized in navigational practice by stating, "The object of a scientific education is

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[114] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. iv.

[115] "Original Papers" (cit. n. 45), p. 301; emphasis in source.

[116] Day, "Inaugural Address" (cit. n. 58), pp. 69-70.

rather to teach *principles*, than the minute rules which are called for in professional practice. The principles should indeed be accompanied with such illustrations and examples as will render it easy for the student to make the applications for himself, whenever occasion shall require."[117]

The anonymous author of the lone journal review of Day's course posed his own plan for accomplishing mental discipline through mathematics education. Like Day, the reviewer advocated more thorough study of the classics, rhetoric, ethics, and mathematics in American colleges, but he disagreed with Day's approach and even the definition of "analysis" provided by Day later in the Yale Report.[118] The reviewer, aware that a "system of mathematics . . . has always been wanting in the public seminaries of our country," did praise Day for presenting such a system, composed of materials abridged and arranged from original authors with "clearness of method, a judicious selection of materials, and perspicuity and neatness of expression."[119] By studying a system of mathematics written in this manner, with the principles placed in their "natural order" and each truth proven when it was introduced, a college student ideally would discipline his intellect, learn to fix his attention, sharpen his powers of invention, and become a lover of truth.[120] The reviewer's complaint was that Day failed to discipline the intellect of students with

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[117] Jeremiah Day, Notes on Navigation and Surveying, Day Family Papers (cit. n. 1), Box 28; emphasis in source.

[118] Review of *A Course of Mathematics, Adapted to the Method of Instruction in the American Colleges;* by Jeremiah Day, *Analectic Magazine* 9 (1817): 441–467, on p. 466. The majority of the review was devoted to educational issues and *An Introduction to Algebra,* leaving less than one page for specific discussion of *Mensuration.* Of course, that book was not even listed at the head of the review and may have been considered less valuable by the author because it dealt with "practical details" rather than "general principles." See p. 441.

[119] Review of *A Course of Mathematics* (cit. n. 118), pp. 442, 444.

[120] Review of *A Course of Mathematics* (cit. n. 118), p. 441.

his method. Rather, Day removed all obstacles for the students and made

mathematics too easy, not realizing that a certain measure of struggle was preferable

in the learning process: "The satisfaction of finding one difficulty surmounted by his

own exertions, will inspire him with new vigour, and confidence in his ability to

overcome others. The illustration which is read and understood in a few minutes,

may be almost as soon forgotten: But those conclusions which are the result of hours

of active labour, on the part of the student, will never be forgotten."[121] To the

reviewer, this inventive process was completely separate from analytical and

synthetical demonstrations, which were absorbed almost entirely passively by

readers.[122] This was an unusual and apparently uninfluential way of classifying

invention, as apart from analysis and synthesis.

Day and Mathematical Styles

Day's textbook series also evoked questions about mathematical style,

making him perhaps the first to raise the "British versus French mathematics"

question in the United States. When he dismissed existing British textbooks —

meaning works by English authors — as either too voluminous or too concise in the

preface to *Introduction to Algebra*, he opened the door to comparisons between the

presentations by these writers and other approaches to mathematics along with

putting forth his own work as limited in length but complete in explanation. Thus,

Day's reviewer associated analysis with the Continent and algebra and synthesis

with England and geometry. He generally preferred the former, critiquing several

textbooks, all by English authors, for being too advanced for learners and

contrasting "the familiar, diffuse manner of Euler and Lacroix" with the "concise,

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[121] Review of *A Course of Mathematics* (cit. n. 118), p. 445.

[122] Review of *A Course of Mathematics* (cit. n. 118), p. 446.

abridged mode of the English writers."[123] In a later footnote, the reviewer explained

further:

> We cannot avoid remarking here, the very different aspect which
>
> science presents, as treated by the best English writers, and by those on
>
> the Continent. The former, by pursuing a method rigidly synthetical,
>
> compel us, indeed, to admit the truth of their conclusions; but leave us
>
> to wonder how they came by them. The latter often take us as their
>
> companions in groping their way through the dusky regions of
>
> analysis. They show us the manner in which they use their tools; and
>
> are not ashamed even to acquaint us with their blunders and
>
> unsuccessful experiments. The former method is best calculated to
>
> inspire the learner with a profound reverence for the talents of the
>
> author; the latter, to give him confidence in his own talents.[124]

At that point, the reviewer echoed Playfair's statements on the differences in

style between French and English mathematics, but neither he nor Day were able to

express wholehearted support for the analytical method associated with Continental

mathematicians. The reviewer noted several times that longer analytical/algebraical

demonstrations could become just as prolix as the ancient synthetical proofs,

confusing the reader and obscuring the practical principles supposedly being

shown.[125] He also argued that Maclaurin had correctly demonstrated that the first

principles of fluxions were geometrical rather than algebraic and that the differential

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[123] Review of *A Course of Mathematics* (cit. n. 118), pp. 443-444. The first American textbook review to compare English mathematics with the Continent may have been: Review of *A Course of Mathematics in Two Volumes*; by Charles Hutton, rev. by Robert Adrain, *General Repository and Review* 4 (1813): 268-282.

[124] Review of *A Course of Mathematics* (cit. n. 118), p. 455n.

[125] Review of *A Course of Mathematics* (cit. n. 118), pp. 447, 457, 461, 464.

notation was merely a convenience.[126] Day was similarly concerned with the

soundness of the foundations of algebra relative to those of geometry, stating,

"Euclid and others have given to the geometrical part a degree of clearness and

precision which would be very desirable, but is hardly to be expected, in algebra."[127]

Day believed, however, that algebra's relative lack of rigor did not interfere with the

learning process; rather, the formality of Euclid's *Elements* was not necessary to

teach mathematics to beginners, and it was permissible to omit the most obvious

steps.[128] Algebra was to be learned first and separately, though, before the student

proceeded on to geometry and the more abstract mode of thinking required for this

discipline. Unlike Playfair, Day was not willing to mix algebra with geometry.

As was often common with Day, though, he departed from his general

philosophical statements when he got down to the business of teaching

mathematics. For instance, Day allowed algebraic equations to illustrate geometrical

relationships in *Mensuration* and thus indicated that he agreed with Playfair's

introduction of algebraic symbols into the *Elements* to make them more

understandable. (See Figure 3.1 for a typical proposition from *Mensuration*.) Day also

required his tutors to cover the theory of proportion in both the freshman algebra

course and the sophomore geometry one. As he explained in *An Introduction to

Algebra*: "The section on *proportion*, will, perhaps be thought useless to those who

read the fifth Book of Euclid. That is sufficient for the purposes of pure *geometrical*

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[126] Review of *A Course of Mathematics* (cit. n. 118), pp. 452-453. The reviewer brought up Scottish names such as Colin Maclaurin and Thomas Reid for the purpose of proving his points about English mathematics, demonstrating perhaps that he did not distinguish a "Scottish" from an "English" mathematics.

[127] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. v.

[128] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), pp. iv-v.

PROBLEM II.

To find the area of a TRIANGLE.

8. RULE 1. *Multiply one side by half the perpendicular from the opposite angle.* Or, multiply half the side by the perpendicular. Or, multiply the whole side by the perpendicular, and take half the product.

The area of the triangle ABC (Fig. 5.) is equal to ½ PC × AB, because a parallelogram of the same base and height is equal to PC × AB, (Art. 2.) and by Euc. 41. 1, the triangle is half the parallelogram.

Ex. 1. If AB (Fig. 5.) be 65 feet, and PC 31.2, what is the area of the triangle ?   Ans. 1014 square feet.

2. What is the surface of a triangular board, whose base is 3 feet 2 inches, and perpendicular height 2 feet 9 inches ?
                    Ans. 4F. 4′ 3″, or 4 feet 51 inches.

9. If two sides of a triangle and the included angle, are given, the perpendicular on one of these sides may be easily found by rectangular trigonometry. And the area may be calculated, in the same manner as the area of a parallelogram in art. 5. In the triangle ABC (Fig. 2.)

                    R : BC :: Sin B : CH

And because the triangle is half the parallelogram of the same base and height,

As radius,
To the sine of any angle of a triangle ;
So is the product of the sides including that angle,
To twice the area of the triangle. (Art. 5.)

Figure 3.1. A typical proposition from *Mensuration*,
giving arithmetical rules. From Jeremiah Day,
Mensuration (New Haven, [1816]), p. 5.

demonstration. But it is important that the propositions should also be presented under the algebraic forms."[129]

Despite the American statements favorable to French analytical mathematics, Helena Pycior assigned Day to the British, synthetic camp with respect to his algebra textbook.[130] Her classification relied largely on Day's lifelong belief in the role of mathematics as a tool of liberal education and was based upon definitions incorporating aspects of the three different usages explored in this study. She characterized "synthetic" as a loosely deductive manner of mathematical writing modelled on Euclidean geometry and "analytical" as developing the principles naturally along the lines of the subject's history.[131] Her central evidence for calling *An Introduction to Algebra* a synthetical work was the attention Day paid to arranging *An Introduction to Algebra* like a geometry textbook, by beginning with the importance of definitions and axioms, attempting to explain algebra in a plain and less abstract manner, and becoming preoccupied with the British problem of the negatives.

Although Pycior's comments about *An Introduction to Algebra* fit that textbook, Day's later textbooks — including *Mensuration* — betray an increasing French influence. Recall that Day had begun to read French mathematical textbooks in the early 1810s. He made an especially careful study of Legendre's *Éléments*, begging leave of Frederick Hall to return his copy later than expected since the copy Day had ordered from Europe was slow to arrive.[132] Indeed, Day referred readers of

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[129] Jeremiah Day, *Introduction to Algebra* (cit. n. 82), p. vii; emphasis in source.

[130] Pycior, "British Synthetic" (cit. n. 77), pp. 126-129.

[131] Pycior, "British Synthetic" (cit. n. 77), p. 146.

[132] Jeremiah Day to Frederick Hall, 31 July 1815, Yale Miscellaneous Manuscripts (cit. n. 85). Karen Parshall and David Rowe concur on this point, describing Day's textbook series as in the style if not containing the content of Euler and Lacroix; Karen Hunger Parshall and David E. Rowe, *The*

the rest of the course to non-English as well as English names: Legendre, Lacroix, Euler, Clairaut, and Lhullier—and proponents of Continental mathematics such as Playfair and Woodhouse—appeared alongside Hutton, Bonnycastle, Simpson, and Wallis. Students at Yale worked concurrently on mastering many of the elements of geometry, including the definitions from which Day drew, and *Mensuration*. In this text, Day justified the steps of his proofs with propositions from Playfair's *Elements* but regularly referred students to Legendre's *Éléments* as a more advanced source imparting more rigorous discussions of the measurement of solids and surfaces.

Day's use of Legendre's geometry textbook was notable for a number of reasons. He was apparently the first American to rely upon Continental sources in writing mathematics textbooks, although admittedly these works were much easier to acquire beginning in 1815. Still, Day began to legitimize Legendre's *Éléments* in the United States, opening the way for Americans to embrace that textbook in a manner never possible in Great Britain, where Playfair's *Elements* was such a central pillar of education. Day adopted the French convention of separating diagrams from proofs, placing the figures at the end of the textbook.[133] Finally, Day recognized a typically French willingness to mix pure geometry and mensuration, perhaps finding that this approach helped him keep his treatise to a more manageable length than heavy reliance upon Hutton's or Bonnycastle's bulky mensuration textbooks would have allowed.[134]

Emergence of the American Mathematical Research Community, 1876-1900: J. J. Sylvester, Felix Klein, and E. H. Moore, History of Mathematics Vol. 8 (American Mathematical Society, 1994), p. 16n.

[133] Day also appears to have initiated a short-lived style—used by Farrar and few others in the United States—of numbering the paragraphs consecutively throughout the text, rather than separating them into books of theorems, problems, corollaries, and scholia.

[134] Similarly, Farrar believed that French geometry textbooks "contain[ed] also all the mensuration that I think necessary. . . ." John Farrar to [John T. Kirkland], 22 January 1817, Hollis Professorshp of Mathematics, Professor Farrar, Letters, Undated, 1813-1827 [UAI.15.963], Harvard University Archives, Cambridge.

Yet, despite the reviewer's comment that Day wrote in "the familiar, diffuse manner of Euler and Lacroix," or the analytical style, which contrasts against Pycior's argument that *An Introduction to Algebra* was penned in the synthetical style because of that book's opening explanations, definitions, and axioms, *Mensuration* lacks analysis and synthesis in the directional sense, as method of proof.[135] The book's structure is neither deductive nor organized according to the historical development of geometry. Rather, it is a list of rules for measurement, divided by topic. In the early pages, especially, Day made no pretense of demonstrating why the student should be convinced the rules were viable. He gave a rule and then explained briefly how it worked or listed the instructions for employing the rule. When he did write proofs, they were not formally structured. (See Figure 3.2 for examples.) As the reviewer noted, Day did not always abide by his own dictum to "not admit of introducing rules and propositions which are not demonstrated," either.[136] Because students would gain their thorough mental training from Playfair's *Elements*, Day could write *Mensuration* at a more facile and pragmatic level.

Day's Legacy in American Geometry Education

Day lived his life as a man of balance and moderation. Among the characteristics fondly remembered by his former students and colleagues was Day's manner of walking around New Haven with a calm, measured step.[137] While Day's illnesses had made him fearful of taxing his heart, his methodical walk also

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[135] Review of *A Course of Mathematics* (cit. n. 118), p. 444; Pycior, "British Synthetic" (cit. n. 77), p. 128.

[136] Preface to Jeremiah Day, *Mensuration* (cit. n. 89); Review of *A Course of Mathematics* (cit. n. 118), p. 464.

[137] Typical was Theodore Dwight Woolsey's appreciation, "President Woolsey's Address" (cit. n. 20).

PROPOSITION XII.

93. *A cube has a greater solidity than any other right parallelopiped the sum of whose length, breadth, and depth is equal to the sum of the corresponding dimensions of the cube.*

The solidity is equal to the product of the length, breadth, and depth. If the length and breadth are unequal, the solidity may be increased, without altering the sum of the three dimensions. For the product of two factors whose sum is given, is the greatest when the factors are equal. (Euc. 27. 6.) In the same manner, if the breadth and depth are unequal, the solidity may be increased, without altering the sum of the three dimensions. Therefore, the solid can not be a maximum, unless its length, breadth, and depth are equal.

PROPOSITION XIII.

94. *If a* PRISM BE DESCRIBED ABOUT A CYLINDER, *the capacities of the two solids are as their surfaces.*

The capacities of the solids are as the *areas* of their bases, that is, as the *perimeters* of their bases. (Art. 88.) But the lateral surfaces are. also as the perimeters of the bases. Therefore the *whole* surfaces are as the solidities.

Cor. . The capacities of different prisms, described about the same right cylinder, are to each other as their surfaces.

Figure 3.2. Examples of Day's informal proof
structure. From Jeremiah Day, *Mensuration*
(New Haven, [1816]), p. 67.

represented his kindly but firm authority at Yale. Following Dwight's example, Day served as a father figure who disciplined and guided the students. He also joined Yale's other professors in gradually expanding the college curriculum. For instance, after Yale adopted Webber's *Mathematics*, mensuration, conic sections, navigation and surveying, spherical geometry, and spherical trigonometry were added as mathematical subjects and fluxions were an option for senior students. Day's successors, though, eventually created a crisis at Yale by uncritically following Day's practice of looking only to Yale graduates for potential professors even after other colleges produced talented candidates and without making the gradual accommodations for changing times that Day had always favored even though the United States after the Civil War was a far different place from the early republic.

In that early republic, Day's use of the concepts which were tied up in the different understandings of "analysis" and "synthesis" reflected his commitment to college liberal education. Day recognized the separate mathematical styles typical of mathematicians in France or Great Britain, seeing the difference mainly as a commitment to algebraic or geometrical techniques. His chief contribution was in publicizing French geometry textbooks through *Mensuration*, especially Legendre's *Éléments*. Day also thought of algebra and geometry as separate disciplines, although he was willing to introduce symbols into geometry to facilitate learning since his students were not destined to become professional mathematicians anyway. While Day did not appeal to analysis or synthesis as a method of proof in *Mensuration*, he did raise analysis as an educational technique. He was not in favor of forcing students to discover mathematical principles on their own, like the original inventors, because this method took too much time. Rather, he wanted students to be presented with an ever-growing body of material and then memorize

general principles of mathematics in order to develop the mental discipline necessary to be careful reasoners in all activities of life.

In all, then, Day was a geometry textbook author who drew parallels in order to help young men develop into gentlemen. The word that Day preferred was "balance" — he talked explicitly about balancing the obvious and obscure in mathematical textbooks, balancing a literary and scientific education, and balancing theory and practice; and more indirectly about balancing the tutor system derived from English examples with the Scottish professorial system or balancing the separate subjects within mathematics with separate textbooks. These aspects of Yale education were in fact "parallels," though, for the two characteristics to be balanced rarely intersected with each other. This became especially true after Yale assigned tutors by subject instead of by class year after 1830. As the author of the first significant American geometry textbook and the president of the institution which was the greatest single role model for other colleges, Day was able to set the agenda — including his interest in these parallels — for American concerns about mathematics education in the early nineteenth century. Though he helped ensure that "mental discipline" would be the central phrase of the discussion and demonstrated that Scottish and French sources could provide inspiration, he did not take the next step, setting aside Playfair's *Elements* for Legendre's *Éléments*. This was accomplished by John Farrar, the subject of Chapter Four.

CHAPTER FOUR

A STRAIGHT LINE IS THE SHORTEST WAY FROM ONE POINT TO ANOTHER:

JOHN FARRAR AND THE ART OF GEOMETRY[*]

How should one summarize the life of an American geometry textbook

author? After several tries in correspondence with her friend and printer, Charles

Folsom, Eliza Rotch Farrar settled on the following for her husband's gravestone:

John Farrar

Professor of Mathematics and Nat. Philo. in Harvard University for 29 Years

A Lucid Eloquent Devout Expositor of Material Laws

Dignified, simple & refined in manners,

Kind & upright in his dealings,

After Fourteen Years of Painful Disease

Borne With Patience and Serenity

He died, as he had lived, an humble Disciple of Jesus Christ.

Born 1779 Died 1853[1]

Although Eliza's memorial was shaped mainly by the fact that most of her life

with Farrar had been spent dealing with his illnesses, she did also allude to his

academic career and scientific writing. Farrar held one of the oldest and most

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[*] An early version of this chapter appeared as "John Farrar: Forgotten Figure of American Mathematics," in *Proceedings of the Canadian Society for the History and Philosophy of Mathematics*, ed. J. J. Tattersall, vol. 11, pp. 63-68.

[1] See the undated drafts and final printed version, as well as related correspondence from Farrar to Folsom, in the Ms. Folsom collection at the Boston Public Library. Folsom had been a businessman and family friend at least since he had seen Eliza Farrar's 1836 *The Young Lady's Friend* through the press, as recorded in Eliza Rotch Farrar to Charles Folsom, 26 July 1836, Ms. Folsom, Boston Public Library, Boston.

respected American professorships during the development of the nineteenth century scientific community and, like Day, was led by a quest to accommodate the college curriculum to the current day to introduce new content into the mathematics and natural philosophy courses. Day and Farrar both worked within the standard American tutor system, but Farrar took a different route by translating and substituting entire textbooks, mostly French publications. For example, Farrar's translation of Legendre's *Éléments* was one tool with which Farrar hoped to move students in a direct line from the beginning to the end of the Harvard curriculum as well as a representation of the nineteenth-century sense of "art" as an essential subject of knowledge. Stymied over the years by factors including his own tendency toward ambivalence which was met in equal measure by Harvard's institutional ambivalence, Farrar's overall influence, though significant at the time, lasted only briefly.

History of Harvard Mathematics

The archetype of what a great American professor of mathematics and natural philosophy could be was John Winthrop (1714-1779) of Harvard, a great-great-grandson of the first governor of the Massachusetts Bay Colony and whose first education was at the Boston Latin School.[2] He graduated from Harvard in 1732

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[2] A brief summary of Harvard mathematics is Julian L. Coolidge, "The Story of Mathematics at Harvard," *Harvard Alumni Bulletin* 26 (1924): 372-378 [HUK 137], Harvard University Archives, Cambridge. Biographies of Winthrop include Frederic K. Kellogg, "A Man and a Method: John Winthrop: Professor of Natural and Experimental Philosophy at Harvard 1738-1779" (Undergraduate Thesis, Harvard University, 1964) [HU92.64.476], Harvard University Archives, Cambridge; "Sketch of Professor John Winthrop," *Popular Science* 39 (1891): 837-842; G. L'E. Turner, "Winthrop, John," in *Dictionary of Scientific Biography* (hereinafter cited *DSB*), ed. Charles Coulston Gillispie, vol. 14 (New York: Charles Scribner's Sons, 1970), pp. 452-453; Frederick Edward Brasch, "Winthrop, John," in *Dictionary of American Biography* (hereinafter cited *DAB*), ed. Allen Johnson, vol. 20 (New York: Charles Scribner's Sons, 1927-1936), pp. 414-416; "Winthrop, John," in *The National Cyclopedia of American Biography* (hereinafter cited *NCAB*), vol. 7 (New York and Clifton, NJ: James T. White and Company, 1898-1984), pp. 165-166; and "Winthrop, John," in *Biographical Dictionary of American Science: The Seventeenth Through the Nineteenth Centuries*, by Clark A. Elliott (Westport, CT: Greenwood Press, 1979), p. 278. See also "Correspondence Between John Adams and Prof. John Winthrop,"

after studying natural philosophy, arithmetic, geometry, algebra, and astronomy

under Harvard's recently inaugurated Hollis Professor of Mathematics and Natural

Philosophy, Isaac Greenwood (1702-1745).[3] Winthrop continued to attend

Greenwood's natural philosophy lectures for two more years and then he returned

to his father's Boston home to study on his own and to amass a library of natural

philosophy.[4] In the meantime, the members of the Harvard Corporation were

probably thinking that they should have learned when Greenwood returned from

London in 1727 without paying debts he owed Thomas Hollis, the benefactor of the

professorship first bestowed on Greenwood despite the debts. Even though he was

acquainted with some of England's leading natural philosophers, prepared a

manuscript for the first algebra textbook by an American author, and taught

Newton's mathematical system of nature, Greenwood's inability to remain sober led

to his censure in 1737 and dismissal on August 30, 1738.[5] Winthrop was inaugurated

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Collections of the Massachusetts Historical Society, 5th ser., 4 (1878): 289-313; and Michael Carter Mathieu, "John Winthrop and Colonial Science Education at Harvard" (Honors Thesis, Harvard University, 1989).

[3] On Greenwood, see David C. Leonard, "Harvard's First Science Professor: A Sketch of Isaac Greenwood's Life and Work," *Harvard Library Bulletin* 29 (1981): 135-168; Lao Genevra Simons, "Algebra at Harvard College in 1730," *American Mathematical Monthly* 32 (1925): 63-70; Brooke Hindle, "Greenwood, Isaac," in *DSB* (cit. n. 2), vol. 5, pp. 519-520; David Eugene Smith, "Greenwood, Isaac," in *DAB* (cit. n. 2), vol. 7, pp. 591-592; and "Greenwood, Isaac," in Elliott, *Biographical Dictionary* (cit. n. 2), p. 109.

[4] Kellogg, "Man and a Method" (cit. n. 2), pp. 6-17; "Sketch" (cit. n. 2), p. 837; Simons, "Algebra at Harvard" (cit. n. 3); Lao Genevra Simons, *Introduction of Algebra into American Schools in the Eighteenth Century*, Bureau of Education Bulletin No. 18 (Washington, DC: Government Printing Office, 1924), pp. 3-17.

[5] "Greenwood" in Elliott, *Biographical Dictionary* (cit. n. 3); Leonard, "Harvard's First" (cit. n. 3); Simons, "Algebra at Harvard" (cit. n. 3); Coolidge, "Mathematics at Harvard" (cit. n. 2). Greenwood went on to establish a school which soon failed and then moved to Philadelphia, where he and Franklin discussed natural philosophy. Sadly, though, Greenwood drank himself to death in 1745. The tutor he left behind at Harvard, Nathan Prince, was also a problem drinker.

as his replacement on January 2, 1739, after the Harvard overseers examined him in mathematics.[6]

Winthrop quickly made his professorship one of the most prestigious in the colonies. In addition to living a personal life beyond reproach, he continued the modern instruction established by Greenwood and also carried on astronomical work. One of his first activities was to observe the transit of Mercury in 1740 and to send his report to the Royal Society, the first of Winthrop's twelve contributions to *Philosophical Transactions*. He involved his students in the observation of Halley's comet in 1759, a trip to Newfoundland for the transit of Venus in 1761, and observations of comets in 1769 and 1770, the transit of Mercury in 1763, and a second transit of Venus in 1769. Winthrop kept a journal of the weather in Cambridge for thirty-five years, where he reported on an earthquake in 1755 and meteors he saw between 1739 and 1760. Winthrop apparently used apparatus which had been donated by Hollis before 1727, including a twenty-four-foot telescope, until a fire in 1764 forced him to go begging for instruments but ultimately improve the quality of the Harvard collection.[7]

In the mathematics courses under Winthrop's supervision, fluxions were the most notable component. Although Winthrop's tutors had to start the students with Euclidean geometry and algebra, Winthrop's own interests lay in training as many students as possible in higher mathematics. In fact, fluxions became the dominant

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[6] "Sketch" (cit. n. 2), p. 837; "Winthrop" in Elliott, *Biographical Dictionary* (cit. n. 2); Brasch, "Winthrop" (cit. n. 2).

[7] Kellogg, "Man and a Method" (cit. n. 2), pp. 18-49; "Sketch" (cit. n. 2), pp. 838-841; Frank R. Freeman, "American Colonial Scientists Who Published in the *Philosophical Transactions* of the Royal Society," *Notes and Records of the Royal Society of London* 39 (1985): 191-206; I. Bernard Cohen, *Some Early Tools of American Science: An Account of the Early Scientific Instruments and Mineralogical and Biological Collections in Harvard University* (New York: Russell & Russell, 1950), pp. 3-50. On Winthrop and other American Newtonians, see Frederick E. Brasch, "The Newtonian Epoch in the American Colonies (1680-1783)," *Proceedings of the American Antiquarian Society*, new ser., 49 (1939): 314-332.

topic of mathematical Commencement theses by 1751, with eight that year repeated verbatim from Newton's *Principia*.[8] In 1771, sixteen of the thirty-nine theses dealt with fluxions.[9] Winthrop's program of study fell into decay near the end of his life, however, as age and his involvement in the American Revolution distracted his attention from grooming students for the study of natural philosophy.[10]

Samuel Williams (1748-1817), who had been one of the Harvard students along on Winthrop's trip to Newfoundland, was elected to succeed Winthrop as Hollis Professor on November 23, 1779.[11] Although Williams made an astronomical expedition to Penobscot Bay for the eclipse of 1780 and officially placed astronomy into the Harvard curriculum in 1785, Williams and his tutor taught at a more elementary level overall than had been the case under Winthrop. In part, Harvard was still experiencing financial difficulty related to the Revolution during Williams's professorship, and there were only half as many graduates per year from the college as there had been before the independence of the United States.[12] In 1788, Williams

[8] Lao Genevra Simons, "The Adoption of the Method of Fluxions in American Schools," *Scripta Mathematica* 4 (1936): 207-219, on p. 208. On teaching methods in eighteenth-century American mathematics, see also Theodore Hornberger, *Scientific Thought in the American Colleges, 1638-1800* (University of Texas, 1946; reprint, New York: Octagon Books, 1968).

[9] Lao Genevra Simons, "Short Stories in Colonial Geometry," *Osiris* 1 (1936): 584-605, on p. 586. Simons also said that a variety of versions of Euclid's *Elements* were used as the geometry textbook, including Dechales's, Winston's, Ham's, and Barrow's. Simson's book may have been adopted in the late eighteenth century.

[10] Brasch, "Winthrop" (cit. n. 2), p. 415; "Sketch" (cit. n. 2), p. 842; Gordon Joseph Schiff, "The Efforts and Accomplishments of Samuel Williams" (Undergraduate Thesis, Harvard University 1981) [HU92.81.774], Harvard University Archives, Cambridge, pp. 4-14. See "Correspondence" (cit. n. 2) for some of Winthrop's concerns during the Revolution and Samuel Eliot Morison, *Three Centuries of Harvard, 1636-1936* (Cambridge: Harvard University Press, 1936), pp. 133-163, for wartime upheaval at Harvard.

[11] One of the few biographies of Williams is Schiff, "Efforts and Accomplishments" (cit. n. 10).

[12] Cohen, *Early Tools* (cit. n. 7), pp. 45-65; Schiff, "Efforts and Accomplishments" (cit. n. 10), pp. 15-59; Morison, *Three Centuries* (cit. n. 10), pp. 133-163.

left Harvard after falling into debt himself, and his tutor, Samuel Webber (1759-1810), was named Hollis Professor. Webber replaced Simson's *The Elements of Euclid* with Playfair's *Elements* some time before 1798 and compiled a two-volume textbook, *Mathematics, Compiled from the Best Authors*, covering the rest of the entire course of mathematics — which no longer included any form of the calculus — in 1801 under the direction of the Harvard Corporation.[13] During this period, Harvard's president, Joseph Willard, restored the discipline and finances of the college.[14]

At the turn of the nineteenth century, the Harvard curriculum still was comprised almost solely of the classical languages and mathematics. Harvard students were divided by class and met with an instructor called a "tutor" for each subject, including algebra in the first year and geometry in the second. The tutor, who generally had graduated from Harvard himself a year or two earlier, heard the students recite from the text and drilled them on mathematical rules. Thus, for geometry, students memorized demonstrations from Playfair's *Elements*. At Harvard, the same tutor taught geography, geometry, natural philosophy, and astronomy. Tutors had been assigned to subjects rather than to classes in 1767, far earlier than in other American colleges. Harvard's first entrance requirement in mathematics was instituted in late 1803. This was arithmetic up to the "Rule of Three," a technique for finding the fourth member of a proportion when the other three numbers were known.[15]

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[13] Simons, "Adoption" (cit. n. 8), p. 218; Samuel Webber, *Mathematics, Compiled From the Best Authors*, 2 vols. (Boston: Thomas & Andrews, 1801). Webber also edited William Enfield, *Institutes of Natural Philosophy, Theoretical and Experimental* (Boston, 1802), which was originally published in London in 1785. Webber's edition was printed five times by 1832.

[14] "Willard, Joseph," in *NCAB* (cit. n. 2), vol. 6, pp. 417-418.

[15] Florian Cajori, *The Teaching and History of Mathematics in the United States* (Washington, D.C.: Government Printing Office, 1890), pp. 44-97; *Harvard College Triennial Catalogues* (Cambridge, MA, 1797, 1812-1824).

John Farrar and Harvard

John Farrar was born July 1, 1779, in Lincoln, Massachusetts.[16] His father was a militia leader during the early years of the American Revolution; he later served as a deacon but farmed for a living. The oldest of Farrar's three older brothers was the family designate to leave the farm for college, but John asked for his own opportunity to continue his education. Since he had already shown an aptitude for academics, his parents sent him to the Phillips Academy in Andover for preparatory work before entering Harvard College in 1799. Farrar, although older than most students in the United States at the turn of the nineteenth century, like them attended college to be trained for ministry.[17] Thus, after graduation in 1803, Farrar joined his eldest brother at Andover Theological Seminary.

However, Farrar's scientific and pedagogic abilities had already become evident during his undergraduate career. In an 1802 classroom competition, he won a copy of the astronomy textbook by the English mathematician, John Bonnycastle. At commencement in 1803, Farrar presented the traditional Latin theses on a less traditional topic, physics.[18] In the meantime, he paid his expenses by teaching school

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[16] Biographical material on Farrar is drawn from several sources, including [John Gorham Palfrey], "Professor Farrar," *Christian Examiner* 55 (1853): 121-136; David Eugene Smith, "Farrar, John," in *DAB* (cit. n. 2), vol. 6, pp. 292-293; Brooke Hindle, "Farrar, John," in *DSB* (cit. n. 2), vol. 4, pp. 546-547; and "John Farrar," in Elliott, *Biographical Dictionary* (cit. n. 2), p. 91. *Christian Examiner* authors were identified by Kenneth Walter Cameron, ed., *Research Keys to the American Renaissance* (Hartford: Transcendental Books, 1967).

[17] Although it appears that many American college students intended to become clergy, the number who actually did so upon completing the liberal arts curriculum had already been declining for some time. Stanley Guralnick reported statistics from Richard Hofstadter's *Academic Freedom in the Age of the College* showing that 60% of Harvard and Yale graduates became ministers around 1700, 40% in the 1740s, and only 20% by 1840; Stanley M. Guralnick, "Sources of Misconception on the Role of Science in the Nineteenth-Century American College," *Isis* 65 (1974): 352-366, on p. 356n.

[18] The titles of Harvard theses in mathematics during Farrar's lifetime may be found in Henry C. Badger, "The Mathematical Theses Preserved in the Library of Harvard University, and Prepared By Members of the Junior and Senior Classes From 1782 to 1839," in *Bibliographical Contributions*, ed. Justin Winsor, vol. 2, no. 32 (Cambridge: Library of Harvard University, 1888).

during the winter breaks. Thus, in 1805, the Harvard administration invited Farrar to become the Greek tutor. He was not expecting this appointment, though, and he deliberated for some time before leaving Andover Seminary. In addition to believing his calling was the Church, Farrar apparently suffered from a nervous constitution. Yet, once he took the job, Farrar proved himself capable and doubled the amount of Greek instruction provided to students. He also found time and opportunity to preach at least once per week, and he began several quarters of service on the committee overseeing the students' commons.[19]

Farrar's combination of Christian piety, expository clarity, and interest in natural philosophy was well-suited to the early nineteenth century American college. When Samuel Webber ascended from the Hollis Professorship to the presidency of Harvard in 1806, apparently largely on his ability to be wholly uncontroversial, the Harvard Corporation first approached a man of greater scientific standing, Nathaniel Bowditch, and then one of greater theological standing, the Reverend Joseph McKean, to replace Webber.[20] When both men refused the position, Farrar was the next choice, an excellent compromise between

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[19] 14 October 1805, Corporation Records (1795-1836), vol. 4 [UAI.5.30.2], Harvard University Archives, Cambridge. As a student, Farrar studied philosophy under Harvard's first and only permanent tutor, Levi Hedge. Hedge espoused Common Sense philosophy and edited *Treatise on the Philosophy of the Human Mind* by the Scottish philosopher, Thomas Brown, at the end of his life. Farrar's writings reveal no conscious connections with the Scottish school of philosophy, however. For a biography of Hedge, see Ernest Sutherland Bates, "Hedge, Levi," in *DAB* (cit. n. 2), vol. 4, part 2, pp. 499.

[20] Cohen, *Early Tools* (cit. n. 7), pp. 45-65. The historian of Harvard, Samuel Eliot Morison, snidely called Webber a "colorless" president. Morison, *Three Centuries* (cit. n. 10), p. 195. See also John C. Greene, *American Science in the Age of Jefferson* (Ames: Iowa State University Press, 1984), pp. 60-90, and the gentler biography of Webber in *NCAB* (cit. n. 2), vol. 6, p. 417. Meanwhile, McKean was named Harvard's Boylston Professor of Rhetoric and Oratory in 1809. See 12 October 1809, Corporation Records (cit. n. 19), vol. 4.

Bowditch's and McKean's personalities.[21] A committee gave Farrar the news two weeks later, but it took him another month to officially accept.[22] He cited "recent disorders" at the college as causing his delay, thanked Webber for his current and future support, and declared, "It will be a delightful task to instruct the youth of our Country in those branches of Science, which are calculated to make them ———— citizens, and to lead their thoughts to those contemplations which are well suited to inspire them with pious and virtuous sentiments."[23] Farrar was installed on June 11, 1807.

At this time, the Hollis Professor did some supervising of the mathematics tutors, although his main responsibility was delivering natural philosophy lectures to the juniors and seniors. Farrar proved quite talented at this task and soon embraced the technical aspects of science as much as its virtuous benefits. With the philosophical apparatus placed under his care in exchange for a one thousand dollar bond, he skillfully performed experiments in class, which were used in the early nineteenth century both to catch students' attention and to demonstrate physical principles to them. He explained concepts clearly, and he emphasized the reasoning process in class.[24] Farrar soon became aware, though, that his own level of learning—not to mention that of Harvard students, ninety percent of whom failed to master even quadratic equations according to Farrar's biographer, John Gorham

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[21] 20 March 1807, Corporation Records (cit. n. 19), vol. 4. Farrar was suggested as a candidate by Theophilus Parsons, one of the members of the Harvard Corporation; Greene, *American Science* (cit. n. 20), p. 77.

[22] 2 April 1807 and 16 April 1807, Corporation Records (cit. n. 19), vol. 4.

[23] John Farrar to Samuel Webber, 4 May 1807, Harvard College Papers (1797-1825), vol. 5 [UAI.5.131.10], Harvard University Archives, Cambridge.

[24] Cohen, *Early Tools* (cit. n. 7), p. 25.

Palfrey[25] — did not compare with contemporary work in European natural philosophy and mathematics. Indeed, the mathematization of natural philosophy accomplished in the eighteenth century had not yet entered the American college curriculum to a full degree. But Farrar now realized that his students needed a thorough and up-to-date training in mathematics before they could comprehend these principles of natural philosophy. He began reading recent publications in European mathematics and science, focusing on the research completed by the French Society of Arcueil.[26]

Farrar's first actions to rebuild Harvard mathematics and natural philosophy were focused on improving the apparatus collection and on establishing an observatory. Farrar began to constantly badger the president for funds with which to update Harvard's scientific instrument collection. For example, in 1810, he wrote to President Webber that the Voltaic "battery is totally inadequate to the performance of the more interesting and important experiments in . . . demonstrating the identity of Galvanism and Electricity by charging an electrical battery. . . . There are also wanting in the apparatus a large glass jar, a doubler and condenser of electricity."[27] He apparently was authorized to purchase a battery for

[25] [Palfrey], "Professor Farrar" (cit. n. 16), p. 126.

[26] On Laplace, Berthollet, and Arcueil, see Maurice Crosland, *The Society of Arcueil: A View of Science at the Time of Napoleon I* (Cambridge: Harvard University Press, 1967).

[27] John Farrar to Samuel Webber, 5 June 1810, Hollis Professorship of Mathematics, Professor Farrar, Letters, Undated, 1813-1827 [UAI.15.963], Harvard University Archives, Cambridge.

one hundred dollars.[28] Later that year, Farrar requested an eight-foot rain gauge and a sundial, items authorized in January 1811.[29]

In addition, Farrar tried to rally institutional support for an observatory at Harvard thirty years before Benjamin Peirce took up the cause and finally raised sufficient funds in 1843.[30] He started in 1812 by trying to replace the telescopes which had been purchased by Winthrop and arguing that one high-quality telescope would be preferable to a number of secondhand ones.[31] By 1816, Farrar had surveyed a possible site for an observatory and convinced the Harvard Corporation to authorize a trip to Philadelphia and Washington, D.C., so that he could examine instruments and apply to the federal government for funds.[32] Although the

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[28] John Farrar to John T. Kirkland, 27 December 1816, Hollis Professorship of Mathematics (cit. n. 27). By this time, Farrar was lobbying to replace that battery and received sixty-five dollars for constructing an addition to the galvanic apparatus. See also 16 December 1816 and 27 December 1816, Corporation Records (cit. n. 19), vol. 5.

[29] John Farrar to Harvard Corporation, 22 November 1810, Harvard College Papers (cit. n. 23), vol. 6; 28 January 1811, Corporation Records (cit. n. 19), vol. 5.

[30] Cohen, Early Tools (cit. n. 7), pp. 45-65. Sara Schechner Genuth, the historian of Harvard astronomy, has skipped over efforts such as Farrar's, saying only, "From time to time [between 1780 and 1839], a succession of Harvard faculty members attempted to revive interest in building an observatory, but as projected costs escalated, state funding dried up." Sara Schechner Genuth, "Blazing Starts, Open Minds, and Loosened Purse Strings: Astronomical Research and Its Early Cambridge Audience," *Journal for the History of Astronomy* 21 (1990): 9-20, on p. 16. Similarly, see Sara Schechner Genuth, "From Heaven's Alarm to Public Appeal: Comets and the Rise of Astronomy at Harvard," in *Science at Harvard University: Historical Perspectives*, ed. Clark A. Elliott and Margaret W. Rossiter (Bethlehem, PA: Lehigh University Press, 1992), pp. 28-54. One hundred forty years earlier, William Mitchell did discuss Farrar as well as two attempts to raise funds by John Quincy Adams in the 1820s, although Bowditch is credited with proposing the observatory in his story. In Harvard records, Bowditch first appears several months after Farrar's research trip. William Mitchell, "The Astronomical Observatory of Harvard University," *Christian Examiner* 50 (1851): 264-279, on pp. 265-266. See also Josiah Quincy, *The History of Harvard University*, vol. 2 (Cambridge: John Owen, 1840), pp. 566-568; and Greene, *American Science* (cit. n. 20), pp. 77-78.

[31] John Farrar to John T. Kirkland, 17 February 1812, Harvard College Papers (cit. n. 23), vol. 7.

[32] John Farrar to John T. Kirkland, August 1815, Harvard College Papers (cit. n. 23), vol. 7; 2 January 1816, Corporation Records (cit. n. 19), vol. 5.

instruments were "not exactly what [Farrar] expected" and President James

Madison foreshadowed the ultimate denial of federal funds by saying that the

national capital was a fine enough location for a national observatory, Farrar

considered his travels a success since he met government surveyors such as

Ferdinand Hassler and General Joseph G. Swift and gained a new appreciation for

the comforts of Cambridge.[33] Yet, no further action on the observatory was taken

until July, when Farrar and Bowditch were authorized to order instruments.[34] In

October, however, costs for the circular instrument surpassed the estimate, and the

observatory was left unbuilt.[35] Farrar lobbied for an observatory again in 1818 and

requested reimbursement for expenses incurred in another discussion of the matter

in 1822, but there would not be a building in operation at Harvard until well after

Farrar retired.[36]

This was the time period when Farrar prepared a series of mathematics

translations which formed the second series of full-length mathematical textbooks

[33] John Farrar to John T. Kirkland, 15 January 1816 and 16 January 1816, Hollis Professorship of Mathematics (cit. n. 27); John Farrar to John T. Kirkland, 26 January 1816, Harvard College Papers (cit. n. 23), vol. 7.

[34] 24 July 1816, Corporation Records (cit. n. 19), vol. 5.

[35] 14 October 1816 and 31 October 1816, Corporation Records (cit. n. 19), vol. 5. William C. Bond, who repaired Harvard's instruments, advised Farrar and Bowditch by traveling to Europe to gather information on instruments, and was to build the circular instrument, became the astronomer when the observatory was finally constructed. Although one of his sons and assistants was named George, this family was apparently not related to the George Bond of Boston who was married to Eliza Ware Rotch Farrar's cousin.

[36] John Farrar, "Extract from a Letter Addressed to the Editor, On the Importance of an Observatory at Cambridge," North American Review 8 (1818): 205-208; John Farrar to John T. Kirkland, 22 August 1822, Harvard College Papers (cit. n. 23), vol. 10. As was seen in Chapter One, Farrar remained interested in building an observatory in Cambridge even after he retired, but his inquiries after 1822 were made only unofficially. Harvard finally raised sufficient funds — Farrar expected the observatory would cost $20,000 in 1818 — in the wake of popular enthusiasm over the comet of 1843, and, although the United States Navy improvised an observatory in the 1830s, Congress also did not authorize the National Observatory until 1842; Greene, American Science (cit. n. 20), pp. 78, 129. North American Review authors were also identified by Cameron, Research Keys (cit. n. 16).

published for American students: *An Elementary Treatise on Arithmetic*, based upon

Silvestre François Lacroix's arithmetic, published in 1818; the 1818 *An Introduction to*

the Elements of Algebra, selected from Leonhard Euler's writing; *Elements of Algebra*,

taken from a textbook by Lacroix and aimed toward more advanced students, also

published in 1818; *Elements of Geometry*, from Adrien-Marie Legendre's *Éléments*,

completed in 1819; *An Elementary Treatise on Plane and Spherical Trigonometry*,

compiled from the writings of Lacroix and Etienne Bézout, published in 1820; *An*

Elementary Treatise on the Application of Trigonometry, which drew on a variety of

sources, appeared in 1822, and was sometimes known by an alternate title,

Topography; Bézout's *First Principles of the Differential and Integral Calculus*, completed

in 1824; and Louis Pierre Marie Bourdon's *Elements of Algebra*, published in 1831.[37]

(See Table 4.1 for a complete list of Farrar's publications.) The books were published

by William Hilliard, who had been hired as college printer in 1802, but who by this

time was publishing as an independent bookseller.[38] Hilliard had always been

interested in textbooks as a profitable possibility, and it became common for

Harvard professors to give manuscripts to Hilliard during John T. Kirkland's

administration — Kirkland (1770-1840) was elected president after Webber's death in

1810.[39]

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[37] In *Application of Trigonometry*, Farrar listed his sources as treatises on trigonometry by Cagnoli and by John Bonnycastle; Delambre's work on astronomy, Bézout's textbook on navigation; and books on topography by Puissant and by Malortie; see John Farrar, *An Elementary Treatise on the Application of Trigonometry to Orthographic and Stereographic Projection, Dialling; Mensuration of Heights and Distances, Navigation, Nautical Astronomy, Surveying and Levelling; Together With Logarithmic and Other Tables*, 4th ed. (Boston: Hilliard, Gray, and Company, 1840).

[38] Max Hall, *Harvard University Press: A History* (Cambridge and London: Harvard Univesrity Press 1986), pp. 4-28.

[39] Madeline B. Stern, *Imprints on History: Book Publishers and American Frontiers* (Bloomington: Indiana University Press, 1956), pp. 24-44. Kirkland became known as Harvard's "beloved" president, who helped spread the school's reputation as a liberal institution. He was the last president to guide the students with a paternal hand; Morison, *Three Centuries* (cit. n. 10), pp. 195-221.

Table 4.1. Publications by John Farrar.

"Observations of the Comet of 1811." *Memoirs of the American Academy of Arts and Sciences* 3 (1809-1815): 308-312.

"Abstract of Meteorological Observations, Made at Cambridge, New England." *Memoirs of the American Academy of Arts and Sciences* 3 (1809-1815): 361-398.

"An Abstract of Meteorological Observations, Made at Andover, Massachusetts." *Memoirs of the American Academy of Arts and Sciences* 3 (1809-1815): 399-412.

"Spots on the Sun," *North American Review* 3 (1816): 36-40.

"Continuation of Spots on the Sun," *North American Review* 3 (1816): 285-287.

"Abstract of Meteorological Observations for August and September, Taken at Cambridge." *North American Review* 6 (1817-1818): 149.

Review of *Ferguson's Astronomy, Explained Upon Sir Isaac Newton's Principles.* By David Brewster. *North American Review* 6 (1817-1818): 205-224.

"Abstract of Meteorological Observations taken at Cambridge for October and November." *North American Review* 6 (1817-1818): 292.

Review of *A Discourse Delivered Before the Literary and Philosophical Society of New York, July 4th, 1814.* By Dewitt Clinton. And *Address Delivered Before the Literary and Philosophical Society of South Carolina on the 19th of November 1817.* By Timothy Ford. *North American Review* 8 (1818): 157-168.

"Extract From a Letter Addressed to the Editor, On the Importance of an Observatory at Cambridge." *North American Review* 8 (1818): 205-208.

An Elementary Treatise on Arithmetic. Silvestre François Lacroix. Boston, 1818.

An Introduction to the Elements of Algebra. Leonhard Euler. Cambridge, 1818.

Elements of Algebra. Silvestre François Lacroix. Cambridge, 1818.

Elements of Geometry. Adrien-Marie Legendre. Cambridge, 1819.

An Elementary Treatise on Plane and Spherical Trigonometry, and On the Application of Algebra to Geometry. Silvestre François Lacroix and Etienne Bézout. Cambridge, 1820.

Table 4.1 (continued)

"An Account of the Violent and Destructive Storm of the 23d of September 1815." *Memoirs of the American Academy of Arts and Sciences* 4 (1821): 92-97.

"An Account of a Singular Electrical Phenomenona." *Memoirs of the American Academy of Arts and Sciences* 4 (1821): 98-102.

Review of *Histoire de L'Astronomie Ancienne et Moderne.* By J. S. Bailly. *North American Review* 12 (1821): 150-174.

Review of *Report upon Weights and Measures.* By John Quincy Adams. *North American Review* 14 (1822): 190-230.

An Elementary Treatise on the Application of Trigonometry to Projection, Mensuration, Navigation, and Surveying. Cambridge, 1822.

First Principles of the Differential and Integral Calculus. Etienne Bézout. Cambridge, 1824.

An Elementary Treatise on Mechanics. Cambridge, 1825.

An Experimental Treatise on Optics. Jean-Baptiste Biot. Cambridge, 1826.

Elements of Electricity, Magnetism, and Electro-Dynamics. Jean-Baptiste Biot. Cambridge, 1826.

Elements of Natural Philosophy. Ernst Gottfried Fischer. Cambridge, 1827.

An Elementary Treatise on Astronomy. Jean-Baptiste Biot. Cambridge, 1827.

Review of *A Discourse on the Advantages of Natural Philosophy and Astronomy.* By Dionysius Lardner. *Christian Examiner* 7 (1829): 261-268.

Elements of Algebra. Louis Pierre Marie Bourdon. Boston, 1831.

Tract on Comets. François Arago. Boston, 1832.

"Arago on Comets." *North American Review* 42 (1836): 196-216.

Farrar was perceived as an esteemed mathematician and natural philosopher

well before the mathematics series was published, for he had been elected to the

American Academy of Arts and Sciences in 1808, the scientific society founded in

Cambridge in 1780 by John Adams on the model of the Paris Academy of Sciences

and as a rival to Philadelphia's American Philosophical Society.[40] He then served as

the Academy's recording secretary for fourteen years (1811-1824) and was on the

committee of publication for fifteen years (1810-1825). From 1829 to 1830, he was

vice-president of the organization. Additional recognition came through honorary

doctorates in law bestowed upon Farrar by Brown University and Bowdoin College

in 1833. Farrar's few scientific research publications also appeared during his active

years in the Academy. First, he quelled fears about a large spot which had appeared

on the sun by showing sun spots were a common occurrence and by describing

William Herschel's observations before concluding that Herschel's theory about the

connection between sun spots and the weather had not been borne out by Farrar's

meteorological measurements.[41] Farrar then reviewed David Brewster's 1817 edition

of *Ferguson's Astronomy, explained upon Sir Isaac Newton's principles*.[42] The text was

popular, but Farrar wished it would be replaced by an intellectual treatise which

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[40] Florian Cajori. *The Early Mathematical Sciences in North and South America* (Boston: The Gorham Press, 1928), pp. 145-148.

[41] [John Farrar], "Spots on the Sun," *North American Review* 3 (1816): 36-40, 235-237. See also Farrar's two reports of meteorological data, [John Farrar], "Abstract of Meteorological Observations for August and September, Taken at Cambridge," *North American Review* 6 (1817-1818): 149; and "Abstract of Meteorological Observations Taken at Cambridge for October and November," *North American Review* 6 (1817-1818): 292.

[42] [John Farrar], review of *Ferguson's Astronomy, explained Upon Sir Isaac Newton's Principles*; ed. David Brewster, *North American Review* 6 (1818): 205-224. See also [John Farrar], review of *A Discourse Delivered Before the Literary and Philosophical Society of New York, July 4th, 1814*; by Dewitt Clinton, and *Address Delivered Before the Literary and Philosophical Society of South Carolina on the 19th of November 1817*; by Timothy Ford, *North American Review* 8 (1818): 157-168; and [John Farrar], review of *Report upon Weights and Measures*; by John Quincy Adams, *North American Review* 14 (1822): 190-230.

reflected advances made since the mid-eighteenth century, and he expressed

surprise that Brewster would edit a work which was so obviously unsatisfactory.

Yet, Farrar's students read *Ferguson's Astronomy* through most of the 1820s, before

Farrar replaced it in 1826 with a textbook by John Gummere, which Farrar reported

was based upon analytical processes and deductions rather than the older book's

"detail of facts," and then with his own translation of Jean-Baptiste Biot's *Traité*

Elémentaire d'Astronomie Physique in 1827.[43] In contrast, in a later journal review,

Farrar praised Jean-Sylvain Bailly for teaching the leading truths of astronomy to

readers who had little scientific background in *Histoire de L'Astronomie.*[44] Farrar

turned to a discussion of the contemporary role of science in American life in a

review of speeches delivered by Dewitt Clinton and Timothy Ford.[45] After his letter

advocating an American observatory at Harvard appeared in *North American Review*,

Farrar published a few more notes on meteorology: "An Account of the Violent and

Destructive Storm of the 23d of September 1815," "An Account of a Singular

Electrical Phenomenon," and "Account of an Apparatus for Determining the Mean

Temperature and the Mean Atmospheric Pressure for Any Period."[46] He also wrote

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[43] 21 September 1821, Corporation Records (cit. n. 19), vol. 6,; Farrar to Board of Overseers, 10 January 1826, Hollis Professorship of Mathematics (cit. n. 27); John Farrar, trans., *An Elementary Treatise on Astronomy, Adapted to the Present Improved State of the Science, Being the Fourth Part of a Course of Natural Philosophy, Compiled for the Use of the Students of the University at Cambridge, New England*, by Jean-Baptiste Biot (Cambridge: Hilliard and Metcalf, 1827).

[44] [John Farrar], review of *Histoire de L'Astronomie Ancienne et Moderne*; by J. S. Bailly, *North American Review* 12 (1821): 150-174. Bailly's volumes were published in the 1770s, so Farrar was again bringing attention to a work he felt was unknown in the United States. Bailly, meanwhile, had been executed during the French Revolution; Jean-Baptiste Biot and A. Beuchot, "Bailly (Jean-Sylvain)," in *Bibliographie Universelle: Ancienne et Moderne*, ed. J. Fr. Michaud, new ed., vol. 2 (Paris, 1854), pp. 643-647.

[45] [Farrar], review of *Discourse* and *Address* (cit. n. 42).

[46] [Farrar], "Extract From a Letter" (cit. n. 36); John Farrar, "An Account of the Violent and Destructive Storm of the 23d of September 1815," and "An Account of a Singular Electrical Phenomenon, Observed During a Snow Storm Accompanied With Thunder,"*Memoirs of the American Academy of Arts and Sciences* 4 (1821): 92-97, 98-102; John Farrar, "Account of an Apparatus for

a lengthy essay advocating the metric system as a review of a Congressional report by John Quincy Adams.[47] Farrar's also reviewed an address on natural philosophy and astronomy by Dionysius Lardner.[48] Lardner delivered the lecture at the opening of the University of London, where he was Professor of Natural Philosophy and Astronomy. Farrar welcomed the new university in the hope that it would encourage scientific investigation in a changing world, unlike old universities which taught only a knowledge of the past, and he quoted Lardner's similar sentiments at length. Finally, Farrar published a discussion of the treatise on comets by François Arago, which Farrar had already translated into English.

Despite his desire to write an original textbook on natural philosophy, Farrar instead prepared a second series of translations. The 1825 *An Elementary Treatise on Mechanics* was compiled from various sources including Biot, Bézout, Poisson, Francœur, Gregory, Whewell, and Leslie.[49] *Elements of Electricity, Magnetism, and Electro-Magnetism* was translated from portions of Biot's *Précis élémentaire de physique* in 1826 and updated by Farrar in 1839 by incorporating later publications by Biot and Cesar Mansuete Despretz. Farrar also completed *An Experimental Treatise on*

Determining the Mean Temperature and the Mean Atmospherical Pressure for Any Period," *Boston Journal of Philosophy and the Arts* 1 (1823-1824): 491-494.

[47] [Farrar], review of *Weights and Measures* (cit. n. 42). Charles Davies later reprinted Adams's report as *The Metric System, Considered with Reference to its Introduction into the United States; Embracing the Reports of the Hon. John Quincy Adams and the Lecture of Sir John Herschel* (New York: A. S. Barnes & Co., 1870). The book appeared in three printings; Davies was at that time the chair of the Committee on Coins, Weights, and Measures of the University Convocation of the State of New York; *The National Union Catalog: Pre-1956 Imprints* (hereinafter cited *NUC*), vol. 134 (London: Mansell, 1976), p. 362. Adams's son thanked Davies for reprinting the report in Charles Francis Adams to Charles Davies, 18 November 1870, Miscellaneous Papers, Special Collections and Archives, United States Military Academy, West Point.

[48] John Farrar, review of *A Discourse on the Advantages of Natural Philosophy and Astronomy;* by Dionysius Lardner, *Christian Examiner* 7 (1829): 261-268.

[49] See [Timothy Walker], review of *Elements of Geometry;* by Adrien-Marie Legendre, trans. John Farrar, *North American Review* 27 (1828): 191-214, on p. 194.

Optics, selected as well from Biot's *Précis élémentaire de physique*, in 1826. An

Elementary Treatise on Astronomy, based upon selections from Biot's *Traité élémentaire*

d'astronomie physique, appeared in 1827. Farrar's version of Biot's 1806 translation

into French of Ernst Gottfried Fischer's *Elements of Natural Philosophy* also was

published in 1827. Finally, Farrar translated Arago's *Tract on Comets* in 1832.

In the meantime, financial problems stemming from Harvard's loss of its state

subsidy in 1823 had a direct effect on Farrar.[50] Shortly thereafter, the Harvard

Corporation directed President Kirkland to consider eliminating at least some of the

tutors by having Farrar and the Chemical Professor assume their duties.[51] Farrar

already believed he was overburdened — he was on the committee for textbooks and

would soon be placed back on the committee which adjusted the cost of commons,

and he guided two or more student natural philosophy clubs in addition to his

existing teaching duties — and that Harvard needed more science and mathematics

instructors, not fewer.[52] It had taken him years to convince the Corporation to hire a

Chemical and Mineralogical Professor.[53] Thus, when Kirkland asked Farrar again in

September 1825 if he would take on more private exercises and recitations, Farrar

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[50] See Morison, *Three Centuries* (cit. n. 10), pp. 195-221.

[51] 14 January 1824, Corporation Records (cit. n. 19), vol. 6. The Chemical Professor was not named but may have been John Gorham.

[52] For some examples of these directives from the Harvard Corporation, see 27 August 1814, 16 May 1815, and 25 February 1824, Corporation Records (cit. n. 19), vols. 5- 6. Additionally, as a bachelor between 1807 and 1820, Farrar had lived in the house Harvard provided to the Hollis Professor. The Harvard Corporation often required him to board upperclassmen (although Farrar chose the students who let rooms) and to give over rooms in his house as classrooms. Farrar paid $160 in rent each year and generally struggled to get the Corporation to authorize repairs. See 22 June 1807, 24 December 1808, 30 June 1817, and 5 September 1817, Corporation Records (cit. n. 19), vols. 4- 5; 1817 Report of the Committee of the Corporation on Rooms, Harvard College Papers (cit. n. 23), vol. 7; and [Palfrey], "Professor Farrar" (cit. n. 16), pp. 129-130.

[53] 31 March 1817, 8 October 1818, and 25 November 1819, Corporation Records (cit. n. 19), vols. 5-6.

flatly refused.[54] After three months, the Corporation gave in and approved a third

tutor.[55] In addition, Farrar had injured his eyes for two to three years in the late

1810s by doing his translations in poor light and over long, uninterrupted periods.

He also took his first leave of absence in 1822 in order to take his ailing wife of two

years on a trip to the Azores; she passed away in September 1824.[56]

Then, Bowditch joined the Harvard Corporation in 1826 and soon turned his

energies to resolving the ongoing financial crisis which continued unabated even

after Kirkland resigned in 1828, in part by instigating the dismissal of the college

treasurer.[57] Farrar cooperated with Bowditch to sell some of the older and smaller

instruments and to use the funds to repair or purchase other apparatus.[58] Yet, it

often did not help Farrar to have a friend on the Corporation. In December 1826,

Farrar published a newspaper announcement of evening lectures in which he would

demonstrate the new apparatus, only to be refused permission to use the

[54] John T. Kirkland to John Farrar, 22 September 1825, President's Papers, John T. Kirkland, Letter Book (1817-1825) [UAI.15.880.5], Harvard University Archives, Cambridge; 28 September 1825, Corporation Records (cit. n. 19), vol. 6. See also tutor James Hayward's argument to the Corporation's committee on expenses of the college that more instructors were needed and that one additional professor would be less expensive but more capable than two tutors; James Hayward to Charles Jackson, April 1824, Harvard College Papers (cit. n. 23), vol. 11.

[55] 10 October 1825, Corporation Records (cit. n. 19), vol. 6; John T. Kirkland to John Farrar, 5 December 1825, President's Papers, John T. Kirkland (cit. n. 54); 7 December 1825, Corporation Records (cit. n. 19), vol. 6.

[56] 15 August 1822, Corporation Records (cit. n. 19), vol. 6; Farrar made arrangements for covering his teaching responsibilities in John Farrar to John T. Kirkland, August 1822, Harvard College Papers (cit. n. 23), vol. 10.

[57] Harvey P. Holden, "Athens and Sparta: The Archivist and Resource Allocators," *Provenance* 5, No. 2 (1987): 37-46, on p. 38.

[58] John Farrar to John T. Kirkland, 5 July 1826, Harvard College Papers, 2d ser. (1826-1838), vol. 1 [UAI.5.131.10], Harvard University Archives, Cambridge; 6 June 1826 and 5 July 1826, Corporation Records (cit. n. 19), vol. 6.

instruments by the Corporation.[59] Farrar then unsuccessfully lobbied to divide

mathematics and natural philosophy into two separate departments; while Farrar

argued that the public desired more education in the mechanical arts and that he

and one tutor could not meet the increasing demands, the Corporation's committee

on reducing expenses responded that Harvard could not afford professors who

taught "only those who have mastered the rudiments" by communicating the

discoveries of other learned men.[60] The following term, the high-strung Farrar fell ill

and was unable to deliver the astronomy lectures to the seniors.[61]

Perhaps the time was right for Farrar's personal life to improve. In 1820, Eliza

Ware Rotch (1791-1870) remained in New Bedford, Massachusetts, after a visit with

her grandfather, the patriarch of a prominent whaling merchant family, while her

parents returned to Wales, where they had settled after being detained by a lawsuit

during the French Revolution.[62] Eliza appreciated the quiet domesticity of New

Bedford, but she also entered Boston society through her cousins, the George Bond

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[59] Newspaper Clipping, 2 December 1826, Biographical Materials, John Farrar [HUG 300], Harvard University Archives, Cambridge; 11 December 1826, Corporation Records (cit. n. 19), vol. 6. Similarly, Farrar's request to give a public evening lecture in Harvard Hall in 1831 was denied, apparently because it was a fire risk; 13 January 1831, Corporation Records (cit. n. 19), vol. 7.

[60] John Farrar to John T. Kirkland, 5 January 1827, Harvard College Papers, 2d ser. (cit. n. 58), vol. 1; 19 April 1827 and 29 April 1827, Corporation Records (cit. n. 19), vol. 7. In August, Farrar's salary was cut by two hundred dollars per year; 28 September 1827, Corporation Records (cit. n. 19), vol. 7.

[61] John Farrar to John T. Kirkland, 23 September 1827, Hollis Professorship of Mathematics (cit. n. 27).

[62] See Joonok Huh, "Farrar, Eliza Ware Rotch," in *American National Biography* (hereinafter cited *ANB*), ed. John A. Garraty and Mark C. Carnes, vol. 7 (New York and Oxford: Oxford University Press, 1999), pp. 737-738; and Eliza Farrar, "Memorials of the Life of Elizabeth Rotch, Being the Recollections of a Mother, By Her Daughter, Eliza Farrar" (Springfield, 1861), chapters 1-4, Rotch Family Papers, Subgroup 12, Eliza Rotch Farrar Papers, 1281-1830, 1861, Old Dartmouth Historical Society, New Bedford, MA.

family.[63] Eliza apparently met Farrar on a visit to Boston in 1827; despite his occasionally disagreeable behavior, she began to compete with another young woman for his affection.[64] Farrar's marriage proposal in May 1828 was accepted in early June after Eliza quizzed the Bonds over his background and Farrar visited her again in New Bedford.[65] A sophisticated and cultured woman—while married to Farrar, Eliza made a name for herself by writing biographies for children and widely-read etiquette books for young ladies[66]—Eliza was still relieved as her extended family became acquainted with Farrar and realized what a "great acquisition he is to the family circle," and she fretted about having to receive visitors as soon as she moved into Farrar's bachelor house after their October wedding.[67]

Unfortunately, the respite afforded by Farrar's marriage was only temporary for him, as his stressful professorship exerted an ever-higher toll. Josiah Quincy (1772-1864) became president of Harvard in 1829 and regimented the recitation system with a Scale of Merit.[68] Although students detested the new grading scale in

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[63] Eliza Farrar to Mrs. George Bond, 15 March 1821 and 23 September 1821, Rotch Family Papers, 1821-1828, Massachusetts Historical Society, Boston.

[64] Eliza to Mrs. Bond, 31 December 1827, Rotch Family Papers (cit. n. 63).

[65] Eliza to Mrs. Bond, 7 May 1828 and 2 June 1828, Rotch Family Papers (cit. n. 63).

[66] Books by Eliza Farrar still held by Harvard University include: *The Children's Robinson Crusoe* (Boston: Hilliard, Gray, Little, and Wilkins, 1830); *The Story of the Life of Lafayette* (Boston: Hilliard, Gray, Little, and Wilkins, 1831); *The Adventures of Congo in Search of His Master: A Tale* (London: J. Harris, 1832); *John Howard* (Cambridge: Brown, Shattuck, 1833); *The Youth's Letter-Writer* (New York: R. Bartlett and S. Raynor, 1834); *The Young Lady's Friend* (Boston: American Stationers' Company, 1836); and *Recollections of Seventy Years* (Boston: Ticknor and Fields, 1866). A revised edition of *Friend* was issued as late as 1880. See also *NUC* (cit. n. 47), vol. 167, pp. 234-235.

[67] Eliza Farrar to Mrs. George Bond, 29 July [1828] and 2 September 182[8], Rotch Family Papers, Subgroup 12 (cit. n. 62). Farrar, his first wife, and her three sisters lived with President Kirkland after spending one year in a parsonage. Farrar designed a house and had it built, but he did not move there until 1825 and did not occupy more than an apartment within the house until his second marriage. See [Palfrey], "Professor Farrar" (cit. n. 16), pp. 130-131.

[68] Morison, *Three Centuries* (cit. n. 10), pp. 246-272. Quincy also tried to discipline the students severely; Farrar was a member of a minority faction of faculty which opposed Quincy's measures;

particular and the method of instruction in general, they enjoyed Farrar as a teacher.

Eliza reported that his lecture room was crowded each day as the seniors from the

previous year and other interested people joined the seniors in the class.[69] Farrar

was flattered by the attention but overwhelmed by the accompanying

responsibilities and his nagging physical and mental problems. According to a

report filed for the academic year from 1828 to 1829, Farrar and the tutor spent the

equivalent of thirty-one forty-hour weeks just in one-to-one contact with the

students.[70] Then, in July, Quincy asked Farrar to increase his recitation hours from

four per week to nine, a requirement Farrar claimed had not existed since

Greenwood was inaugurated as Hollis Professor in 1728. Farrar had considered

resigning his professorship to travel to Europe at least as early as 1828, and he now

threatened that, if he were not allowed to relinquish $500 of his salary to fund two

proctors for the recitation, "I am ready to withdraw from the College entirely."[71]

Seymour Martin Lipset, "From the Post-Revolution Era to the Civil War," in "Political Controversies at Harvard, 1636-1974," in *Education and Politics at Harvard*, Seymour Martin Lipset and David Reisman (New York: McGraw-Hill Book Company, 1975), pp. 47-90, on p. 69.

[69] Eliza Farrar to Anna Rotch, 28 April 1830, Rotch Family Papers, Subgroup 12 (cit. r. 62). John Gorham Palfrey illustrated Farrar's popularity with students despite the difficult subjects he taught with a verse from a Harvard drinking song:

> "When to Farrar we drink, let no heel-taps remain,
> For so worthy a soul we shall ne'er see again;
> Who never grudged any the credit his due,
> And *like a true gentleman* handled the screw." [Palfrey], "Professor Farrar" (cit. n. 16),
> p. 129; emphasis in source.

[70] 1828-1829 Financial Report, Harvard College Papers, 2d ser. (cit. n. 58), vol. 4. When Eliza described what appears to have been a typical day for Farrar to her cousin, she explained that he had a recitation section before breakfast, delivered a lecture at 11 o'clock, administered a two-hour student examination in the afternoon, reviewed mathematics, attended a governmental meeting, and looked over the mathematics lesson for the next day at 9 o'clock before bed, all the while suffering from a toothache; Eliza Farrar to Anna Rotch, 22 April 1830, Rotch Family Papers, Subgroup 12 (cit. n. 62).

[71] John Farrar to Josiah Quincy, 28 July 1830, Harvard College Papers, 2d ser. (cit. n. 58), vol. 4. On Farrar's possible trip to Europe, see George Ticknor to George Bancroft, 5 January 1828, George Bancroft Papers, 1822-1834, Massachusetts Historical Society, Boston.

Even though Farrar did receive the help of an assistant, his physical and mental health continued to break down. In May 1831, he reported to Quincy that he needed a break from teaching and planned to travel to England. Farrar suggested that he could give all the natural philosophy lectures for the year in the last term after his return and offered again to resign if this was not possible.[72] By the end of July, arrangements had been negotiated for Farrar to give one lecture at 11 o'clock and one in the evening, which actually would have been convenient for the astronomy course—Farrar indeed planned to deliver those talks outdoors. In addition, the Corporation directed Farrar to shop for a planetarium and for apparatus for the Rumford Professorship during his trip.[73] By the end of 1831, the Farrars departed for a stay of nearly six months in Great Britain, where they spent their time purchasing optical instruments and acquainting Farrar with Eliza's parents.[74] Of the visit, Eliza wrote: "[Farrar's] refined appearance, good manners, and gentle nature endeared him to my mother, whilst my father delighted in adding to his own knowledge of Natural Philosophy, the more accurate information and late discoveries, which the Professor could give him. . . . My mother's feelings, both as an American, and a parent, were highly gratified by the attentions her Yankee

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[72] John Farrar to Josiah Quincy, 18 May 1831, Harvard College Papers, 2d ser. (cit. n. 58), vol. 4.

[73] John Farrar to Josiah Quincy, 22 May 1831 and 21 July 1831, Harvard College Papers, 2d ser. (cit. n. 58), vol. 4; 19 May 1831, 23 June 1831, 21 July 1831, 19 January 1832, 21 June 1832, Corporation Records (cit. n. 19), vol. 7.

[74] John Farrar to Josiah Quincy, 12 November 1831, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5; John Farrar to Joseph E. Worcester, 28 February 1832, Joseph E. Worcester Papers, vol. II (C-G), p. 140, Massachusetts Historical Society, Boston. Farrar wanted to attend the first meeting of the British Association for the Advancement of Science held in York on 26 September 1831, but he did not get there; Jack B. Morrell and Arnold Thackray, *Gentlemen of Science: Early Years of the British Association for the Advancement of Science* (Oxford, 1981), p. 69.

son-in-law received from the scientific world in England, and she would often urge

him to accept invitations which his feeble health obliged him to decline."[75]

When Farrar returned to Cambridge in 1832, it soon became clear that the

next generation was ready to assume control of Harvard instruction in mathematics

and natural philosophy. Benjamin Peirce (1809-1880) had become friends with

Bowditch's son as a teenager in grammar school, and he began to help proofread

and correct Bowditch's translation of *Mécanique céleste* before graduating from

Harvard in 1829.[76] After teaching at George Bancroft's prestigious Round Hill

School, Peirce was called back to Harvard in 1831 to be the tutor who taught all the

recitations during Farrar's absence. In recognition of his responsibilities, a

professorship was founded for Peirce in 1833.[77] Farrar had never overseen any more

recitations since before he left for Great Britain, permanently accepting a one-third

reduction in salary.[78] Despite the reduced workload, however, Farrar proved unable

to resume any significant scientific activity.[79] Finally, his doctor ordered him to

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[75] Farrar, "Memorials," Rotch Family Papers, Subgroup 12 (cit. n. 62), pp. 127-129.

[76] The four volumes of *Celestial Mechanics* appeared in 1828, 1832, 1834, and, after Bowditch's death in 1838, in 1839. For biographies of Peirce, who is often considered the first truly professional mathematician in the United States, see Amy Ackerberg, "Peirce, Benjamin," in *ANB* (cit. n. 62), vol. 17, pp. 250-251; and Raymond Clare Archibald, "Peirce, Benjamin," in *DAB* (cit. n. 2), vol. 14, pp. 393-397. The famous friendship between Peirce and Bowditch is also covered in I. Bernard Cohen, ed., *Benjamin Peirce: "Father of Pure Mathematics" in America* (New York: Arno Press, 1980). On Peirce's mathematics, see Helena M. Pycior, "Benjamin Peirce's *Linear Associative Algebra,*" *Isis* 70 (1979): 537-551; and Ivor Grattan-Guinness, "Benjamin Peirce's *Linear Associative Algebra* (1870): New Light On Its Preparation and 'Publication,'" *Annals of Science* 54 (1997): 597-606.

[77] 16 March 1833, Corporation Records (cit. n. 19), vol. 7. Peirce's title was "University Professor of Mathematics and Natural Philosophy." Since this position was unendowed, Peirce transferred into the Perkins Professorship of Astronomy and Mathematics when it was established in 1842.

[78] See, for example, Financial Report, August 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 6.

[79] For example, Farrar wrote to Elias Loomis that he had been laid up twice after attempting to spend a few nights observing a comet; John Farrar to Elias Loomis, 23 October 1835, Loomis Papers, vol. E-G (4), Beinecke Rare Book & Manuscript Library, Yale University, New Haven.

resign due to his "frequent attacks of nervous debility and slow fever" in 1836.[80] At

that time, Farrar transferred responsibility for the philosophical apparatus to Joseph

Lovering, a tutor of mathematics and natural philosophy who was not replaced

when he was officially elected as Hollis Professor in September 1837.[81] This meant

that, for a time, all Harvard mathematics courses were taught by professors.[82]

The Farrars returned to Europe in the summer of 1836, where they lived and

traveled rather comfortably on the profits from their books, despite an economic

recession and the difficulties inherent in transacting their affairs from overseas via

an agent.[83] Farrar in fact left Harvard hopeful that he would recover relatively

quickly, telling Nathan Hale that, "I expect to return by the middle of April [1837]

when I presume the lectures [which Hale had invited Farrar to deliver to the Society

for the Diffusion of Useful Knowledge] will be nearly or quite completed," and

discussing an updated edition of *Elements of Electricity, Magnetism, and Electro-*

Dynamics with his agent, Willard Phillips.[84] The Farrars remained in Europe until

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[80] [Palfrey], "Professor Farrar" (cit. n. 16), p. 131. See also John Farrar to Josiah Quincy, 15 June 1836, Harvard College Papers, 2d ser. (cit. n. 58), vol. 7; and 16 June 1836, Corporation Records (cit. n. 19), vol. 7.

[81] 21 July 1836, Corporation Records (cit. n. 19), vol. 7; 21 September 1837, Overseer's Records (1788-1812, 1824-1847) [UAII.5.5.2], vol. 8, Harvard University Archives, Cambridge.

[82] *Harvard College Triennial Catalogues* (Cambridge, 1830-1839).

[83] The agent was Willard Phillips (1769-1875), a lawyer and former Harvard tutor who served in the Massachusetts state legislature, wrote books on insurance and law, was president of the New England Mutual Life Insurance Company, and later edited *American Jurist and Law Magaizine* and *North American Review*. See the series of letters from Farrar to Phillips: 31 July 1836, December 1836, 4 February 1837, 16 March 1837, 8 June 1837, 18 June 1837, 6 October 1837, 19 November 1837, 29 December 1837, 23 February 1838, 17 May 1838, 27 July 1838, 31 August 1838, 28 September 1838, 16 January 1839, 15 October 1839, and 27 January 1840; Willard Phillips Papers, Boxes 9-10, 1835-1841, Massachusetts Historical Society, Boston.

[84] John Farrar to Nathan Hale, 22 July [1836], Boston Society for the Diffusion of Useful Knowledge Papers II, Massachusetts Historical Society, Boston; John Farrar to Willard Phillips, 16 March 1837, 18 June 1837, and 29 December 1837, Willard Phillips Papers (cit. n. 83).

1840, however, and so he was never able to lecture at the Society. Farrar was able to

participate in physical activities such as hikes during a stay in the Mediterranean in

1838, but then Eliza fell ill in 1839 while Farrar suffered a paralyzing tremor in his

right hand and was so disturbed as a result that Eliza kept the death of her father

from him.[85] She took Farrar to an asylum her brother had founded, but Farrar did

not believe his health improved there and indeed it did decline for a time when

doctors prescribed him opiates and morphine.[86] Their funds diminishing and lonely

for their Cambridge friends, the Farrars finally returned to Harvard once Eliza

found a companion for her mother.[87]

Farrar served on the examining committee in physics at Harvard with Oliver

Wendell Holmes and others the following winter, in 1841.[88] His last surviving letter,

in which Farrar confirmed to Loomis that the only magnetic dipping needle known

to have been used by Samuel Williams was an unreliable instrument, was written

that July.[89] Farrar continued to suffer relapses of his physical and mental ailments,

eventually becoming unable to control his movements, until he died on May 8, 1853.

During his illness, Eliza sacrificed her own writing career and her interest in the

abolitionist cause to care for Farrar and conserve their funds, although he was

comfortable enough for her to leave Cambridge and visit her mother in the summer

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[85] John Farrar to Willard Phillips, 16 January 1839 and 15 October 1839, Willard Phillips Papers (cit. n. 83); Farrar, "Memorials," Rotch Family Papers, Subgroup 12 (cit. n. 62), chapter 18.

[86] John Farrar to Willard Phillips, 27 January 1840, Willard Phillips Papers (cit. n. 83); [Palfrey], "Professor Farrar" (cit. n. 16), pp. 133-134.

[87] Farrar, "Memorials," Rotch Family Papers, Subgroup 12 (cit. n. 62), chapter 19. Eliza felt Farrar arrived in Massachusetts in worse health than when they left four years earlier; Farrar, *Recollections* (cit. n. 66), p. 263.

[88] 11 February 1841, Overseer's Records (cit. n. 81), vol. 8.

[89] John Farrar to Elias Loomis, 13 July 1841, Loomis Papers (cit. n. 79).

before he died.[90] Eliza then erected the grave monument described at the beginning

of this chapter, spent the next several years in Europe to be nearer to her mother,

who died in 1857, and passed her last years in the United States, during which time

she donated all of Farrar's books to the Harvard Library.[91]

The Convoluted Development of *Elements of Geometry*

Farrar never set out to translate mathematics textbooks. When he assumed

the Hollis Professorship in 1807, the mathematics curriculum was centered around

Webber's *Mathematics* and Playfair's *Elements*. President Webber prepared a second

printing of his compendium, which appeared in 1808.[92] Three years later, members

of the Harvard Board of Overseers asked Farrar and Kirkland, then Harvard's new

president, to investigate printing just the chapter on arithmetic from Webber's first

volume for sale to the public.[93] This appeared as *A System of Arithmetic* in 1812.

Nothing more was said on mathematics textbooks until 1816, when the Harvard

[90] See Eliza Farrar to Maria Weston Chapman, 5 September 1844 and 5 December 1844 ; and Eliza Farrar to Unidentified, 25 November 1844 [Ms. A.9.2 vol. 20, nos. 58, 105, 117], Boston Public Library, Boston; Farrar, *Recollections* (cit. n. 66), pp. 263-264.

[91] For example, Harvard's copies of Jeremiah Day, *A Practical Application of the Principles of Geometry to the Mensuration of Superficies and Solids* (New Haven: Oliver Steele, [1816]); and [Thomas Carlyle, trans.], *Elements of Geometry and Trigonometry; With Notes. Translated from the French of A. M. Legendre*, ed. David Brewster (Edinburgh: Oliver & Boyd, 1824), were owned by Farrar. Unfortunately, Farrar's library is now disbursed among Harvard's collection. Farrar, "Memorials," Rotch Family Papers, Subgroup 12 (cit. n. 62), chapters 22-27; C. C. Felton to Eliza Rotch Farrar, 27 June 1861, C. C. Felton Papers, Harvard University Archives, Cambridge.

[92] Webber contracted for the second printing on October 7, 1806, but the printer needed him to buy type first; 7 October 1806 and 16 December 1806, Corporation Records (cit. n. 19), vol. 4. Harvard purchased copies of Playfair's *Elements* from Philadelphia bookseller Francis Nichols, the first to reprint the work in the United States.

[93] Since Webber had just passed away in July 1810, this may also have been in part a relief effort for his widow. In 1812, the Corporation voted to give her 6 1/4¢ for each arithmetic printed and sold. See 14 January 1811 and 7 February 1812, Corporation Records (cit. n. 19), vol. 5. Yet, the Corporation had also directed Farrar and Kirkland to consider raising the admission requirements in mathematics above the current standard, arithmetic as far as the Rule of Three; 27 May 1811, Corporation Records (cit. n. 19), vol. 5.

Corporation established a committee to hear Farrar's recommendations on a mathematics textbook.[94] Farrar's original directive was to make additions to the algebra chapter of Webber's *Mathematics* to bring the book up to date, but by the time he reported to President Kirkland in January 1817, Farrar had decided that the desired revisions were so difficult to incorporate into Webber's old-fashioned textbook as to be impossible. Based on a conversation with Kirkland, Farrar "considered [him]self as authorized to substitute new treatises both of algebra and trigonometry" and was already teaching from and ready to print a translation from French into English of Leonhard Euler's *Introduction to the Elements of Algebra*.[95] Furthermore, Farrar had found that the other mathematical subjects covered in *Mathematics* could not be salvaged, either: "I have been looking forward also to the other branches but I have been utterly unable to execute the plan proposed in any tolerable manner. The book will not be of a piece. The parts will not connect with each other. . . . Besides all this the present Text Book in those parts which we proposed to retain is prepared upon a plan altogether different from that which we now think the state of the college and of the country requires."[96] The main fault with *Mathematics*, according to Farrar, was that it contained too many examples and too few general principles. Webber's language was outmoded, and his demonstrations were in the notation of Newtonian fluxions.

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[94] 8 May 1816, Corporation Records (cit. n. 19), vol. 5; *NUC* (cit. n. 47), vol. 652, p. 97. At this time, all of arithmetic finally became an entrance requirement; Cajori, *Teaching and History* (cit. n. 15), p. 60.

[95] John Farrar to [John T. Kirkland], 22 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

[96] John Farrar to [John T. Kirkland], 22 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

Yet, at this point, Farrar was rather ambivalent about the alternate course of action he would advise the Harvard Corporation to follow. He suggested at first the preparation of a two-volume textbook similar to Webber's *Mathematics*, with a full treatment of algebra and geometry in the first volume and plane and spherical trigonometry, applications of trigonometry to navigation and nautical astronomy, stereographic, orthographic, and gnomonic projections; and applications of algebra to geometry including a treatment of conic sections in the second volume.[97] Within a few days, though, Farrar was telling Kirkland that he had not started work on this new compilation because he thought the Corporation would go ahead and have another edition of *Mathematics* printed instead. Since it would take a considerable amount of time to prepare a new compilation and Farrar preferred to spend his time writing a textbook on natural philosophy, Farrar suggested two alternatives: the publication of separate treatises in English such "as are best suited to our purpose on each of the several branches of mathematics," or adoption of Jeremiah Day's series, supplemented by the "republishing [of] such English works as may be necessary to complete the course of mathematical instruction."[98] However, three months later, Farrar wrote to Day to express his concern that the Yale professor was not going to finish his course of mathematics and to inform Day that, since "old institutions and large bodies are slow in making improvements," the Harvard Corporation had rather decided to print another edition of Webber's *Mathematics* with alterations and substitutions by Farrar and thus adopted neither of Farrar's

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[97] John Farrar to [John T. Kirkland], 22 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

[98] John Farrar to [John T. Kirkland], 26 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

alternatives.[99] Farrar expected this project to take a year to complete, a fact he

reiterated after Day expressed interest in the proposed textbook.[100] Nearly one year

later, Francis Nichols was warning Day that it would be "some years" before Farrar

and Harvard furnished the work, while Farrar had expanded his proposed

compendium to five volumes, including a one-volume work of arithmetic and

algebra to be mastered by prospective Harvard students, the two volumes outlined

above, and two volumes for Harvard students who wanted to study additional

mathematics, covering second parts of algebra and geometry in addition to the

subject of perspective and either fluxions or integral and differential calculus.[101]

In the end, Farrar apparently stalled the Corporation long enough on the

compendium that Harvard was forced to replace Webber's *Mathematics* with a

mathematical series, albeit one prepared by Farrar himself. In other words, Farrar

both did and did not get what he desired. He brought superior material into the

curriculum — although each of these "new" works were already at least twenty years

old themselves — but he invested a significant amount of time in translating or

supervising translation work in return. He also had to publish the translation of

Euler's *Introduction to Algebra* at his own expense.[102] By October 1818, without any

further discussion of mathematics textbooks being preserved in Harvard records,

Farrar won over members of the Corporation, likely by demonstrating that the

students responded well to the alternate textbooks. Harvard funded the publication

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[99] John Farrar to Jeremiah Day, 15 April 1817, Day Papers: Letters to Jeremiah Day, Beinecke Rare Book & Manuscript Library, Yale University, New Haven.

[100] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 99).

[101] Francis Nichols to Jeremiah Day, 23 March 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99); John Farrar to John T. Kirkland, 20 May 1818, Harvard College Papers (cit. n. 23), vol. 7.

[102] Francis Nichols to Jeremiah Day, 23 March 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99).

of Farrar's translations into English of Silvestre François Lacroix's *Elements of Algebra* and *Elementary Treatise of Arithmetic*.[103] The rest of the series, popularly known as the "Cambridge Course of Mathematics," appeared in subsequent fashion, beginning with the 1819 translation of Legendre's *Éléments*.

Farrar's idea to translate Legendre's *Éléments de Géométrie* evolved both within the development of the series and independently. As with the other elementary mathematical subjects, Farrar originally intended to leave geometry as part of the revised two-volume compendium; yet, he wanted to substitute a new treatise for the geometrical portions of Webber's *Mathematics* and also Playfair's *Elements*.[104] He developed a list of several reasons why it was preferable to replace Playfair's version of the Euclidean system. For example, he told Day that he wanted a modern treatise in geometry because "[t]here is certainly a good deal that is superfluous in Euclid & a good deal that we do not find there. We need no longer study pure mathematics as a matter of curiosity or for the gratification of perceiving abstract truth. Phys. applications are sufficiently numerous and important to exhaust the time allotted for an elementary course of instruction."[105] To Kirkland, Farrar wrote, "But it is not so much for the superfluous matter contained in Euclid

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[103] Francis Nichols to Jeremiah Day, 23 March 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99). The new books gained institutional approval despite the facts that Nichols thought poorly of Farrar's choices and that Webber's *Mathematics* seemed to refuse to go away: "Farrar's course cannot succeed, and ought not to prevent or retard the completion of [Day's series]. . . . A son of the late Dr. Webber proposes to print certain parts of W's Mathematics, with alterations and improvements." Francis Nichols to Jeremiah Day, 3 October 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99). Of course, Nichols may have begun to become disgruntled after Farrar had discouraged the Harvard Corporation from purchasing an algebra textbook from Nichols; 22 January 1818, Corporation Records (cit. n. 19), vol. 5.

[104] John Farrar to [John T. Kirkland], 22 January 1817 and 26 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

[105] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 99).

as for its deficiency that a change is necessary."[106] While Farrar paid homage to

Euclid's *Elements* as a "specimen of clear and close reasoning [that] has rarely if ever

been surpassed," he argued that versions of this work did not include the

achievements of recent researchers in geometry, who had applied their results to

"almost every branch of physical science," including chemistry and mineralogy.[107]

He believed that a treatise such as Lacroix's *Élémens de géométrie* was modeled on

Euclid's *Elements* in structure but was written in more concise and definite modern

language. The Corporation accepted Farrar's textbook recommendation on June 30,

1817.[108] By the next year, though, unidentified persons had suggested that there was

a more comprehensive French geometry textbook available, for Nichols reported to

Day later in 1818 that Farrar had been convinced to set aside Lacroix's textbook in

favor of Legendre's *Éléments* and that "a young man [was] translating the work."[109]

This book was published as the individual volume, *Elements of Geometry*, in 1819.

Like Thomas Carlyle's later translation, Farrar's *Elements of Geometry* was

based upon the 1817 eleventh edition of Legendre's *Éléments*. In general, Farrar

included all of Legendre's definitions and almost all of the propositions. He

removed three theorems on spherical isoperimetrical polygons from Legendre's

Book VI and Legendre's appendix to this book, on the five regular polyhedra. Farrar

restored these propositions at the end of his 1825 second edition, along with the

demonstration of the solidity of a triangular pyramid by Queret that Legendre

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[106] John Farrar to [John T. Kirkland], 23 June 1817, Hollis Professorship of Mathematics (cit. n. 27).

[107] John Farrar to [John T. Kirkland], 23 June 1817, Hollis Professorship of Mathematics (cit. n. 27).

[108] 30 June 1817, Corporation Records (cit. n. 19), vol. 5.

[109] Francis Nichols to Jeremiah Day, 3 October 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99).

incorporated into the 1823 twelfth edition of *Éléments* and sent to David Brewster for inclusion in Carlyle's translation. But after presenting nearly all of the main portion of the text, Farrar included only Legendre's first endnote and a portion of the second. In between, Farrar added his own note clarifying the English terms for the names used by Legendre. He then stopped in the middle of Legendre's note on the parallel postulate before Legendre's attempts at proofs and referred the reader to the third edition of Leslie's *Elements of Geometry* — where the disagreement between Legendre and Leslie was playing out.[110]

Just as Carlyle would three years later, Farrar expected that his readers would not have encountered an algebraic treatment of the theory of proportion before.[111] Farrar chose to take an explanation from Lacroix's *Élémens de géométrie* and include it as an introductory section. He also elaborated on new geometrical vocabulary. For example, he noted that a perpendicular from the center to one of the sides of a polygon was called an "apothême" by Legendre but that there was no English equivalent to the word.[112] Although Legendre used "solidity" to indicate the measure of a solid, Farrar commented that "volume" was less ambiguous and beginning to gain favor with mathematicians.[113] Many of Farrar's changes or additions, however, were stylistic. Like Day, he numbered each proposition

[110] John Farrar, trans., *Elements of Geometry by A. M. Legendre*, 2d ed. (Cambridge: Harvard University Press, 1825), pp. 219-222. Farrar also omitted Legendre's section on trigonometry. The preface in Farrar's *Elements of Geometry* reads like Legendre's writing, but it is not clear whether this brief essay on the nature of geometry is from the eleventh edition of *Éléments*, for that book was not available for this study. See *NUC* (cit. n. 47), vol. 323, p. 660.

[111] Advertisement to [John Farrar, trans.], *Elements of Geometry*, by Adrien-Marie Legendre (Cambridge: Cummings & Hilliard, 1819).

[112] [Farrar], *Elements of Geometry* (cit. n. 111), p. 162; see Adrien-Marie Legendre, *Éléments de Géométrie, avec des Notes*, 9th ed. (Paris: Firmin Didot, 1812), p. 135.

[113] [Farrar], *Elements of Geometry* (cit. n. 111), p. 137; Legendre, *Éléments* (cit. n. 112), pp. 181-182.

successively, rather than starting over in each book, which he renamed as a "section." Farrar also added references to other textbooks in the series, such as his translation of Lacroix's *Elements of Algebra.*

It is not clear how influential *Elements of Geometry* was in the short run. Although Farrar's translation was eventually printed ten times, eight of those printings were between 1829 and 1841.[114] Of the twenty-four colleges examined by Florian Cajori, only two besides Harvard permanently adopted the textbook — Bowdoin some time before 1855 and Alabama in 1833 — while five others used an unspecified version of Legendre's *Éléments.*[115] Indeed, even Harvard tutors did not consistently teach from the textbook. For example, the inscription in Harvard University's own copy of the 1795 first edition of Playfair's *Elements* reveals that it was used there as a textbook in 1826 by Charles Sumner, Harvard Class of 1830.[116] In 1833, the mathematics tutor was teaching from the course by Day and supplementing it with Playfair's *Elements.*[117] In the early 1820s, on the other hand, former tutor Caleb Cushing reported that students had taken more pride in their mathematical studies since Farrar's course was introduced.[118] During the period that

[114] See *NUC* (cit. n. 47), vol. 323, p. 661. One reason for the spurt in publication was that Farrar sold the copyright to his two textbook series in 1828, and *Elements of Geometry* was stereotyped in 1830. See George Ticknor to George Bancroft, 5 January 1828, Bancroft Papers (cit. n. 71).

[115] Cajori, *Teaching and History* (cit. n. 15). Three of these institutions were the University of Virginia, the United States Military Academy, and Columbia College, which all later adopted Charles Davies's adapted translation of *Éléments;* these three institutions were also listed by [Walker] review of *Elements of Geometry* (cit. n. 49), p. 193. Farrar named the same colleges and added Brown University in John Farrar to Josiah Quincy, 24 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5.

[116] John Playfair, *Elements of Geometry; Containing the First Six Books of Euclid, With Two Books on the Geometry of Solids* (Edinburgh: Bell & Bradfute, 1795); held in Special Collections, Monroe C. Gutman Library, Graduate School of Education, Harvard University, Cambridge.

[117] Farrar's book was bound with Day's *Mensuration* at least once itself; this copy is also held in the textbook collection within Special Collections in the Gutman Library at Harvard.

[118] Caleb Cushing to John Farrar, 18 June 1824, Harvard College Papers (cit. n. 23), vol. 11.

Farrar's *Elements of Geometry* was a required text, Harvard freshmen in the third

term learned and presented two pages from the first part of the book on the

blackboard and with only a book of diagrams as aid in daily sessions lasting

between thirty and forty-five minutes, following the same process as sophomores in

the first term of the next academic year in order to master the second part of the

book.[119]

Reviews of Farrar's series were also mixed. The first to appear was in *North

American Review*, where George Barrell Emerson (1797-1881) generally considered

the course to be "well adapted, and a great addition to our means of instruction."[120]

Emerson outlined all of *Elements of Geometry*, which he did not do with any of the

other books in Farrar's series. The others had been translated at least partially into

English previously, so Legendre's *Éléments* was likely the least familiar of the four

textbooks covered in the review. A second review of what became commonly

known as the "Cambridge Course of Mathematics" appeared in *American Journal of

Science* in 1822 and 1823.[121] Jasper Adams believed the French books to help

inculcate mental discipline but to fail to provide practical examples. He also

[119] For instance, see May 1824, Harvard College Papers (cit. n. 23), vol. 11; and John Farrar to John T. Kirkland, 26 December 1825, Harvard College Papers (cit. n. 23), vol. 11. Note also that an 1823 pamphlet signed by George E. Winthrop and containing the plates from *Elements of Geometry, Trigonometry, Applications of Trigonometry*, and *Topography* is held by the Massachusetts Historical Society, Boston.

[120] [George Barrell Emerson], review of *An Elementary Treatise on Arithmetic, An Introduction to the Elements of Algebra, Elements of Algebra*, and *Elements of Geometry*; trans. John Farrar, *North American Review* 13 (1821): 363-380, on p. 366. Emerson served as a tutor under Farrar and helped complete Farrar's translation of Bézout's calculus textbook.

[121] [Jasper Adams], "Review of the Cambridge Course of Mathematics" and "Elements of Geometry," *American Journal of Science* 5 (1822): 304-326; 6 (1823): 283-302. Farrar identified the author as "Professor Adams" and described the career of Jasper Adams, who was professor of mathematics and natural philosophy at Brown University from 1819 to 1824, when he became president of Charleston College for more than ten years; John Farrar to Josiah Quincy, 24 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5. See also Harris Elwood Starr, "Adams, Jasper," in *DAB* (cit. n. 2), vol. 1, part 1, p. 72.

provided a detailed outline of Legendre's *Éléments*.[122] Finally, the *North American*

Review printed a review of the second edition of Farrar's translation of *Elements of*

Geometry in 1828, which appears unusual at first glance, considering that the only

change Farrar made was to include the few propositions he omitted in the first

edition. However, Timothy Walker (1802-1856), the author of the review, published

his own geometry textbook two years later and in fact called for a textbook like the

one he published in the closing statements of the review: "Of this, however, we are

assured, that the wants of the public really do require a work on geometry less

amplified than Legendre, and at the same time rendered more practical; and we

know of no treatise which would so well serve for the basis of such a work, as that

which we have attempted to review."[123] Walker spent most of the review musing on

the meanings of geometrical definitions and listing propositions contained in

Legendre's *Éléments* which were not necessary for students. These theorems and

problems were among those omitted in his 1830 abridged version of Legendre's

Éléments.[124] All three reviewers, though, considered the translation to be a faithful

rendering of Legendre's book. Emerson said, "It is rare to find a mathematical book,

from the English or French presses, so uniformly free from errors," while Adams

concluded that, "American mathematical science, is under great obligations to the

translator, for giving Legendre's elements in so handsome an English dress."[125]

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[122] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 291-301.

[123] [Walker], review of *Elements of Geometry*, (cit. n. 49), p. 214.

[124] Timothy Walker, *Elements of Geometry With Practical Applications, For the Use of Schools*, 3d ed. (Boston: Richardson, Lord & Holbrook, 1831).

[125] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 380; [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 301.

Yet, Walker's self-serving review foreshadowed the undercutting of Farrar's labors in the textbook series by his own former students in the early 1830s. Toward the end of Farrar's European trip of convalescence, Peirce, Quincy, and Bowditch formed a committee on mathematical instruction and apparently determined to replace Farrar's translation of Legendre's *Éléments* with Timothy Walker's *Elements of Geometry.*[126] Like Peirce, Walker was a Harvard graduate whom Farrar had recommended for a teaching position at the Round Hill School and who repaid the favor with the review of Farrar's second edition of Legendre's *Éléments* essentially advocating his own textbook.[127] Shortly after his return from Europe, Farrar defended Lacroix and Legendre as elementary writers and reiterated that his translations were accurate.[128] Then, he focused in on the heart of his objections — notably, the Farrar who emphasized all of the ways in which geometry could be applied to the physical sciences in 1818 advocated geometry's role in the development of mental discipline in 1832:

> With respect to the proposed substitution of Walker's little book for Legendre's book I cannot but express my utter astonishment. . . . What would be tho't of a proposition to substitute . . . Corderius & Utropius for Livy & Tacitus? . . . They do not task the understanding sufficiently. They do not call forth all the energies of the youthful mind.
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> To the general student, the —— object of the study of Geometry is not any direct application of geometrical truths. Its only

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[126] 25 August 1831, Corporation Records (cit. n. 19), vol. 7.

[127] John Farrar to George Bancroft, 25 February 1826, George Bancroft Papers (cit. n. 71).

[128] John Farrar to Josiah Quincy, 24 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5. Parenthetically, Farrar also noted in this letter that Bowditch had corrected the translation of Bourdon's algebra textbook completed by Emerson under Farrar's supervision in 1831.

recommendation is that it invigorates & develops the intellectual powers. ... [Legendre's textbook] was written for such institutions as ours. It is perfectly suited to the comprehension of about nine out [of] ten of our students, as I well know by experience.[129]

At Harvard, though, it was too late to be a proponent of mental discipline. Peirce countered to Quincy that while Legendre's *Éléments* deserved its great reputation, teaching it took up too much of the time allotted to pure mathematics. Further, training students in the rigor of demonstration injured their minds and gave them a tendency to skepticism. In any event, Walker's book contained the most important theorems of Legendre's, so, lacking Farrar's emotional and financial attachment to his translation, Peirce seems to have considered the entire argument over retaining Farrar's textbook instead of adopting Walker's to have been somewhat insignificant.[130] The Corporation directed in August 1832 that Farrar's translation of Legendre's *Éléments* be replaced by Walker's textbook if sufficient copies of the newer work could be found.[131] Harvard records indicate Farrar wrote four more letters about geometry textbooks at that time, but these missives were not preserved among Harvard College papers. Peirce had by then completely replaced Farrar as the voice of Harvard elementary mathematical instruction, and he himself wrote a geometry textbook for Harvard in 1837.

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[129] John Farrar to Josiah Quincy, 24 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5.

[130] Benjamin Peirce to Josiah Quincy, 26 June 1832, Harvard College Papers, 2d ser, (cit. n. 58), vol. 5.

[131] 24 August 1832, Corporation Records (cit. n. 19), vol. 7. Note from above, though, that the tutor used Day's *Mensuration* and Playfair's *Elements* in 1833.

Farrar and Method of Proof

Once again, the framework of analysis and synthesis provides a way of considering what mathematics, geometry, and education meant to Farrar. For example, although it is not clear that he understood all the subtleties involved, Farrar was among the few Americans to recognize Legendre's concern for the language of geometry and, where one sense of analysis and synthesis enters in, the relationship between language and the organization of geometry into a system. In the preface by Legendre which Farrar included and Carlyle later omitted, Legendre discussed the definition of a straight line as "being the most important of the elements."[132] To avoid a detailed philosophical study to distinguish between a straight line between two points and the shortest line between two points and to allow the rest of the elements of geometry to rest upon the notion of a straight line, Legendre decided to treat his description of a straight line as both an *a priori* definition and a self-evident axiom. In other words, Legendre called a straight line "that which is the shortest between two points," and he assumed that that line was the only one between those two points.[133]

While Legendre sacrificed some of his desired exactness and precision in defining the straight line, he introduced other vocabulary to increase the accuracy of his systematization of elementary geometry. He explained the new definitions in the only endnote Farrar translated in its entirety.[134] For example, Legendre criticized the word "parallelogram" as not necessarily referring to a figure of four sides and for just being too long ("parallelogramme" in French). He would replace the term with

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[132] [Farrar], *Elements of Geometry* (cit. n. 111), p. v.

[133] [Farrar], *Elements of Geometry* (cit. n. 111), p. v.

[134] [Farrar], *Elements of Geometry* (cit. n. 111), pp. 200-204.

"rhomb." Legendre also distinguished between "equal," "symmetrical," and "equivalent" figures, reserving "equal" for situations where two figures, when one was applied to the other, coincided. He additionally pointed out the difference between an angle and its vertex. In general, Legendre was troubled when definitions contained statements which implied as true propositions which had not yet been demonstrated. Thus, early in the note, Legendre raised the possibility that "instead of putting the definitions, as is usual, at the head of a section, we distribute them through the section each in the place where the proposition implied is demonstrated."[135] His aim was to reduce the amount of material that readers were required to accept without proof.

Farrar added his own input to Legendre's concern for language by penning another endnote, which admittedly was more concerned with matters of translation than with the structure or placement of definitions. After noting that he had "carefully preserved" Legendre's improvements to the language of geometry, Farrar clarified the English words which he had to use in a different sense than was customary ("polygon," "polyedron," and "quadrilateral"), and he noted the English words ("rhombus" and "trapezoid") he employed to translate the French "lozenge" and "trapéze." He also explained why he chose "lune," coined by Charles Hutton, rather than "lunary surface" or Legendre's "fuseau" for the portion of the surface of a sphere between the semi-circumferences of two great circles: "as lune properly stands for the surface comprehended between two unequal circular curves, [it] was thought the least exceptionable."[136]

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[135] [Farrar], Elements of Geometry (cit. n. 111), p. 200.

[136] [Farrar], Elements of Geometry (cit. n. 111), p. 204.

Although questions of definition were occasionally raised by American professors in journal reviews, Farrar's students were more likely to be exposed to analysis and synthesis as a method of proof via the sense of proof techniques and, specifically, reference to particular diagrams. Farrar followed Legendre by putting the figures at the back of the textbook and by referring to the labels in the diagrams when stating the theorems and problems in the textbook (for instance, "If from a point O (fig. 24), within a triangle ABC, there be drawn straight lines OB, OC, to the extremities of BC, one of its sides, the sum of these lines will be less than that of AB, AC, the two other sides."[137]). (Farrar and Legendre's Figure 24 is shown in Figure 4.1.) Timothy Walker explained the viewpoint, commonly held by mathematicians associated with Harvard, that reference to particular diagrams was beneficial for beginning students by avoiding the difficulty of conceiving of generalization and abstraction:

> But let any one examine the enunciations of Legendre, in this point of view, and he will find a manifest superiority in them. This may be chiefly owing to the fact, that in Legendre each one [proposition] is rendered specific and definite by the introduction of letters, referring every part immediately to the diagram; whereas in Euclid the enunciations are all general and without letters.[138]

While Charles Davies would avoid reference to particular diagrams a decade later, Farrar was apparently not concerned that students would focus on the diagram used to illustrate any given proposition at the expense of the body of the proof and thus fail to comprehend that the proposition was generally true. In fact, the diagrams for

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[137] [Farrar], *Elements of Geometry* (cit. n. 111), p. 8.

[138] [Walker], review of *Elements of Geometry*, (cit. n. 49), p. 198.
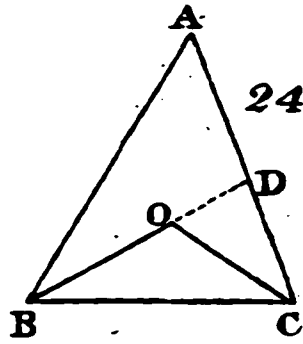
THEOREM.

Fig. 24. 41. *If from a point* O (fig. 24), *within a triangle* ABC, *there be drawn straight lines* OB, OC, *to the extremities of* BC, *one of its sides, the sum of these lines will be less than that of* AB, AC, *the two other sides.*

Demonstration. Let BO be produced till it meet the side AC in D; the straight line OC is less than $OD + DC$; to each of these add BO, and $BO + OC < BO + OD + DC$; that is

$$BO + OC < BD + DC.$$

Again, $BD < BA + AD$; to each of these add DC, and we shall have $BD + DC < BA + AC$. But it has just been shown that $BO + OC < BD + DC$, much more then is

$$BO + OC < BA + AC.$$

Figure 4.1. Farrar and Legendre referred directly to the diagram, although the diagram was placed in the end matter of the textbook. From [John Farrar, trans.], *Elements of Geometry* (Cambridge, 1819), p. 8 and Plate 1.

Elements of Geometry were printed separately so the students could bring them to class as an aid with recitation. As at Yale, Harvard students recited from the diagram only and without having the written proof in front of them.[139]

Farrar and Mathematical Styles

American intellectuals already perceived mathematicians in Great Britain and on the Continent as adherents to separate styles of mathematical practice by the time Farrar translated *Elements of Geometry*. They were generally culturally aware, and, like Day, many imported foreign works and textbooks in mathematics and science if they were not able to travel to Europe and purchase books directly. Americans also read discussions of the so-called "British decline," such as the articles by Playfair in the *Edinburgh Review*. For example, Emerson echoed Playfair as he voiced his belief that Americans had been caught up in the so-called "British decline" and praised Farrar's translation of Lacroix's algebra textbook as "one step, and a very considerable one, towards removing the reproach, to which, from community of language, we have been obnoxious, together with the English, of being almost a century behind the rest of the world in all that relates to mathematical and physical science."[140] Although they were thus exposed to celebrations of the superiority of

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[139] See, for example, John Farrar to John T. Kirkland, 26 December 1825, Harvard College Papers (cit. n. 23), vol. 11; where Farrar reported that the only books allowed in recitation were the pamphlets of diagrams, that even these were allowed only for the most complicated figures, and that "the student being required for the most part to draw the figure upon a black board, & to demonstrate the proposition without other aid."

[140] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 374. Farrar did not often discuss the issues which have been incorporated under the rubric of analysis and synthesis in this study, so this section and the following one rely more heavily on the accounts of the reviewers of Farrar's series than the chapters on Day and Davies do. Emerson, especially, can be considered to speak in agreement with Farrar, for he worked closely with his former professor. See Helena M. Pycior, "British Synthetic Vs. French Analytic Styles of Algebra in the Early American Republic," in *The History of Modern Mathematics*, ed. David E. Rowe and John McCleary, vol. 1 (San Diego: Academic Press, Inc., 1989), pp. 125-154, on p. 149. In addition, Farrar and his reviewers all tended to speak only of "English" mathematics. They meant, however, "British" mathematics in the sense that "British" is used in this project, to refer to English and Scottish influences taken together.

French achievements in mathematics, Americans still made their own decisions about whether to align with the side which preferred the differential and integral calculus to fluxions and, more generally, an algebraic form of mathematics founded on abstract rigor and accompanied by formal operations. Specifically, although they became especially vocal on the issue in the late 1810s and 1820s, American mathematics professors did not mimic the actions of the Cambridge Analytical Society. Americans did not adopt the Lagrangian form of the differential and integral calculus, and they were willing to replace Euclid's *Elements* with Legendre's *Éléments*.

For instance, Farrar's textbook series provided a venue for comparing British and French mathematics and mathematical textbooks in the native language of most Americans.[141] While Day had urged his readers to consult Legendre's *Éléments*; Americans could read the work without additional knowledge with Farrar's *Elements of Geometry*. But although they all endorsed the superiority of French mathematics, Farrar and his reviewers spent surprisingly little time setting out definitions of "British" and "French" mathematical styles to assist other professors and students with making the comparison. They tended to presume that their readers were familiar with textbooks currently in use in the United States, most from or based on British works, and they assumed that French textbooks were more soundly founded upon modern mathematics, which had the advantage of algebraic tools over ancient, pre-algebraic techniques. As Walker said, "The ancients were entirely unacquainted with algebra, and hence were in want of one powerful

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[141] This is not to say that Farrar was the first person to translate Continental mathematics textbooks into English. His translation of Euler's *Elements of Algebra* was based upon one by Francis Horner, and the Cambridge Analytical Society translated Book I of Laplace's *Mécanique céleste* in 1814 and Lacroix's *Traité du calcul différential et du calcul intégral* in 1816. See Pycior, "British Synthetic" (cit. n. 140), p. 131.

instrument which the moderns possess."[142] Their greatest emphasis, then, was on

the perception of French textbooks as analytical in the sense that the authors laid out

each mathematical discipline in a "natural order," which followed the path by which

great mathematicians had originally discovered the results which formed the

discipline. This conception of analysis as the method of invention will be addressed

in the following section.

Farrar did, however, criticize British influences and publicize French

mathematics and science both directly and indirectly in his journal publications. For

example, during the period that Farrar was discussing whether to repair or replace

Webber's *Mathematics*, he demonstrated his growing realization that Americans

could not depend on British authors to understand the needs of contemporary

students. Farrar therefore criticized what he saw as David Brewster's sloppy work in

republishing an outdated and nonmathematical astronomy textbook and only

crudely incorporating recent advances.[143] This text, popularly known as *Ferguson's*

Astronomy, was still taught at Harvard for several years after Farrar's review, but

Farrar's dissatisfaction with the book eventually led to his translation of Biot's

Elementary Treatise on Astronomy.[144] Additionally, in 1822, Farrar publicized John

Quincy Adams's five-year-old report on the relative merits of the French and

English systems of measure, in which Adams expressed hope that the United States

would improve upon and adopt the more uniform metric system.[145] Farrar quoted

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[142] [Walker], review of *Elements of Geometry*, (cit. n. 49), p. 199.

[143] [John Farrar], review of *Ferguson's Astronomy* (cit. n. 42).

[144] Farrar, *Elementary Treatise on Astronomy* (cit. n. 43).

[145] [Farrar], review of *Report on Weights and Measures* (cit. n. 42). The scientific interests of John Quincy Adams (1767-1848) may be an under-reported topic. One of the most recent scholarly biographies of Adams, Paul C. Nagel, *John Quincy Adams: A Public Life, A Private Life* (New York: Alfred A. Knopf, Inc., 1997), is typical by remaining rather spare in this regard and on the ways in which Adams continually crossed paths with Harvard after his studies there in 1786 and 1787 and his

Adams at length on the development and advantages of the French system of measurement with a few of his own comments on how to overcome resistance to a decimal system and his belief that the "new system of France is the fruit of an enlightened philosophy."[146]

Farrar and his readers were not overly concerned with the debate over the relative merits of algebra and geometry. For instance, although they taught the theory of proportion in the geometry course, they used arithmetic and symbols to convey the material just as Playfair and Legendre did. Since Legendre went directly to the more advanced proportional relationships between plane figures in *Éléments*, Farrar added a short treatise on proportion as an introduction to *Elements of Geometry*, noting only that, "[a]s the reader is supposed to be acquainted with algebraical signs and the theory of proportions, a brief explanation of these, taken chiefly from Lacroix's geometry, is prefixed to the work under the title of an introduction."[147] The material Farrar drew from Lacroix's *Élémens de géométrie* consisted of an explanation of arithmetical symbols, a description of how to raise numbers to powers, and illustrations of several of the possible manipulations of the terms of proportions.[148] Among the reviewers of Farrar's series, Emerson noted the presence of the introduction on proportion without further comment, and Walker

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brief tenure in 1805 as the first Boylston Professor of Rhetoric and Oratory despite noting that Adams favored mathematics and science and especially astronomy and algebra on pp. 45, 151. Farrar and Adams were probably not close friends, though. Farrar was mentioned only once in the excerpts of Adams's diary published by Adams's son, when he showed Harvard's philosophical apparatus to Adams—who was a member of the committee to examine the library—on 14 July 1836; Charles Francis Adams, ed., *Memoirs of John Quincy Adams*, vol. 9 (Boston, 1874-1877; reprint, Freeport, NY: Books for Libraries Press, 1969), p. 304.

[146] [Farrar], review of *Report on Weights and Measures* (cit. n. 42), p. 201.

[147] [Farrar], *Elements of Geometry* (cit. n. 111), p. iii.

[148] [Farrar], *Elements of Geometry* (cit. n. 111), pp. ix-xv.

ignored the issue, more concerned with outlining his proposed abridgement of

Legendre's *Éléments*.[149] Adams did find it necessary to defend the algebraic

treatment of proportion, mainly because he was troubled that William Wallace had

relied on Legendre's *Éléments* for the article on "Geometry" in the *Edinburgh*

Encyclopaedia but then had inserted a section on ratios and proportion based on the

geometrical methods of Euclid's *Elements*. In Adams's opinion, Wallace's action was

inappropriate: "But for us, who are in possession of algebraic methods, at once easy

and elegant, to pursue the same course [as Euclid], is entirely a different thing."[150]

He saw no reason for continuing to force students through the traditional approach

to the theory of proportion, since algebra was more flexible than geometry for this

topic and made proportion easier for beginners.

Farrar and Educational Technique

As was noted above, Farrar and his reviewers defined the analytical structure

of French mathematics textbooks as following the route mathematicians took in

discovering the truths of the disciplines taught to students, providing complete

explanations and a limited number of examples. Emerson and Adams both named

Lacroix as the archetypal disciple of "simple methods" and "natural order":

> Next to this object, of which he never loses sight, some of the rules by
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> which Lacroix seems to have been guided in composing the books
>
> before, were; 1°, making use of the analytical method, to pursue, as
>
> nearly as possible, the steps of invention; 2°, always to select the most
>
> general method; 3°, never to go over the same ground twice, either in
>
> his reasoning, or his explanations; 4°, to adapt the elements as he

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[149] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), 379-380.

[150] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 289.

professes to do, to the great works, which contain all that is most important in science.[151]

On the other hand, they viewed British mathematicians as "obscure" and adhering in all of mathematics to the synthetical method, which laid out all of the evidence and proceeded from simple to complex but which also failed "almost entirely in communicating to the mathematical reader, that spirit of invention, which may enable him, after perusing what is most valuable in the writings of others, to open a new track for himself."[152]

This sense of analysis led Farrar and his reviewers to frame their thoughts on education in terms of several contrasts. For instance, sometimes they indicated analysis and synthesis as the distinction between mathematics by moderns or by ancients. One of the reasons Farrar favored Legendre's Éléments was that the book was "thought to unite the advantages of modern discoveries and improvements with the strictness of the ancient method."[153] An enthusiast for "progress," Farrar was concerned that students be made aware of up-to-date results.[154] In 1817, while explaining why a replacement text for Playfair's Elements was needed, he remarked to Kirkland on the irony that, "There is scarcely anything in which our superiority over the ancients is more manifest and palpable than in mathematics and yet this is

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[151] [Emerson], review of An Elementary Treatise, etc. (cit. n. 120), pp. 366-367. This was not a unanimous view beyond the northeastern United States; in Philadelphia, Francis Nichols wholly disdained "the French mode of writing mathematics, which often mixes rules, operations, and explanations promiscuously together;" Francis Nichols to Jeremiah Day, 23 March 1818, Day Papers: Letters to Jeremiah Day (cit. n. 99).

[152] [Emerson], review of An Elementary Treatise, etc. (cit. n. 120), p. 369; [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 311-312.

[153] [Farrar], Elements of Geometry (cit. n. 111), p. iii.

[154] One of Farrar's waxes upon "progress" may be found in [Farrar], review of Discourse and Address (cit. n. 42), pp. 161ff.

almost the only branch of knowledge in which we continue to acknowledge them as our teachers."[155] Mastery of modern textbooks was deemed a necessary step for proceeding to a study of French astronomy and physics.[156]

Another contrast drawn by these authors was the difference between basing a mathematics textbook on general principles or on particular examples. For instance, Farrar said of Webber's *Mathematics* that, "It consists especially in the parts above mentioned [algebra, geometry, mensuration, fluxions] almost entirely of examples. I would reverse this and make them to consist almost entirely of principles with only such examples as are in themselves instructive or illustrative of a general truth."[157] In a related matter, Farrar proudly noted that his students were able to recite from Legendre's textbook and from "the analytical processes & demonstrations of Gummere [the natural philosophy textbook] . . . without any other aid than that afforded by the figure."[158] In addition, one of the drawbacks of British textbooks was that they relied on the ancients who had been too wed to particular cases. As Adams explained, "It is time to distrust this predilection for particular methods, under the idea that they are more elementary than general methods; whereas the truth is, that they are preferred because more ancient, and more agreeable to habits previously acquired, and which are not easily reformed."[159]

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[155] John Farrar to [John T. Kirkland], 23 June 1817, Hollis Professorship of Mathematics (cit. n. 27).

[156] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 374.

[157] John Farrar to [John T. Kirkland], 22 January 1817, Hollis Professorship of Mathematics (cit. n. 27).

[158] John Farrar to John T. Kirkland, 26 December 1825, Harvard College Papers (cit. n. 23), vol. 11.

[159] [Adams], "Review of the Cambridge Course" (cit. n. 121), 5 (1822): 311.

In these cases, Farrar and his readers associated modern treatments founded on general principles with French textbooks such as Legendre's *Éléments*. One extended discussion of the differences between the French (analytical) and British (synthetical) styles of writing and teaching mathematics was given by Adams. He argued that the textbooks by Lacroix and Euler rigorously demonstrated every rule and principle which was introduced, while American and British arithmetics and algebras listed the rules without investigating them. Adams also favored Lacroix's mathematical writing because it avoided repetition — the French author added recent results rather than present several different proofs of the same proposition. Lacroix chose general over particular methods, used the analytical methods of invention, and prepared students to master higher mathematical treatises.[160] Adams believed it was important to equip mathematics readers to make discoveries and that everyone should be prepared to study higher mathematics even if all students did not "devote a considerable part of [their lives] to mathematical learning."[161] When he turned to Legendre's *Éléments*, Adams began with the reasons why that textbook could be preferable to Euclid's *Elements* according to his view of what a geometrical treatise should contain.[162] An excellent guide for students should be filled with propositions which were widely applicable, be rigorous yet concise, be constructed in a deductive chain and in natural order, avoid *reductio ad absurdum*, and be synthetical and uniform in style.[163] In these respects, Legendre's and Lacroix's *Éléments* corrected

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[160] [Adams], "Review of the Cambridge Course" (cit. n. 121), 5 (1822): 310-313.

[161] [Adams], "Review of the Cambridge Course" (cit. n. 121), 5 (1822): 312.

[162] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 285.

[163] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 286.

many of the imperfections in Euclid's *Elements*.[164] For example, Adams disapproved of the many isolated or subsidiary propositions in Euclid's *Elements*, preferring the arrangements by Legendre and Lacroix, where all the propositions not only depended on the previous propositions but also were depended upon by succeeding propositions.[165] The French textbooks were also clearer and more complete on the geometry of solids than Simson's *The Elements of Euclid* or Playfair's *Elements*.[166]

Another distinction made by American professors and often cited by historians of American science was the separation of abstract theory from practical knowledge. With his emphasis on general principles, Farrar generally favored theory over practice. Early in the 1810s, Farrar explained that he did not teach memorization of astronomical tables because the general science of chemistry was more important and, in a later letter, noted that it was better to teach an understanding of the tables rather than the tables themselves.[167] Furthermore, in a discussion of Day's series, Farrar said the books contained too many applications to enable students to advance as far or as fast as they could with other textbooks. He believed Day's works were best suited for academy students but could be read by Harvard freshmen and sophomores who did not wish to extend their mathematical knowledge.[168] As noted previously, Farrar disdained the absence of mathematics in

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[164] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 288. In arguing that Euclid had come to be too highly revered, Adams quoted Playfair from *Edinburgh Review* as an example of the absurd praise bestowed on the ancients after quoting Playfair's "Dissertation" in order to emphasize human progress since ancient times. Adams apparently did not know that Playfair was the author of the *Edinburgh Review* article he cited and was thus unaware of the irony.

[165] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 289.

[166] [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 294.

[167] 5 March 1811, Corporation Records (cit. n. 19), vol. 5; John Farrar to John T. Kirkland, 24 March 1812, Harvard College Papers (cit. n. 23), vol. 7.

[168] Farrar to Unidentified, n. d., Hollis Professorship of Mathematics (cit. n. 27).

Brewster's version of *Ferguson's Astronomy*, especially in an age when all Americans were turning their attention to invention and speculation.[169] This illustrates that theory and practice were not mutually exclusive to Farrar. In proposing changes to the curriculum, Farrar cited public demand for more training in the liberal arts, and in discussing his understanding of progress, he remarked upon the positive social influence of mathematical theory: "An important relation has been found to subsist even between physical phenomena, and the abstract truths of geometry, that had long been regarded as merely curious. . . . The sciences, by their influence upon the arts, and especially that of navigation, have changed the face of the world and the condition of human existence."[170]

Encompassing these aspects of analysis as general principles arranged according to historical development, though, was the usage with respect to educational technique shown in Playfair's career: should students discover the solutions to problems on their own or should the textbook and tutor set out all the information for them? Farrar and his reviewers preferred French mathematics textbooks in general, but they did not do so because they wanted to teach with analytical methods in all mathematical subjects. To them, "analytical" could merely connote "convoluted," as in Farrar's description of the sections on spherics in Webber's *Mathematics*: "The basis of the demonstrations thus are given in as obscure & difficult to learners as that of the best analytical treatise & I believe more so."[171] Furthermore, most commentators remained convinced that a major reason for teaching mathematics was to train the reasoning abilities of students, at the expense

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[169] [Walker], review of *Elements of Geometry*, (cit. n. 49), pp. 206, 211.

[170] John Farrar to John T. Kirkland, 5 January 1827, Harvard College Papers, 2d ser. (cit. n. 58), vol. 1; [Farrar], review of *Discourse* and *Address* (cit. n. 42), p.162.

[171] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 99).

of preparation for higher mathematics or the making of applications if necessary. Farrar was one of the American men most interested in professional mathematics during his day, but he was at the same time true to the focus on mental discipline of his age. As he wrote while defending his textbook series late in his career, geometry's "only recommendation is that it invigorates & develops the intellectual powers. It is allowed to offer the finest specimens of logic, the most perfect instances of strict reasoning that man has attained to."[172] Jasper Adams also stated, "But it is particularly with a view to the development of the mental powers, that a course of mathematics is important."[173] The issue was which type of textbooks best accomplished that purpose. Emerson was rather enamored of Lacroix's "natural method, the light it throws on the logic of mathematics, and its completely analytical form . . . . making the parts succeed each other in the same order in which they might be supposed to have occurred to an original inventor."[174] Its character and higher goal separated it from other elementary treatises which gave algebra in synthetic form and were designed to make calculators.[175] In addition, Emerson claimed that Lacroix's arithmetic and algebra were ideal for training the reasoning abilities; Emerson and Adams both cited Lacroix's *Essais sur l'enseignement* as evidence that French mathematicians also saw development of the intellectual faculties as "an essential part of education."[176] Lacroix's analytical method of

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[172] John Farrar to Josiah Quincy, 24 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5.

[173] [Adams], "Review of the Cambridge Course" (cit. n. 121), 5 (1822): 308. See also [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 366, for a similar statement by Emerson.

[174] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 373.

[175] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 373.

[176] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 366. See also [Adams], "Review of the Cambridge Course" (cit. n. 121), 5 (1822): 308-309.

invention was too complex for anyone besides teachers and parents, however, and these people should master the general observations, fill in examples of the principles themselves, and then explain the rules of mathematics by teaching this expanded version to children.[177]

Indeed, analytical textbooks were not viewed as a type of "cure-all" appropriate for all mathematical students and subjects. Although students should be exposed to "at least one example of the instrument which Newton and Laplace have employed in their sublime discoveries," most elementary textbooks had to be synthetical, according to Emerson.[178] Most notably, the importance of teaching Euclidean geometry for these writers continued to lie in its synthetical proofs, which were "more strikingly and irresistibly convincing than perhaps any other."[179] Since the proofs in Legendre's *Éléments* were in fact written in this synthetical style, leading students through theorems which were already demonstrated and problems which were already solved, the reviewers had to point out specific differences between the synthetic modes of editions of Euclid's *Elements* and of Legendre's *Éléments*.[180] Thus, even though Emerson considered Playfair's *Elements* to be the best of the mathematics textbooks used in American colleges, he noted that "some advantages would . . . be gained, by the substitution of Legendre."[181] Emerson actually hoped that the first part of Farrar's translation of Legendre's *Éléments* could

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[177] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), pp. 366-367.

[178] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), pp. 365, 373.

[179] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 379.

[180] The exception to this form of synthetic style was Legendre's theory of parallels—the arithmetical language used for proportion appeared within proofs synthetic in reasoning. For example, Adams praised Euclid's treatment of parallel lines, saying that Legendre's analytical demonstration of this subject could not be followed by students who were unfamiliar with the theory of equations and functions; [Adams], "Elements of Geometry" (cit. n. 121), 6 (1823): 294.

[181] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), p. 374.

be taught at the very beginning of the mathematics course rather than in the sophomore year in order to train students in reasoning right away. This would be possible because Legendre had removed the difficulty of the first propositions in Euclid's *Elements* by omitting several of the axioms and postulates and by deducing the parallel postulate from the equality of triangles with three equal sides.[182]

Finally, Farrar's interest in education also led him into a minor role in the curriculum reform movement generally believed to have been delayed until the 1820s. Farrar was one of the first American professors to propose elective courses. At least as early as 1817, Farrar suggested a departure from the fixed college course. He proposed that students could master elementary mathematics in their first two years—perhaps with the aid of Day's textbook series—and then be allowed to choose between a course of higher algebra, trigonometry, fluxions, and physical astronomy or one in modern languages or one in natural history.[183] Nothing seems to have come of this suggestion, and the textbooks Farrar translated pushed all Harvard students to master mathematical subjects at a fairly sophisticated level. It is not clear what Farrar's opinions were regarding George Ticknor's attempt in the mid-1820s to render the curriculum less superficial by reorganizing the college into departments and dividing recitation sections by ability, an effort which was sabotaged by the faculty members who still felt the public's demand for introductory courses.[184] Feeling other pressures, Farrar said nothing further on

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[182] [Emerson], review of *An Elementary Treatise, etc.* (cit. n. 120), pp. 376, 379.

[183] John Farrar to Jeremiah Day, 15 April 1817, Day Papers: Letters to Jeremiah Day (cit. n. 99). See also an undated letter which he outlines the same plan in John Farrar to Unidentified, n.d., Hollis Professorship of Mathematics (cit. n. 27).

[184] Morison, *Three Centuries* (cit. n. 10), pp. 222-245; Lipset, "Post-Revolution Era" (cit. n. 68). One of Farrar's reports on the experimental physics and natural history course from this period indicates that seniors had the option to omit part of the course; John Farrar to John T. Kirkland, 26 December 1825, Harvard College Papers (cit. n. 23), vol. 11. Another report three weeks later shows that the senior course in chemistry was entirely voluntary; Department of Natural Philosophy to

reorganizing the curriculum except for a brief mention in 1832 to Quincy of a plan he had which was similar to the one he described in 1817.[185] Farrar also tried unsuccessfully in the 1810s to have the Harvard Corporation require prospective students to have mastered Books I through III of Euclid's *Elements*.[186] Instead, geometry did not become an entrance requirement at Harvard until 1844.

The Limits to Farrar's Influence

As Harvard's Hollis Professor, Farrar restored prestige to an important American college chair and found himself near the center of the emerging scientific community. While raising his own level of knowledge to the former standards of his professorship, he purchased improved equipment for classroom experiments and fought for the construction of an observatory at Harvard. He wrote journal reviews and articles on meteorology. He was active in the American Academy of Arts and Sciences and on various Harvard committees. Like Day, Farrar was beloved as a teacher, and he made similar suggestions for gradual modifications to the traditional liberal arts curriculum. He found an excellent partner in his equally intelligent and respected second wife, Eliza Ware Rotch. Most of Farrar's lasting fame, though, stemmed from the role he rather accidentally assumed, as the translator of mathematics and science textbooks. He introduced entire works, most from France and the Society of Arcueil, which helped sharpen Americans' tastes for mathematics and science.

Board of Overseers, 10 January 1826, Hollis Professorship of Mathematics (cit. n. 27). Thirty-six of the fifty juniors were judged competent to study fluxions that year, and four of those students substituted a course in the theory of perspective for ancient and modern languages; Annual Report, 10 May 1826, Hollis Professorship of Mathematics (cit. n. 27).

[185] John Farrar to Josiah Quincy, 28 June 1832, Harvard College Papers, 2d ser. (cit. n. 58), vol. 5.

[186] John Farrar to Jeremiah Day, 16 June 1817, Day Papers: Letters to Jeremiah Day (cit. n. 99).

One of these books, *Elements of Geometry*, the translation of Legendre's *Éléments*, was viewed in this chapter as an illustration of the concerns Farrar and his colleagues had for the various understandings of analysis and synthesis. With respect to analysis and synthesis as a method of proof, Farrar highlighted Legendre's attention to vocabulary which was a manifestation of his desire to properly organize geometry as a systematic body of knowledge. Farrar also tacitly accepted reference to particular diagrams as a proof technique. Farrar and his reviewers often treated "analysis" as equivalent to algebra and the mathematical style prevalent in France, which they preferred to the synthetic style they associated with England. Farrar was willing to intermingle algebra and geometry and taught the theory of proportion with an algebraic treatment. Farrar and his reviewers were not fond of the understanding of "analysis" as original invention in mathematics and a method of teaching with discovery. Although Farrar supported the use of geometry in making physical applications in the 1810s and made a brief foray into curriculum reform, the Harvard curriculum did not contain separate practical courses for such subjects as mensuration or navigation and surveying during his active career.

Instead, Farrar's influence upon Harvard mathematics was quickly overshadowed by the dominating personality of Benjamin Peirce. Peirce wrote his own geometry textbook in 1837 and proposed a new version of an elective system in mathematics in May 1838, whereby students would choose from three separate courses of study after the freshman year.[187] The textbook, while used at Harvard for

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[187] Benjamin Peirce, *An Elementary Treatise on Plane and Solid Geometry* (Boston: James Munroe and Company, 1837). Peirce did successfully divide the Harvard course into three tracks for a brief time in the 1840s during the life of the Lawrence Scientific School; academic majors were not instituted in American colleges until after the Civil War. Under Peirce's system, Harvard freshmen chose between a one-year practical course of mathematics, a one-year theoretical course for prospective schoolteachers, and a three-year course for mathematicians; Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community, 1876-1900: J. J.*

a number of years, followed Peirce's other textbooks in being "mathematically

intriguing but pedagogically painful," as mathematician George M. Rosenstein, Jr.,

put it.[188] In curriculum reform, Peirce was chiefly interested in ensuring that he only

saw the students who were interested in mathematics. Thus, by the time geometry

became an entrance requirement at Harvard in 1844, Peirce had spent several years

trying to establish an advanced school in science and engineering, succeeded in a

fashion with the Lawrence Scientific School, and escaped undergraduate education

in his new role as the Perkins Professor.[189] In all his endeavors as a teacher, Peirce

ignored the efforts of his predecessors such as Farrar and with the exception of

Bowditch, as was common of the Lazzaroni, the loose group of professional

scientists who organized the American Association for the Advancement of

Science.[190] The only time Peirce mentioned Farrar graciously in public was at the

annual meeting of the American Academy of Arts and Sciences after the older man's

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Sylvester, Felix Klein, and E. H. Moore, History of Mathematics Vol. 8 (American Mathematical Society, 1994), p. 17n. On Harvard mathematics under Peirce, see also Mary Ann James, "Engineering an Environment for Change: Bigelow, Peirce, and Early Nineteenth-Century Practical Education at Harvard," in Elliott and Rossiter, *Science at Harvard University* (cit. n. 30), pp. 55-75; I. Bernard Cohen, "Harvard and the Scientific Spirit," *Harvard Alumni Bulletin* 50 (1948): 393-398 [HUK 137], Harvard University Archives, Cambridge; Garrett Birkhoff, "Mathematics at Harvard, 1836-1944," in *A Century of Mathematics in America*, ed. Peter Duren, vol. 2 (Providence, RI: American Mathematical Society, 1989), pp. 3-58; and Robert A. McCaughey, "The Transformation of American Academic Life: Harvard University 1821-1892," *Perspectives in American History* 8 (1974): 237-332.

[188] George M. Rosenstein, Jr., "The Best Method. American Calculus Textbooks of the Nineteenth Century," in Duren, *Century of Mathematics* (cit. n. 187), vol. 3, pp. 77-109.

[189] James, "Engineering an Environment" (cit. n. 187), pp. 55-75; McCaughey, "Transformation" (cit. n. 187). The Lawrence Scientific School only had two students in 1849, and mathematics was again made mandatory for sophomores after 1850; Parshall and Rowe, *American Mathematical Research Community* (cit. n. 187), p. 19.

[190] On Peirce and the Lazzaroni, see Lillian B. Miller, *The Lazzaroni: Science and Scientists in Mid-Nineteenth Century America* (Washington, DC: Smithsonian Institution Press, 1972).

death, where he "ascribed to [Farrar], more than to any other man, the adoption of the present admirable system of instruction in the mathematical sciences."[191]

In a like manner, Farrar's wider influence was substantial at first but proved shallow over the duration. His impact mirrored his career, which was not a straight line like the one Farrar envisioned Harvard students travelling from the beginning to the end of their mathematics courses by employing tools such as *Elements of Geometry* to learn the arts or essential subjects of knowledge which prepared men for productive lives. Rather, his efforts took a sharp upswing in the late 1810s in a burst of energy but then traced a long and steady decline as his achievements became overshadowed by bouts with illness. The path of Farrar's career contrasted with the impact of Charles Davies, who often merely skimmed the intellectual surface but then found a deep well of popularity for his textbook series. Even when he was healthy, Farrar was ambivalent about his goals and was unable to move fully from exposition into research activities, leaving himself open to criticism from those of his former students who realized that the United States remained behind Europe in mathematics, but probably accomplishing as much as he could given his circumstances personally and at Harvard. Yet, even though Day's definition of college liberal education was cited for decades and many American colleges continued to use Playfair's *Elements* as a textbook until "*Davies's Legendre*" appeared while the "Cambridge Series of Mathematics" soon passed away, Farrar filled a necessary role in the expansion and development of nineteenth-century geometry teaching as the first complete translator of Legendre's *Éléments* into English.

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[191] "Communication Presented by Joseph Lovering at 380th Meeting," *Proceedings of the American Academy of Arts and Sciences* 3 (1852-1857): 38-40, on p. 39.

CHAPTER FIVE

THE TWO CIRCLES WILL TOUCH EACH OTHER INTERNALLY:

CHARLES DAVIES AT THE ART AND BUSINESS OF TEACHING GEOMETRY

"[W]ith scientific attainments fit for a head schoolmaster, he [Charles Davies]

has exactly the kind of talent fit for making Columbia & its Board the instrument of

his own selfish needs."[1] "[W]ith all his selfishness and mischief making, he is, and

always was a fool. I have known him from boyhood, before he became a professor at

West Point, and when he was brought into Columbia, I felt assured that his

influence would ruin all your hopes, for he is essentially a [small] minded man, and

incapable of generous impulses or enlarged views."[2] The average person who

managed to so antagonize prominent men at a time when one would expect them to

have been more concerned with the American Civil War would likely have found

his career at an end. The situation proved to be no impediment to a born salesman

and teacher like Davies, however. He not only survived this episode with his

reputation intact to continue to write and revise wildly successful mathematics

textbooks, he eventually talked the Columbia Board into granting him the prestige

of emeritus status when he retired in 1865, a title Davies used proudly to the end of

his life.

Many of the same characteristics which irritated some acquaintances were the

features which distinguished Davies from Jeremiah Day and John Farrar. Most

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[1] J. B. Barnard to G. Kemble, 26 October 1863, Hamilton Fish Papers, Rare Book and Manuscript Library, Columbia University, New York.

[2] G. Kemble to Hamilton Fish, 31 October 1863, Hamilton Fish Papers (cit. n. 1).

notably, Davies viewed writing textbooks as one of his primary occupations, rather than as something he did out of necessity or for a short time only. He then actively and openly marketed his books, building up his own reputation in a manner that Day and Farrar would have rejected as unseemly even though they lived just as comfortably on the profits from their textbook series. An exceptionally prolific author, Davies also differed from Day and Farrar because he was educated in a military environment and then moved around to several institutions during his career, he revised his textbooks over time, and he wrote separate books for children, secondary students, and college students. Like Day and Farrar, though, Davies was largely self-taught in mathematics, was a young man when he became a professor, and was beloved in the classroom. In his geometry textbook, *Elements of Geometry and Trigonometry*, Davies mixed together content from French, Scottish, and English textbooks with the goal of training the intellect. While his treatment of geometry could be superficial at times, the influence of *"Davies's Legendre"* ran deep as it was overwhelmingly adopted in the United States in the mid-nineteenth century. Yet, overall, what shaped all these aspects of Davies's career was the fact that he saw himself simultaneously as a professor and a businessman, like two touching circles with one inside the other.

The Shallow Roots of West Point Mathematics

Military education in the United States did not command the inherent prestige Harvard and Yale held through those institutions' longevity. Rather, after two other short-lived attempts at formal, hierarchical training, a military school was not opened until 1801. At West Point in the Hudson Highlands of New York, a Revolutionary War fort (the plans of which Benedict Arnold had attempted to sell to the British), the school's twelve cadets were instructed by George Baron, who previously taught at the Royal Military Academy in England with Charles Hutton

and who was later known for nitpicking over the computational details in Nathaniel

Bowditch's *Navigator*. Baron's contentious personality led to his firing in February

1802.[3] The school was then replaced by the United States Military Academy, as

established by the Military Peace Establishment Act passed by Congress on March

16, 1802, to create the Army Corps of Engineers.[4] President Thomas Jefferson used

the powers granted him by the Act to choose officers and cadets whom he believed

would model the Academy on France's *École Polytechnique*.[5]

By July, William A. Barron, Jared Mansfield (1759-1830), and Colonel

Jonathan Williams (1750-1815) were settled at West Point and offering courses.

Barron was a Harvard graduate and tutor who had worked closely with Samuel

Webber, while Williams was perhaps most likely to introduce the French influence,

given his background and position as Superintendent of the Academy. He had

worked for Benjamin Franklin in Europe in the 1780s and returned to the United

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[3] Albert L. Mills, "Introduction," in *The Centennial of the United States Military Academy: 1802-1902*, vol. 1 (Washington, DC: Government Printing Office, 1904), pp. 1-10; Edward R. Hogan, "George Baron and the *Mathematical Correspondent*," *Historia Mathematica* 3 (1976): 403-415.

[4] Histories of the Military Academy include Stephen E. Ambrose, *Duty, Honor, Country: A History of West Point* (Baltimore: The Johns Hopkins University Press, 1966); and *Centennial* (cit. n. 3). On the founding of this institution, see also Allan R. Millett and Peter Maslowski, *For the Common Defense: A Military History of the United States of America*, rev. ed. (New York: The Free Press, 1994), pp. 75, 104.

[5] While Jefferson had an educational connection to Scottish mathematics — William Small, one of Jefferson's teachers at William and Mary, emphasized Euclid's *Elements* as the foundation of mathematics and sound reasoning of any sort and trained Jefferson both in Newtonian science and Common Sense philosophy — his influence upon the United States Military Academy was mainly limited to hiring Jonathan Williams and endorsing the Academy's scientific society; I. Bernard Cohen, *Science and the Founding Fathers: Science in the Political Thought of Jefferson, Franklin, Adams, and Madison* (New York: W. W. Norton & Co., 1995), pp 68-72. See also Florian Cajori, *The Teaching and History of Mathematics in the United States* (Washington, DC: Government Printing Office, 1890), p. 84; Peter Michael Molloy, "Technical Education and the Young Republic: West Point as America's École Polytechnique, 1802-1833" (Ph.D. diss., Brown University, 1975), pp. 172, 273-274. On Jefferson's direct mathematical influence on the University of Virginia, see David Eugene Smith, "Thomas Jefferson and Mathematics," *Scripta Mathematica* 1 (1932-1933): 3-14; and "A Note on Thomas Jefferson," *Scripta Mathematica* 1 (1932-1933): 87-90.

States to become an active member of the American Philosophical Society.[6]

Williams's collection of technical books formed the basis of the Academy's library. A

Yale graduate, Mansfield was already known as a surveyor and as the author of

what some historians have called the first piece of American mathematical research,

the 1801 *Essays, Mathematical and Philosophical.*[7] The War Department bestowed a

military commission as captain of engineering upon him so that he could teach

engineering and mathematics at the Academy. In the mathematics part of the course,

Mansfield taught algebra and Barron covered geometry, both from Charles Hutton's

A Course of Mathematics. Stephen Ambrose called the entire Academy course of study

"superficial," noting that cadets took the graduation examination whenever they felt

they were ready.[8] The course material remained at this same informal level into the

1810s.[9] From 1802 to 1813, there was a Military Philosophical Society organized by

Williams, but most members were civilian and held their bi-weekly meetings away

from West Point. Only the top cadets attended the discussions of military

engineering with the faculty.[10]

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[6] Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 21-22, 25. An extended biography of Williams is Molloy, "Technical Education" (cit. n. 5), pp. 199-238.

[7] Biographies of Mansfield include "Mansfield, Jared," in *The National Cyclopedia of American Biography* (hereinafter cited *NCAB*), vol. 3 (New York and Clifton, NH: James T. White and Company, 1898-1984), p. 214; and Alois F. Kovarik, "Mansfield, Jared," in *Dictionary of American Biography* (hereinafter cited *DAB*), ed. Allen Johnson, vol. 6, part 2 (New York: Charles Scribner's Sons, 1927-1936), pp. 256-257.

[8] Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 24-37.

[9] Samuel E. Tillman, "The Academic History of the Military Academy, 1802-1902," in *Centennial* (cit. n. 3), pp. 223-241, on pp. 223-230.

[10] Molloy, "Technical Education" (cit. n. 5), pp. 271-308; Ambrose, *Duty, Honor, Country* (cit. n. 4), p. 30. See also Sidney Forman, "The United States Military Philosophical Society, 1802-1813," *William and Mary Quarterly* 2 (1945): 273-285.

Academy faculty generally did not remain at the institution for long. Mansfield was relieved in 1803 when he was appointed surveyor-general of the Northwest Territory, while Williams resigned briefly in the same year. Williams departed a second time in 1812, after the Academy had begun to suffer from the actions of a hostile Secretary of War, William Eustice. For instance, in 1810 Eustice took away commissions from Academy graduates, forcing them to enter active service as privates, and transferred several hundred artillerymen into the cadets' and faculty's quarters at West Point. Another faculty member who remained at the Academy only briefly was the Swiss-born Ferdinand Hassler, who replaced Barron in 1807 but then himself left to set up the United States Coast Survey in 1810. Thus, there were in fact no instructors — or students — remaining in West Point when professional military leaders were needed for the War of 1812 except for Alden Partridge (1785-1854), who had become assistant professor in mathematics in 1806 after studying for three years at Dartmouth College and one at the Academy. One of the first two graduates of the Academy, Joseph G. Swift, succeeded Williams as Superintendent but stayed away from West Point pursuing his interests in military politics and then the war effort. Swift was also Chief of Engineers for the United States Army.

Reacting in shock and concern at the nation's military unreadiness, members of Congress began to act to regularize the institution. The number of cadets had already been optimistically limited to 250 on April 29, 1812 (there had been only eighty-eight graduates in the Academy's first ten years), and an Academic Board was granted the authority to confer degrees. Reading, writing, and arithmetic were established as entrance requirements. Furthermore, formal provisions were made for a permanent and enlarged faculty. Partridge was the first to accept a full professorship, of civil and military engineering, in 1813. He was then named

Superintendent of the Academy on January 31, 1815; he had been serving as the
acting superintendent in Swift's absence. In the meantime, Mansfield was called
back to West Point in 1812 as Professor of Natural and Experimental Philosophy,
taking up his post in 1814, while Andrew Ellicott was hired as Professor of
Mathematics to succeed Partridge. Although he also used only Hutton's *A Course of
Mathematics* as a textbook, Ellicott took his students all the way from algebra and
geometry through trigonometry and logarithms to calculus during the three hours
of mathematical instruction held each day.[11] Yet, cadets continued to enter the
Academy at their convenience throughout the year and nearly always left or were
commissioned long before they completed the four-year course.

Davies's Mobile Career

Meanwhile, Charles Davies was born in Washington, Connecticut, on January
22, 1798.[12] While he was still quite young, his parents moved to St. Lawrence
County, New York, where his father farmed and became county sheriff. Davies
attended public schools near his home and helped his father until the War of 1812.
At that time, General Swift became acquainted with the Davies family while he was
supervising fortification construction along the St. Lawrence River. Swift was

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[11] Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 38-50; Silvio A. Bedini, *Thinkers and Tinkers: Early American Men of Science* (New York: Charles Scribner's Sons, 1975), p. 364; Tillman, "Academic History" (cit. n. 9), p. 227, 241; Mills, "Introduction" (cit. n. 3), p. 10.

[12] Biographies of Davies include: "Davies, Charles," in *NCAB* (cit. n. 7), vol. 3, p. 26; "Charles Davies," in *Biographical Register of the Officers and Graduates of the U. S. Military Academy at West Point, N.Y.*, George W. Cullum, 3d ed., vol. 1 (Boston and New York: Houghton, Mifflin and Company, 1891), pp. 151-155; E. D. Mansfield, "Charles Davies," in *Eighth Annual Reunion of the Association of the Graduates of the United States Military Academy at West Point, New York, June 14, 1877* (New York: A. S. Barnes & Co., 1877), pp. 23-27; "Presidents of New York State Teachers' Association," *American Journal of Education* 15 (1865): 477-487; and "Charles Davies," in *Davies Memoir: A Genealogical and Biographical Monograph on the Family and Descendants of John Davies of Litchfield, Connecticut*, Henry Eugene Davies (Privately Printed, 1895), pp. 67-73. See also "Prof. Charles Davies," *American Journal of Science* 112 (1876): 320.

favorably impressed by the young Davies and recommended him for appointment to the Military Academy, which was accomplished on December 27, 1813.[13]

The uncooperative New England militia and failed American offensives on the Canadian front certainly left Swift feeling the urgent need for training new officers, but the Davies family was not as anxious to send their teenager downstate to an institution which offered no consistent system of instruction.[14] Swift had written the family in November 1813 to advise them of Davies's impending admittance. He wrote another letter on February 18, 1814, asking for Davies's acceptance or nonacceptance of his appointment and requesting Davies to report to West Point by April 1.[15] Davies did then begin his studies, but, despite the end of the war in the winter of 1814-1815, Davies graduated and was commissioned as a Brevet Second Lieutenant of Light Artillery in December 1815 without completing the entire Academy course.[16] Ellicott sent him to service in New England with a testimonial: "As a regular student Mr. Davies had but few equals in this institution, and his progress was such, that it may be considered both unfortunate to himself,

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[13] Register of Cadets Admitted, Special Collections and Archives, United States Military Academy, West Point; "Davies," *Davies Memoir* (cit. n. 12), pp. 67-68; "Davies," *Eighth Annual Reunion* (cit. n. 12), p. 23.

[14] On the course of the War of 1812, see Millett and Maslowski, *For the Common Defense* (cit. n. 4), pp. 106-119.

[15] Joseph G. Swift to Thomas John Davies, 18 February 1814, Charles Davies, Miscellaneous Papers, Special Collections and Archives, United States Military Academy, West Point. This letter mentioned Swift's previous missive.

[16] "Davies," *Davies Memoir* (cit n. 12), p. 68. "Davies," *Eighth Annual Reunion* (cit. n. 12), pp. 23-24. Davies was one of forty Academy graduates in 1815; there was no class in the following year; *Register of Graduates and Former Cadets of the United States Military Academy* (West Point, NY: The West Point Alumni Foundation, Inc., 1964).

and his country, that he was called away before he had completed a full course of scientific education."[17]

Indeed, the professors at the Military Academy held Davies in high enough esteem that, after he was promoted to Second Lieutenant and transferred to the Corps of Engineering on August 31, 1816, he was invited to return to West Point as an assistant professor of mathematics when John Wright resigned in December 1816.[18] Davies, though, found himself in the midst of a storm when he arrived at the Academy. Partridge had proven to be a rather autocratic superintendent, putting the cadets through endless drills and administering stern punishments. He lost the support of Mansfield and Ellicott even further by trying to force his novel educational ideas upon the other men.[19] Meanwhile, the War Department was clarifying the Academy's regulations, or organizational principles, and published the new version in July 1816.[20] When it appeared by 1817 that Partridge was balking at implementing the new rules, President Monroe appointed Sylvanus Thayer (1785-1872) to replace Partridge upon Thayer's return from a two-year study of military schools, armies, and fortifications in Europe.[21] Partridge left the Academy on leave

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[17] Andrew Ellicott about Charles Davies, 20 December 1815, Charles Davies, Miscellaneous Papers (cit. n. 15).

[18] George Graham to Joseph G. Swift, 8 January 1817, Thayer Papers, *The West Point Thayer Papers, 1808-1872*, ed. Cindy Adams, et al, vol. 2 (West Point, NY: Association of Graduates, 1965), Special Collections and Archives, United States Military Academy, West Point; "Charles Davies," *Biographical* Register (cit. n. 12), p. 151.

[19] Cajori, *Teaching and History* (cit. n. 5), p. 86; Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 38-48. The biography of Partridge in *NCAB* glosses over this portion of Partridge's career, while Thomas M. Spaulding in *DAB* was more critical. "Partridge, Alden," in *NCAB* (cit. n. 7), vol. 18, pp. 322-323; and Thomas Marshall Spaulding, "Partridge, Alden," in *DAB* (cit. n. 7), vol. 7, part 2, pp. 281-282.

[20] Regulations, 2 July 1816 (West Point: U. S. Military Academy, 1802-1816), Special Collections and Archives, United States Military Academy, West Point.

[21] Like Partridge, Thayer had completed the classical course at Dartmouth and then spent a year at the Academy, graduating in 1808. He assisted with construction of coastal fortifications in

after the appointment was announced, but he returned shortly before Thayer's appointment began on July 28, 1817, convinced the cadets to take his side, incited them to arrest the professors, and seized command.[22]

Although Thayer quickly enlisted reinforcements including General Swift, garnered an order to arrest Partridge, and restored order at the Academy, it took some time to sort out Davies's role in the affair.[23] On Partridge's recommendation, General Swift had given him the temporary appointment as assistant professor of mathematics, which was rubber-stamped by George Graham, the acting Secretary of War, in January 1817.[24] After his inauguration, President Madison received complaints about the manner in which Davies was given the position and ordered through Graham that Davies's qualifications be verified through examination.[25] Then, already suspected to be a crony of Partridge, Davies had the misfortune to be away from the Academy when Partridge seized control and arrested the professors at the end of July.[26] The War Department believed that Davies had helped Partridge

New England and New York until Swift sent him to Europe in 1815. See Gustav Joseph Fiebeger, "Thayer, Sylvanus," in *DAB* (cit. n. 7), vol. 9, part 2, pp. 410-411; and "Thayer, Sylvanus," in *NCAB* (cit. n. 7), vol. 7, p. 87; as well as *Register of Graduates* (cit. n. 16), a memorial volume devoted to Thayer.

[22] Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 50-61; Tillman, "Academic History" (cit. n. 9), p. 241; Sylvanus Thayer to George Graham, 4 August 1817, Thayer Papers (cit. n. 18), vol. 2.

[23] Spaulding, "Partridge" (cit. n. 19).

[24] George Graham to Joseph G. Swift, 8 January 1817; Jared Mansfield to John O'Connor, [3] September 1817, both in Thayer Papers (cit. n. 18), vol. 2.

[25] George Graham to Joseph G. Swift, 14 February 1817, Thayer Papers (cit. n. 18), vol. 2.

[26] Sylvanus Thayer to George Graham, 14 August 1817, Thayer Papers (cit. n. 18), vol. 2. George Pappus reported that Davies was with the cadets who welcomed Partridge when he docked at the Academy; George S. Pappus, *To the Point: The United States Military Academy, 1802-1902* (Westport, CT: Pareger Publishers, 1993), p. 92. Apparently, though, Davies saw Partridge as a friend and mentor and, lacking teaching experience at that time, did not fully appreciate the differences between Partridge and the professors. He was, after all, only nineteen years old.

and consequently ordered the young man's arrest. Swift and Mansfield intervened on his behalf, reporting that he was not involved, while Swift also asked Thayer to merely tell Davies he was arrested rather than to take him into custody, in order to save him "too much mortification."[27] Ultimately, Davies was called as a witness at Partridge's court martial and was to undergo an investigation himself during the October trial.[28] Partridge was convicted, but his November 1817 sentence was remitted by President Madison. Partridge resigned from the Army on April 15, 1818, to go on to found a series of short-lived military preparatory schools.[29] Davies had already been carrying on with his duties for several months by that point and the controversy died away so much that it was not mentioned in biographies of Davies.[30]

Historians nearly unanimously agree that a new era began at the Academy when Thayer became Superintendent. Thayer took Williams's somewhat vague conception of an "American École Polytechnique" and threw himself into replicating the features he had observed in France. As a first step, he implemented the reforms called for by the 1816 Regulations: he established a five-member Board of Visitors to attend the two general examinations to be held each year, set up the chain of command at the Academy, and brought in a course of instruction for leading

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[27] Daniel Parker to Joseph G. Swift, 3 September 1817; Jared Mansfield to John O'Connor, [3] September 1817; Joseph G. Swift to Sylvanus Thayer, 8 September 1817, all in Thayer Papers (cit. n. 18), vol. 2. Yet, despite his defense of his protégé, Swift was apparently willing to sacrifice Davies if that would help establish control over the cadets. See Joseph G. Swift to Sylvanus Thayer, 13 September 1817, Thayer Papers (cit. n. 18), vol. 2.

[28] Joseph G. Swift to Sylvanus Thayer, 20 September 1817; George Graham to Sylvanus Thayer, 25 September 1817; George Graham to Joseph G. Swift, 25 September 1817, all in Thayer Papers (cit. n. 18), vol. 2.

[29] Spaulding, "Partridge" (cit. n. 19).

[30] See, for example, a September 28, 1817, report on the progress of the cadets to the War Department signed by Davies as well as the rest of the faculty in Thayer Papers (cit. n. 18), vol. 2.

American cadets through French textbooks in engineering and military science.[31] The entrance examinations were given orally beginning in 1818. Recitation sessions were regularized, divided by ability, and organized around the blackboard. Rather than only passively hearing lectures, cadets additionally would be required to master a portion of the assigned text and recite from it to the instructor. Professors, on the other hand, were freed from overseeing an individual program for each cadet once there was a uniform admission calendar. To keep track of student performance in the recitation sections, Thayer imposed a detailed arithmetical grading system based upon the French model. The students were marked in each class each day on a scale from "perfect" (3.0) to "complete failure" (0.0). These scores were totaled in weekly reports, weighted by subject from engineering, natural philosophy, and mathematics (2.0) to French (0.5), and combined with remarks on the amount of work completed by each student and on individual character. Finally, the aggregate marks, together with scores on the semi-annual examinations, were used to sort the cadets by order of merit.[32] The cadets resisted Thayer's disciplined system for years, trying tactics from a mutiny in 1818 to a failed attempt to launch a cannonball into Thayer's house in 1821, but Thayer's reforms were wholly implemented by the mid-1820s.[33]

Perhaps Thayer's most sound move in recreating the *École Polytechnique* was encouraging the former student in the institution who had found himself at West

[31] Regulations, 2 July 1816 (cit. n. 20). See also Keith Hoskin, "Textbooks and the Mathematisation of American Reality: The Role of Charles Davies and the US Military Academy at West Point," *Paradigm* no. 13 (1994): 11–41, on pp. 26-30.

[32] On Thayer's grading system, see Tillman, "Academic History" (cit. n. 9), pp. 223-241; Hoskin, "Textbooks" (cit. n. 31), pp. 27-28. For examples of the system in use, see Staff Records, No. 1, 1818-1835, Special Collections and Archives, United States Military Academy, West Point.

[33] Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 62-86; Hoskin, "Textbooks" (cit. n. 31), p. 27.

Point. Named Professor of Engineering on March 6, 1817, Claudius Crozet (1790-1864) was a natural to establish the twin pillars of Thayer's course, the French language and mathematics. Crozet soon discovered that the cadets were not prepared to learn the advanced engineering he wanted to teach. He had to review elementary mathematics with them first, although, as Thayer's system became normal practice at the Academy and as the cadets adjusted to reading mathematics textbooks in French, at least the best section of students was able to complete the mathematical subjects listed in the 1816 *Regulations* (logarithms, algebra, geometry, plane and spherical trigonometry and applications to problems, infinite series, conic sections, fluxions, analytical geometry, and mensuration) and learn analytical trigonometry, differential and integral calculus, and mechanical principles from Crozet by the early 1820s.[34] Crozet also introduced the subject of descriptive geometry developed by Gaspard Monge, beginning a tradition at West Point by writing a textbook on the subject in 1821.[35]

Davies embraced this renewed curriculum while he carried out his duties in the department of mathematics and, from October 31, 1821, to April 29, 1823, as assistant professor of natural and experimental philosophy under Mansfield.[36] Ellicott, the professor of mathematics, had been succeeded by his son-in-law, David Douglass, in 1820. When Crozet left the Academy in 1823, Douglass became

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[34] Regulations, 2 July 1816 (cit. n. 20); "Crozet, Claudius," in *NCAB* (cit. n. 7), vol. 18, pp. 393-394.

[35] Tillman, "Academic History" (cit. n. 9), pp. 223-241; Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 87-105; "Crozet" in *NCAB* (cit. n. 34); Thomas Marshall Spaulding, "Crozet, Claude," in *DAB* (cit. n. 7), vol. 2, part 2, pp. 580-581; Claude Crozet, *Treatise on Descriptive Geometry* (1821).

[36] Keith Hoskin argues that Davies was so young—not quite nineteen when he became an assistant professor—that Thayer could work his magic of disciplinarity as well on him as on the cadets. Hoskin, "Textbooks" (cit. n. 31), p. 30.

Professor of Engineering, and Davies was promoted to Professor of Mathematics.[37]

In this capacity, Davies proved as hardworking as the other mathematics professors,

Day and Farrar. Most notably, Davies followed Crozet's example and produced a

series of textbooks in the late 1820s and early 1830s by translating or co-opting

others' translations of French mathematics textbooks. (See Table 5.1 for a list of

Davies's publications.[38]) Among Davies's daily duties, he supervised four to five

assistants, perhaps dropping in on their sections as Ellicott had, standardizing the

instruction offered across sections, and apparently also teaching some of the classes

himself.[39] He and the Academy faculty were persistent over the years in requesting

that the assistants be drawn from graduated officers rather than from the elder

cadets, so that they would be better able to keep order and explain the lesson in the

recitation room.[40] All his labors still wore on Davies, though, and rumors circulated

as early as 1834 that he planned to leave West Point.[41] In late 1836, suffering from a

bronchial infection, he embarked on a recuperation tour of historical sites in Europe

which lasted at least seven months, while his family stayed with the Mansfields,

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[37] Tillman, "Academic History" (cit. n. 9), pp. 241-245.

[38] Davies published so many textbooks with such similar titles that compiling a list of them is a challenge. The titles and dates in Table 5.1 are "best guesses" based on *The National Union Catalog* (hereinafter cited *NUC*), 753 vols. (London: Mansell, 1976); Henry Barnard, "American Textbooks," *American Journal of Education* 13 (1863): 625-640; Joe Albree, David C. Arney, and V. Frederick Rickey, *A Station Favorable to the Pursuits of Science: Primary Materials in the History of Mathematics at the United States Military Academy*, History of Mathematics Vol. 18 (American Mathematical Society and London Mathematical Society, 2000); and copies of Davies's textbooks which were available during this study.

[39] Cajori, *Teaching and History* (cit. n. 5), p. 115; Staff Records, 15 June 1835 (cit. n. 32).

[40] For instance, see Staff Records, 9 September 1833 and 22 September 1835 (cit. n. 32).

[41] Sylvanus Thayer to Charles Gratiot, 6 July 1834, Thayer Papers (cit. n. 18), vol. 6.

Table 5.1. Publications by Charles Davies.

"Demonstration of a Problem in Conic Sections." *American Journal of Science* 6 (1823):
280-282.

Elements of Descriptive Geometry. Philadelphia, 1826. 28 printings.

Elements of Geometry and Trigonometry. New York, 1828. More than 33 printings.

Elements of Surveying With the Necessary Tables. New York: J. & J. Harper, 1830. 7
printings. 26 printings as *Elements of Surveying and Navigation* after 1841. 8
printings as *Elements of Surveying, Navigation, and Levelling* after 1870.

Treatise on Shades and Shadows, and Linear Perspective. New York: J. & J. Harper, 1832.
18 printings.

Common School Arithmetic. Hartford, 1833. 4 printings.

Elements of Algebra: Translated from M. Bourdon. New York: Wiley and Long, 1835. 41
printings.

Elements of Analytical Geometry. New York: Wiley and Long, 1836. 25 printings.

Elements of the Differential and Integral Calculus. New York: Wiley and Long, 1836. 16
printings. 3 printings as *Elements of Analytical Geometry and of Differential and
Integral Calculus* after 1859.

Mental and Practical Arithmetic. Hartford: A. S. Barnes & Co., 1838. 3 printings.

First Lessons in Algebra. New York: Wiley and Long, 1838. 5 printings.

First Lessons in Geometry. Hartford: A. S. Barnes & Co., 1839. 2 printings.

Practical Geometry. Philadelphia: A. S. Barnes & Co., 1839. 7 printings.

First Lessons in Arithmetic. Hartford: A. S. Barnes & Co., 1840. 13 printings.

Elementary Geometry. Philadelphia: A. S. Barnes & Co., 1841. 9 printings.

Arithmetic, Designed for Academies and Schools. Hartford, 1841. 12 printings.

Elementary Algebra. Philadelphia: A. S. Barnes & Co., 1842. 22 printings.

Table 5.1. (continued)

Key to Davies's Elementary Algebra. Philadelphia: A. S. Barnes & Co., 1842. 2 printings.

Key, Containing the Statements and Solutions of Questions in Davies's Elementary Algebra. Philadelphia: A. S. Barnes & Co., 1844. 14 printings.

Key to Davies's Arithmetic. Philadelphia: A. S. Barnes & Co., 1844. 6 printings.

The Case of Frederick Emerson versus Charles Davies and Alfred S. Barnes. Boston, 1845.

University Arithmetic. New York: A. S. Barnes & Co., 1846. 21 printings. .

Key to University Arithmetic. New York: A. S. Barnes & Co., 1846. 10 printings.

Elements of Drawing and Mensuration. New York: A. S. Barnes & Co., 1846.

The Arithmetic Table Book. New York: A. S. Barnes & Co., 1848. 3 printings.

The Logic and Utility of Mathematics, With the Best Methods of Instruction Explained and Illustrated. New York: A. S. Barnes & Co., 1850. 8 printings. 1 printing as *The Nature and Utility of Mathematics* in 1873.

Grammar of Arithmetic. New York: A. S. Barnes & Co., 1850. 3 printings.

Lecture on the Duties and Relations of Parents, Teachers and Pupils, as Connected with Education. Detroit: Free Press Book and Job Office Print., 1852.

Practical Mathematics with Drawing and Mensuration. New York: A. S. Barnes and Co., 1852. 6 printings.

School Arithmetic, Analytical and Practical. New York: A. S. Barnes and Co., 1852. 14 printings.

Intellectual Arithmetic. New York: A. S. Barnes and Co., 1854. 14 printings.

Primary Arithmetic and Table Book. New York: A. S. Barnes and Co., 1855. 3 printings.

With William Guy Peck. *Mathematical Dictionary and Cyclopedia of Mathematical Science.* New York: A. S. Barnes & Burr, 1855. 10 printings.

Key to Bourdon's Elements of Algebra. New York, 1856. 7 printings.

Table 5.1. (continued)

Key to Arithmetic for Academies and Schools. New York, 1856.

Key to School Arithmetic, Analytical and Practical. New York: A. S. Barnes and Co., 1856. 7 printings.

New University Arithmetic. New York, 1856. 8 printings.

Key to New School Arithmetic. New York: A. S. Barnes and Co., 1856. 3 printings.

"Inaugural Address, Charles Davies, LL.D." In *Addresses of the Newly-Appointed Professors of Columbia College.* Intro. William Betts. New York, 1858. Pp. 117-151.

University Algebra. New York: A. S. Barnes and Co., 1858. 14 printings.

Key to Davies's University Algebra. New York: A. S. Barnes and Co., 1859. 2 printings.

New Elementary Algebra. New York: A. S. Barnes and Co., 1859. 16 printings.

Differential and Integral Calculus. New York, [1860]. 3 printings.

Primary Arithmetic. New York: A. S. Barnes and Co., 1862. 5 printings.

Practical Arithmetic. New York: A. S. Barnes and Co., 1863. 13 printings.

Elements of Written Arithmetic. New York: A. S. Barnes and Co., 1863. 5 printings.

Metric System Explained. New York: A. S. Barnes and Co., 1867. 1 printing.

Outlines of Mathematical Science. New York: A. S. Barnes and Co., 1867. 1 printing.

Metric System of Weights and Measures. New York: A. S. Barnes and Co., 1870. 3 printings.

Key to the Practical Arithmetic. New York: A. S. Barnes and Co., 1870. 2 printings.

An Examination of the Demonstrations of Davies's Legendre. New York: A. S. Barnes and Co., 1873. 1 printing.

Table 5.1. (continued)

Address to *Fifth Annual Reunion of the Association of the Graduates to the United States Military Academy at West Point, New York. June 11, 1874.* New York: A. S. Barnes & Co., 1874.

Address to *Sixth Annual Reunion of the Association of the Graduates to the United States Military Academy at West Point, New York. June 17, 1875.* New York: A. S. Barnes & Co., 1875.

First Book in Arithmetic. New York: A. S. Barnes and Co., 1877. 1 printing.

The Complete Arithmetic. New York: A. S. Barnes and Co., 1877. 1 printing.

who had retired to Cincinnati in 1828.[42] When he returned at the end of May 1837, Davies officially resigned from the Academy, and the Davies clan moved to Hartford, Connecticut. He was succeeded at the Academy by Albert E. Church, one of the assistant professors. By 1839, all the mathematical textbooks taught at the Academy had been edited by Davies: Bourdon's *Elements of Algebra* (published in 1835), Legendre's *Éléments* with plane and spherical geometry (published in 1828), *Elements of Descriptive Geometry* (published in 1826), *A Treatise on Shades and Shadows* (published in 1832), *Elements of Surveying* (published in 1830), *Elements of Analytical Geometry* (published in 1836), and *Elements of the Differential and Integral Calculus* (published in 1836).[43] Although he left West Point with the intention of focusing on writing more textbooks, Davies also spent a good portion of each day in Connecticut

[42] Charles Davies to Mary Ann Davies, 10 December 1836 and 5 May 1837, Charles Davies, Letters, Special Collections and Archives, United States Military Academy, West Point. Although Henry Davies said that Davies was also studying current research in mathematics during the European trip, Davies's letters to his wife do not mention any scholarly pursuits; "Davies," *Davies Memoir* (cit. n. 12), p. 69. See also Ambrose, *Duty, Honor, Country* (cit. n. 4), pp. 87-105.

[43] Tillman, "Academic History" (cit. n. 9), p. 244.

teaching his three children.[44] One wonders how they responded to the mathematics

lessons!

The Academy textbooks which appeared under Davies's name soon were

purchased by colleges as well, and Davies additionally began to write arithmetics

and at least one guide to geometry aimed at children. He seems to have issued

several schoolbooks from a printing firm he established together with a relative of

his eventual son-in-law, William Guy Peck.[45] In the meantime, Alfred S. Barnes, who

moved from Hartford to New York in the early 1830s to try to establish himself as a

publisher, had decided that his niche in the business should be to publish only the

best textbooks.[46] When he met Davies in 1838, the pair naturally partnered. Well-

versed in mathematics himself, Barnes suggested that Davies's works be marketed

as a national series of standard books. A. S. Barnes & Co. immediately issued its own

printings of Davies's college textbooks: *Elements of Algebra* and *Elements of the*

Differential and Integral Calculus in 1838, and *Elements of Geometry and Trigonometry*,

Elements of Analytical Geometry, *Treatise on Shades and Shadows*, and *Elements of*

Surveying in 1839.[47] Barnes then took the textbooks on the road to show them

directly to colleges and academies.[48] His salesmanship and teachers' perception of

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[44] "Davies" in *Biographical Register* (cit. n. 12), p. 153; Mary Ann Davies to Elizabeth Mansfield, 28 November 1837, Charles Davies, Letters (cit. n. 42).

[45] This printing house apparently also sold other mathematics textbooks, for Hezekiah Howe sold the copyright for Day's *An Introduction to Algebra* to Davies & Peck and Collins, Keen & Co. in 1838; Hezekiah Howe to Leavitt, Lord & Company, 8 January 1838, Hezekiah Howe & Co. Letter Book, 1833-1838, Beinecke Rare Book & Manuscript Library, Yale University, New Haven.

[46] Hellmut Lehmann-Haupt, Ruth Shepard Granniss, and Lawrence C. Wroth, *The Book in America: A History of the Making, the Selling, and the Collecting of Books in the United States* (New York: R. R. Bowker Company, 1939), pp. 187-188.

[47] *Elements of Descriptive Geometry* was not published by A. S. Barnes & Co. until 1844.

[48] John Barnes Pratt, *A Century of Book Publishing 1838-1938* (New York: A. S. Barnes and Co., 1938), pp. 1-7; Lehmann-Haupt, Granniss, and Wroth, *Book in America* (cit. n. 46), pp. 187-188. Keith

the superior content of Davies's works made them unusually popular. Even as

Barnes moved his publishing house to Philadelphia in 1840 and then back to New

York in 1845, the business continued to grow.

Unable to stay out of the classroom, Davies became professor of mathematics

at Trinity College in Hartford in 1839 — ironically, Trinity had been founded several

years earlier in a controversial outreach effort by Yale during Day's presidency.

Davies had also become more deeply involved in politics and legislation, an interest

which dated back to his days as a professor at the Academy developing contacts as

well as enlisting the support of lawmakers for improvements and salary increases.[49]

For example, Davies campaigned for William Henry Harrison in the 1840

presidential election and then was one of the few Whigs to urge for conciliation with

John Tyler when the former Democrat ascended to the presidency.[50] A. S. Barnes &

Co. had also published its first new Davies schoolbook in 1840, *First Lessons in*

Arithmetic. Around the same time in 1841 that the next textbook appeared, Davies

had fallen ill again and resigned his professorship at Trinity. He then served on the

Board of Visitors at the Military Academy in the summer of 1841 and was

recommissioned in the Army as a paymaster in November in order to return to West

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Hoskin names other "market opportunities" for the textbooks, including the Academy's pre-eminence in science, the inner virtue of the textbooks, and the link between the Academy and the first modern high school, Central High in Philadelphia; Hoskin, "Textbooks" (cit. n. 31), pp. 32-34.

[49] Examples of letters between Davies and the Academy faculty and members of Congress include: Faculty to John C. Calhoun, 20 January 1818, Thayer Papers (cit. n. 18), vol. 3; and Henry Clay to Charles Davies, C. F. Smith, and N. Tillinghash, 6 February 1834, Charles Davies, Letters (cit. n. 42).

[50] Joseph Trumbull to Charles Davies, 28 February 1840 and 19 April 1840; John Tyler to Charles Davies, 1 October 1841, all in Charles Davies, Letters (cit. n. 42).

Point permanently as the Academy's treasurer.[51] While serving in his new post, Davies authored several more schoolbooks.

Davies retired from the Academy a second time in September 1845. Even though he purchased a country home at Fishkill-on-the-Hudson in New York and wrote more schoolbooks, Davies allowed himself to be called back to work again in September 1848 for a one year appointment as professor of mathematics at the University of the City of New York.[52] In 1850, he traveled to Europe for another six-month tour of points of interest, this time accompanied by his wife and one of his daughters.[53] Davies continued to produce textbooks in the 1850s, and he also found time to finish correcting a calculus textbook left behind by his former student and colleague at the Academy, Edward Courtenay (1803-1853).[54] In addition, he served as president of the New York Teachers' Association from 1853 to 1854.[55]

The strength of Davies's reputation led to a professorship at Columbia College in 1857. The same course of study was in place there for most of the

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[51] Keith Hoskin drops a tantalizing hint by observing that Davies was also one of three members of an 1841 inspection board which recommended that Daniel Tyler's efficiency measures at the Springfield Armory be adopted; Hoskin, "Textbooks" (cit. n. 31), pp. 37-38.

[52] See L. Bruwish, William Noys, and Robert Kelly (of the University of the City of New York) to Charles Davies, 19 September 1848, Charles Davies, Letters (cit. n. 42); and "Davies" in Biographical Register (cit. n. 12), p. 154. Davies sometimes referred to the location of his home as "Fishkill-on-the-Hudson" and sometimes as "Fishkill Landing." The site of his house is now lost.

[53] "Davies," Davies Memoir (cit. n. 12), p. 70.

[54] Sylvanus Thayer to Charles Davies, 18 March 1855, Charles Davies, Letters (cit. n. 42). Courtenay also taught mathematics at the University of Virginia for eleven years; the textbook was the 1855 Treatise on Differential and Integral Calculus and Calculus of Variations. "Courteney [sic], Edward Henry," in NCAB (cit. n. 7), vol. 5, pp. 519-520.

[55] "Presidents of New York" (cit. n. 12), p. 480. Apparently, Davies remembered this date incorrectly as 1843 to 1844 when he submitted his biography to George Cullum for the first edition of Biographical Register (cit. n. 12) (Charles Davies to George W. Cullum, 12 June 1867, Charles Davies, Letters (cit. n. 42)), and the mistake is repeated through all existing biographies of Davies. He could not have been president in the 1840s, for the Teachers' Association had not yet been founded.

nineteenth century, although a new effort was made in the 1830s to ensure that there was truly content rather than just lip service to the subjects.[56] In mathematics, first- and second-year students studied algebra and geometry, while third-year students turned to spherical trigonometry, conic sections, analytical geometry, and fluxions. Students could continue to study fluxions in the fourth year if they wished.[57] Robert Adrain, the American editor of Hutton's *A Course of Mathematics*, was Professor of Mathematics and Natural Philosophy there early in the nineteenth century, but the professorship of mathematics and astronomy was divided into two positions shortly after Davies was hired as the "Professor of Higher Mathematics." The 156 students attending Columbia in 1858 followed the required course for the first three years in order that their minds might be disciplined and invigorated. Seniors were then to apply their skills toward acquiring a permanent body of knowledge in either the School of Letters, the School of Science, or the School of Jurisprudence.[58] When he began to teach in February 1858, Davies delivered an address on the nature, language, and uses of mathematical science.[59]

Davies took his son-in-law, William Guy Peck, with him to Columbia as an assistant professor.[60] When he suggested in October 1863 that the Trustees reorganize the Department of Higher and Pure Mathematics so that Davies would fill merely a supervisory role at a reduced salary, a full professorship for another of

[56] J. Howard Van Amringe, "History of Columbia University," in *Universities and Their Sons*, ed. Joshua L. Chamberlain, vol. 1 (Boston: R. Herndon Company, 1898), pp. 569-731, on pp. 569-656.

[57] *Statutes of Columbia College, Revised and Passed by the Board of Trustees, March, 1821* (New York, 1821).

[58] Van Amringe, "History of Columbia" (cit. n. 56), pp. 657-666.

[59] [Charles Davies], "Inaugural Address, Charles Davies, LL.D.," in *Addresses of the Newly-Appointed Professors of Columbia College*, intro. William Betts (New York, 1858), pp. 117-151.

[60] Van Amringe, "History of Columbia" (cit. n. 56), p. 666.

his assistants, J. Howard Van Amringe (1835-1915), would be created, and Peck

would be promoted into a vacant professorship in the Department of Mechanics and

Physics, Davies helped set off a chain of events which ultimately led to his own

resignation.[61] When he learned of the proposed plan, General J. B. Barnard, the

brother of another candidate for the physics position, angrily argued to members of

the Board of Trustees that Peck had few scientific qualifications, if any, and

complained that Davies was successfully stalling the Board's decision.[62] Apparently,

most people involved believed that Charles King, president of Columbia, was

planning to retire soon and that the Professor of Physics would have the inside track

to succeed him. Frederick A. P. Barnard was not elected to the physics professorship,

but he was hired for the presidency when King did retire in the spring of 1864.[63]

Davies threatened to leave Columbia before the whole affair was settled, and even

after college politics quieted down, he became increasingly reluctant to leave his

country home and an ailing daughter who would pass away in late 1865.[64] When

Columbia's Committee on Faculty decided that Davies's $2,000 annual salary was

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[61] Charles Davies to Hamilton Fish, 13 October 1863, Hamilton Fish Papers (cit. n. 1). Fish (1808-1893) was a prominent lawyer, former governor of New York, and nearly permanent member of the Columbia Board of Trustees. See Joseph V. Fuller, "Fish, Hamilton," in *DAB* (cit. n. 7), vol. 3, part 2, pp. 397-400. On Van Amringe, who was later an interim president of Columbia, see Milton Halsey Thomas, "Van Amringe, John Howard," in *DAB* (cit. n. 7), vol. 10, part 1, pp. 148-149; and "Van Amringe, J[ohn] Howard," in *NCAB* (cit. n. 7), vol. 29, pp. 137-138.

[62] J. B. Barnard to G. Kemble, 24 October 1863 and 26 October 1863, Hamilton Fish Papers (cit. n. 1).

[63] G. Kemble to Hamilton Fish, 23 January 1864 and 20 May 1864, Hamilton Fish Papers (cit. n. 1); Van Amringe, "History of Columbia" (cit. n. 56), pp. 657-670. Barnard had to survive a controversy of his own when his previous position at a college in Mississippi called his loyalty to the Union into question. See G. M. Ogden to Fish, 30 August 1864, 8 September 1864, and 20 September 1864; Charles King to Hamilton Fish, 31 August 1864; Frederick A. P. Barnard to Hamilton Fish, 29 September 1864, all in Hamilton Fish Papers (cit. n. 1).

[64] Charles Davies to Hamilton Fish, 8 January 1864, Hamilton Fish Papers (cit. n. 1); Eunice Davies to E. D. Mansfield, 29 September 1864, Charles Davies, Letters (cit. n. 42).

not worth the two hours of teaching per week he was providing, he submitted his resignation.[65] It was accepted in September 1865, and an emeritus professorship accompanied by no salary or duties was arranged for him.[66]

Although Davies wrote few textbooks after his resignation and as well tapered off in making revisions for new printings of his works, he continued to be a busy man during his retirement. Due to his contacts from the Military Academy, fame as a textbook author, and prominent brothers—one a judge in the New York Court of Appeals and one a military engineer—Davies was called upon to advise legislators on mathematical and scientific issues.[67] He also found himself in demand as a speaker. For example, he addressed the cadets at the Virginia Military Institute in 1875.[68] Davies additionally instituted a yearly tradition of lectures at the annual reunion of the Military Academy, an event begun in 1869 in part to reunite Army officers who had found themselves on opposite sides in the Civil War. The most famous of these speeches was Davies's address in 1875 on the one hundredth anniversary of the Battle of Bunker Hill.[69] Davies worked a plea for reconciliation of the Union into his speech. Fittingly, Generals Sherman and Johnston both reviewed

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[65] Charles Davies to Hamilton Fish, 6 June 1865; and Beverly Haight to Hamilton Fish, 9 June 1865, Hamilton Fish Papers (cit. n. 1). The daughter was Louisa H. Scudder, the third of Davies's five children.

[66] William A. C. Bartlett to Hamilton Fish, 20 September 1865 and 27 September 1865, Hamilton Fish Papers (cit. n. 1).

[67] Davies reported on his impressions of the nation's capital during one of his lengthy consulting trips in Charles Davies to [Eunice Davies or Alice Davies], 15 January 1866 and 29 March 1866, Charles Davies, Letters (cit. n. 42).

[68] Charles Davies to Mary Ann Davies, 23 June 1875, Charles Davies, Miscellaneous Papers (cit. n. 15).

[69] Charles Davies, "Address," in *Sixth Annual Reunion of the Association of the Graduates to the United States Military Academy at West Point, New York, June 17, 1875* (New York: A. S. Barnes & Co., 1875), pp. 9-18.

and approved a draft of the lecture.[70] Davies died at his country home on September 17, 1876.

Development of *Elements of Geometry and Trigonometry*

Despite Thayer's and Crozet's desires to rely on French mathematics textbooks, the Academy curriculum lacked a firm textual foundation for several years. The mathematics professors may have read the works taught at the *École Polytechnique* in preparing the material they taught, but cadets did not purchase their own copies of the textbooks. Certainly it was inherently difficult to guide first-year cadets simultaneously through the French language and mathematics. Furthermore, there are no records of a separate geometry textbook in the Academy curriculum.[71] Thus, while Crozet's *Treatise of Descriptive Geometry* may have been the only mathematics textbook written in English officially used at the Academy when Davies ascended to the professorship of mathematics in 1823, the other assistant professors of mathematics for the sections containing the less able cadets had continued to teach from Hutton's *A Course of Mathematics* in 1818 and 1819 and Samuel Tillman, in a history of the department of mathematics, noted that Hutton's *A Course of Mathematics* was not entirely discarded until the years 1823 to 1825.[72] It

[70] See William Tecumseh Sherman to Charles Davies, 26 February 1875; and Joseph E. Johnston to Charles Davies, 1 May 1875 and 22 May 1875, all in Charles Davies, Letters (cit. n. 42).

[71] Edward S. Holden and W. L. Ostrander, "A Tentative List of Text-Books Used in the United States Military Academy at West Point From 1802-1902," in *Centennial* (cit. n. 3), pp. 439-466; listed no such book before Farrar's translation of Legendre's *Éléments* was adopted in 1823, including no edition of Playfair's *Elements* or Legendre's *Éléments*. The Military Academy Library does contain the 1813 tenth edition of Legendre's *Éléments*, brought from Paris by Thayer, which may have served as a reference for Academy instructors. With respect to the Library's holdings, the author was greatly aided by an unpublished catalogue of mathematics books at the United States Military Academy which was compiled by Joe Albree, David C. Arney, and V. Frederick Rickey. This catalogue is now generally available as Albree, Arney, and Rickey, *Station Favorable* (cit. n. 38).

[72] Tillman, "Academic History" (cit. n. 9), pp. 242-244; Cajori, *Teaching and History* (cit. n. 5), pp. 115-116. In one of his last actions as assistant professor of natural and experimental philosophy, Davies submitted his only mathematical or scientific article to *American Journal of Science* — a correction of one of Hutton's proofs in the conic sections chapter of *A Course of Mathematics* to make it general;

was not until 1823 that the Academy implemented the 1821 advice of Harvard

professors Andrews Norton and George Ticknor and adopted Farrar's series of

translations: *Elements of Algebra*, based upon Lacroix's textbook; Legendre's *Éléments*;

and *An Elementary Treatise on Plane and Spherical Trigonometry*, from writings by

Lacroix and Bézout, for the more advanced of the first-year cadets.[73] Students in the

second year turned to Biot's *Géométrie analytique* and Lacroix's *Traité élémentaire du*

calcul, in French, along with Crozet's *Treatise of Descriptive Geometry*, and the cadets

studied the principles of perspective, shades, and shadows.[74]

Although these textbooks worked out well enough and the Academy's

reputation for offering an excellent scientific and practical education increased

dramatically in the 1820s as all the cadets were required to study the more advanced

textbooks, Davies gradually became interested in putting his own name on

textbooks. For one thing, he married Mansfield's daughter, Mary Ann (1807-1897),

in 1825, and she gave birth to two of their five children by the end of 1828.[75]

Charles Davies, "Demonstration of a Problem in Conic Sections," *American Journal of Science* 6 (1823): 280-282.

[73] Helena M. Pycior, "British Synthetic Vs. French Analytic Styles of Algebra in the Early American Republic," in *The History of Modern Mathematics*, ed. David E. Rowe and John McCleary, vol. 1 (San Diego: Academic Press, 1989), p. 138.

[74] Although Thayer purchased the Cambridge Analytical Society's English translation of Lacroix's calculus textbook during his sojourn in Europe, Albree, Arney, and Rickey believe that the cadets studied from an 1820 Paris edition of the book. The materials used for perspective, shades, and shadows were probably manuscript notes by Crozet which no longer exist. Albree, Arney, and Rickey, *Station Favorable* (cit. n.38), pp. 15, 153; *An Elementary Treatise on the Differential and Integral Calculus* (Cambridge: J. Deighton and Sons, 1816); Silvestre-François Lacroix, *Traité élémentaire de calcul différential et de calcul intégral* (Paris, 1802).

[75] Mary Ann studied at Emma Willard's Troy Female Seminary, and she was actually scolded by her mother for choosing to study Paley's *Natural Theology* over Euclid's *Elements*; Elizabeth Mansfield to Mary Ann Davies, 9 March 1823, Charles Davies, Letters (cit. n. 42). Like Eliza Farrar, Mary Ann fretted to her mother that Davies preferred another young woman. Elizabeth responded, "I have heard the same report respecting Mr. Davies' attentions to Mary Piston that you have, but do not believe that there is any other foundation for it than his having rode out with her several times." She went on to describe Davies: "[H]is standing in society is highly respectable for one of his years and if he is not an adonis in his person or a Chesterfield in his manners, I have no doubt that he will

Therefore, Davies may have desired an additional route to financial security, especially if one considers that he had earned only ten dollars per month as an assistant professor.[76] Davies's first publication, *Elements of Descriptive Geometry*, was based upon Crozet's *Treatise* and appeared in 1826.[77] In addition, he and his own assistant, Edward C. Ross, became disenchanted with some of Farrar's translations. To remedy the translation errors in *Elements of Algebra* which had been pointed out by Jasper Adams and because the material in Lacroix's textbook had become dated, Ross prepared a translation of Louis Pierre Marie Bourdon's textbook on algebra which was published in 1831.[78] Davies's response was slightly different and reflected his preparation of *Elements of Descriptive Geometry*. He chose to seek out textbooks and existing translations which he could adapt and republish, as he even did with Ross's own book in 1835 (after Ross had left the Academy to enter active service in 1833).[79]

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make an excellent husband. He is besides all this a man of business. . . ." Elizabeth Mansfield to Mary Ann Davies, May 1823, Charles Davies, Letters (cit. n. 42); emphasis in source.

[76] Regulations, 2 July 1816 (cit. n. 20). Davies had supplemented that income at a woolen manufacturer in Glenham, New York, a few miles north of West Point and across the Hudson River. See Charles Davies to Mary Ann Davies, 8 September 1823, Charles Davies, Letters (cit. n. 42). He had even been admitted to the New York bar in 1828, but he apparently only ever argued one case. Davies also spent the 1820s developing contacts by recommending friends and former students for jobs and by traveling to Albany and Washington, D.C. See Charles Davies to Samuel Southard, 18 January 1825, Charles Davies, Miscellaneous Papers (cit. n. 15); Charles Davies to Sylvanus Thayer, 26 July 1825 and 22 June 1827; Edward H. Courtenay to Sylvanus Thayer, 17 June 1827, all in Thayer Papers (cit. n. 18), vol. 4; Charles Davies to Elizabeth Mansfield, 18 October 1825; Marian Foot to Mary Ann Davies, 15 January 1828, both in Charles Davies, Letters (cit. n. 42).

[77] Charles Davies, *Elements of Descriptive Geometry* (Philadelphia: Carey and Lea, 1826).

[78] *Elements of Algebra, Translated from the French of M. Bourdon, for the Use of the Cadets of the U. S. Military Academy*, trans. Lt. Edward C. Ross (New York: E. B. Clayton, 1831). See Pycior, "British Synthetic" (cit. n. 73), p. 138. Farrar published a shorter translation of the same textbook in the same year: *Elements of Algebra* (Boston: Hilliard, Gray, Little and Wilkins, 1831). For a biography of Ross, see Robert Kelly, *Life and Character of the Late Prof. Edward C. Ross, LL.D.* (New York, 1851).

[79] Charles Davies, *Elements of Algebra: Translated from the French of M. Bourdon*, New York: Wiley and Long, 1835.

For example, Davies's second textbook was a republication of Thomas Carlyle's translation, *Elements of Geometry and Trigonometry*, which was published in New York in 1828 by James Ryan and sold by a variety of booksellers. While Helena Pycior argued that Davies freely modified the content of Ross's translation, *Elements of Algebra* to, as Davies put it, "unite . . . the scientific discussions of the French, with the practical methods of the English school,"[80] Davies was more conservative in the first American printing of *Elements of Geometry and Trigonometry*. Davies wrote a new preface, in which he admitted the responsibility of altering such a celebrated work. He claimed that, unlike Legendre, Brewster, and Farrar, he avoided reference to particular diagrams so that beginners could improve their faculty of abstraction. In other words, Davies removed diagram labels from the statement of theorems so the theorems would be general (he would say "every straight line" rather than "every straight line CD"). Indeed, in general, "Geometry is not studied merely for the facts which it teaches . . . but, because it disciplines the untrained intellect, and conducts the untaught mind to the temple of truth."[81] Yet, otherwise, the only changes Davies made to Carlyle's translation were cosmetic and made his *Elements of Geometry and Trigonometry* resemble Farrar's translation: in addition to several minor alterations in wording, Davies renumbered all the propositions continuously and moved the three theorems and appendix at the end of Book VII to the appendix in Book VIII, just as Farrar had.[82] The diagrams were re-engraved, with the shading that had been in

[80] Davies, *Elements of Algebra* (cit. n. 79), pp. iii-iv; quoted in Pycior, "British Synthetic" (cit. n. 73), p. 139. For a less positive view of Davies with respect to Ross's efforts, see Lao Genevra Simons, *Bibliography of Early American Textbooks on Algebra*, Scripta Mathematica Studies No. 1 (New York: Scripta Mathematica, 1936). See "Davies and Mathematical Styles" in this chapter for a discussion of whether Davies truly meant "English" or "British" in the contrast he drew.

[81] [Charles Davies, ed.], *Elements of Geometry and Trigonometry; With Notes. Translated From the French of A. M. Legendre, Revised and Altered for the Use of the Military Academy at West Point*, [trans.Thomas Carlyle], ed. David Brewster (New York: James Ryan, 1828), pp. i-v.

Carlyle's translation removed.[83] Davies lastly added four corollaries or scholia and gave the division of the circle in sexagesimal instead of decimal notation in an abridged treatise on trigonometry.

Davies's slightly-edited *Elements of Geometry and Trigonometry* was received more warmly in the United States than the Brewster and Carlyle version was in Great Britain. Even though James Ryan assembled several mathematics textbooks at his press, a different New York firm, White, Gallagher, and White, reissued *Elements of Geometry and Trigonometry* in 1830.[84] The names of N. & J. White; Collins & Hanney; Collins & Co.; and James Ryan appeared on the 1832 third printing. Then, in 1834, Davies sold the right to publish to another firm, Harper & Brothers, which had purchased Davies's next work, *Elements of Surveying*, in 1830.[85] James and John Harper also issued Davies's *Treatise on Shades and Shadows* in 1832.[86] Although Davies signed over exclusive rights for *Elements of Geometry and Trigonometry* to the Harpers for ten years, his next three college textbooks were sold to Wiley & Long as Davies began to produce volumes in a rush of activity — *Elements of Algebra* in 1835 (the revision/co-option of Ross's translation of Bourdon's textbook), *Elements of Analytical Geometry* in 1836, and *Elements of the Differential and Integral Calculus* in

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[82] [Davies], *Elements of Geometry and Trigonometry*, 1828 (cit. n. 81); [John Farrar, trans.], *Elements of Geometry*, by Adrien-Marie Legendre (Cambridge, MA: Cummings & Hilliard, 1819).

[83] [Davies], *Elements of Geometry and Trigonometry*, 1828 (cit. n. 81), pp. 121-142.

[84] Although Ryan's name appeared on several nineteenth-century textbooks, and he was also the person behind the short-lived mathematics journal, *Mathematical Diary* (c. 1832), no known biographies of him exist.

[85] Contract for *Elements of Surveying*, 1830, Harper & Brothers Archives, Rare Book and Manuscript Library, Columbia University, New York. The book sold as Charles Davies, *Elements of Surveying with the Necessary Tables*, New York: J. & J. Harper, 1830.

[86] Charles Davies, *Treatise on Shades and Shadows, and Linear Perspective*, New York: J. & J. Harper, 1832.

1836 — making Davies Wiley's second mathematical author, after Day.[87] Indeed,

Harper & Brothers' contract with Davies apparently soon fell apart even though the

firm was otherwise quite profitable before the Civil War.[88] Wiley & Long also

printed and sold the 1834 *Elements of Geometry and Trigonometry*, as did young

bookseller Alfred S. Barnes, who was soon to be Davies's business partner.[89]

Davies became a more significant editor with the 1834 printing of *Elements of*

Geometry and Trigonometry.[90] First of all, Davies restored the traditional numbering

system by book for the propositions. He substituted in material from *Edinburgh*

Encyclopaedia in Book V and from *Encyclopaedia Metropolitana* in Book II, also adding

sixty-two pages of logarithmic and trigonometric tables and a table similar to those

in Carlyle's translation comparing the organization of propositions in Euclid's

Elements with that of Legendre's *Éléments*.[91] Davies added eight axioms, several from

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[87] Contract for *Legendre's Geometry and Trigonometry*, 1 April 1834, Harper & Brothers Archives (cit. n. 85); H. B. Phillips, Guilford L. Spencer, and Dick Wick Hall, "Numbers: Books on Mathematics," in *First One Hundred Fifty Years: A History of John Wiley and Sons, Incorporated 1807-1957* (New York: John Wiley & Sons, Inc., 1957), pp. 69-74, on p. 70. Davies is mentioned repeatedly as one of the first Wiley textbook authors in various technical fields. See also, for example, Richard S. Kirby, "Graphic Communication: Books on Drawing and Descriptive Geometry," pp. 83-89, on p. 84; and James Kip Finch, "The World of Construction: Books on Civil Engineering," pp. 110-122, on p. 120. Davies also published *Common School Arithmetic* on his own in Hartford in 1834 as his first school textbook.

[88] Lehmann-Haupt, Granniss, and Wroth, *Book in America* (cit. n. 46), pp. 171-172.

[89] According to the catalogue by Albree, Arney, and Rickey (cit. n. 38), p. 65; the Harper & Brothers version was thirty pages shorter than the copies of *Elements of Geometry and Trigonometry* sold by other printers and booksellers; the chief difference was that it did not contain the two sections Davies added on mensuration, (cit. n. 38), p. 65. The plates for *Elements of Geometry and Trigonometry* remained Davies's property, which may have aided him in simultaneously selling the textbook to other printers; Contract for *Legendre's Geometry and Trigonometry*, 1 April 1834, Harper & Brothers Archives (cit. n. 85).

[90] Charles Davies, ed., *Elements of Geometry and Trigonometry Translated From the French of A. M. Legendre by David Brewster, Revised and Adapted to the Course of Mathematical Instruction in the United States* (Philadelphia: A. S. Barnes and Co., [1834]).

[91] Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. iv.

Playfair's *Elements* or Simson's *The Elements of Euclid*: six statements on the relationships between equals and unequals, "Through the same point, only one straight line can be drawn which shall be parallel to a given line," and "All right angles are equal to each other," eliminating the first proposition from Carlyle's translation, which proved right angles are equal.[92] Repeatedly, Davies elaborated on the reasoning process used to arrive at conclusions within proofs or in additional scholia. He also rearranged a very few definitions and propositions in the earlier books.

The significant change was his inclusion of a new book between Books I and II to contain expanded and rewritten material on proportion. In other words, this and all subsequent editions of *Elements of Geometry and Trigonometry* were published with nine instead of eight books. Davies also removed the appendix on isoperimetry which previously followed the book on regular polygons. He omitted four propositions on planes and solid angles and revised the book on polyedrons. Davies switched the positions of the last two books, so that Book VIII in 1828, on the three round bodies, remained Book VIII in 1834. Although most of the propositions remained, Davies rewrote many of the proofs. Davies recast what became the final book, from a focus on the sphere to an examination of spherical triangles and polygons. He reduced the number of definitions from fifteen to seven, kept half of the original twenty-four propositions, and added nine theorems. Davies had also either completely rewritten or borrowed the section on plane and spherical trigonometry. Finally, Davies added five pages on the application of algebra to geometry from Hutton's work on the same topic and a section with the rules for measuring surfaces and solids. This version was printed at least eleven times and

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[92] Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. 13.

was taught in colleges including Dartmouth, the University of Alabama, and the

University of Michigan.[93]

Davies next revised *Elements of Geometry and Trigonometry* in 1851.[94] The title

page for this book contained one of the first times Davies added "LL.D." after his

name to reflect an honorary doctorate awarded by Geneva College in 1849.[95] He also

explained in the preface to this edition that he used Legendre's *Éléments* as

translated by Carlyle as a model and guide but not as a standard, changing the

language and arrangement of arguments. He put an increased emphasis on

problems for the student to work out and on uniting pure geometry with

mensuration, stating, "Practical examples cannot fail to point out the generality and

utility of abstract science."[96] Davies had removed Carlyle's essay on proportion from

the beginning of the textbook in the previous edition, and he now inserted his own

introduction on extension.[97] He then revised the list of definitions in Book I and

made an effort to simplify his writing, reflecting the younger audience *Elements of

Geometry and Trigonometry* had been gaining in the academies and recently-invented

high schools. Davies retained essentially the same body of theorems and proofs,

though.[98] After making similarly minor changes in Book II, on proportion, Davies

changed a theorem into a postulate in Book III, on the circle and measurement of

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[93] *NUC* (cit. n. 38); Cajori, *Teaching and History* (cit. n. 5).

[94] Charles Davies, ed., *Elements of Geometry and Trigonometry From the Works of A. M. Legendre, Revised and Adopted to the Course of Mathematical Instruction in the United States* (New York: A. S. Barnes & Co., 1851).

[95] "Davies" in *NCAB* (cit. n. 12).

[96] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. iv.

[97] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 1-12.

[98] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 13-46.

angles, and rewrote several corollaries and scholia.[99] In Book IV, the proportion of figures, Davies invented a symbol for equivalence of proportions, but it was never adopted by other mathematicians.[100] Davies revised Book V by adding six theorems and removing three to demonstrate the value of π in a more traditional fashion. He then converted two more theorems to postulates in Book VI, the principles that a perpendicular to a plane could be drawn from either a point within or a point without the plane.[101] This book was renamed "Planes and Polyedral Angles" from "Planes and Solid Angles." After making minor changes to the final three books, Davies wrote a brief essay on direct and indirect proofs for the notes. Davies redivided the trigonometry section into plane, analytical plane, and spherical trigonometry, but he made no changes to the appendix on mensuration.[102] This version went through seven printings, with the University of Mississippi among the colleges which adopted it.

Davies revised *Elements of Geometry and Trigonometry* once more shortly before accepting the professorship at Columbia in 1858, and he made only a few additional modifications in 1862, the last revision of the textbook published during his lifetime.[103] In this edition, he called Legendre's *Éléments* the "preeminent treatise of elementary geometry of the past 100 years" and revised his comments about the

[99] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 57-86.

[100] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 87-134.

[101] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 135-173.

[102] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 245-370.

[103] Charles Davies, *Elements of Geometry and Trigonometry, From the Works of A. M. Legendre* (New York: A. S. Barnes & Co., 1857); Charles Davies, *Elements of Geometry and Trigonometry, From the Works of A. M. Legendre, Adapted to the Course of Instruction in the United States* (New York: A. S. Barnes & Co., 1862). Davies wrote a new preface in 1875, but this was not printed until 1882; [J. Howard Van Amringe, ed.], *Elements of Geometry From Davies' Legendre* (New York andChicago: A. S. Barnes & Co., [1882]).

reference to particular diagrams to explain that he stated each proposition in general terms and then referred to the figure. Davies also replaced the term "solidity" with "volume." For the first time, Davies was aided by an assistant editor, William Guy Peck.[104] Davies rewrote the textbook's introduction to focus on the four types of quantity in geometry: lines, surfaces, volumes, and angles. He also said that operations in geometry were done with signs, as they were in analysis.[105] He reorganized the definitions in Book I and replaced two axioms while removing one. He added an eighth postulate, that it was possible to draw a line parallel to another line. He rearranged some propositions and revised the scholia and corollaries.[106] He abridged Book II, allowed multiplication by a ratio, and removed four propositions while adding two.[107] In contrast, Davies put several symbols back into words in a reorganized Book III and removed his equivalence symbol from Book IV. He also removed nearly half of the problems which had survived since Davies had first reprinted Carlyle's translation.[108] Although Books V through IX covered the same material as before, Davies significantly revised the structure of each book, substituting in new proofs and/or new theorems.[109] He no longer included any notes on the propositions, but he wrote new instructions for the trigonometry tables and combined the mensuration section into one unit. "Davies's Legendre," as the book had come to be called, had sold three hundred thousand copies by 1862, when it was priced at $2.25. After 1862, at least ten additional printings of Elements of Geometry

[104] Preface to Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103).

[105] Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103), pp. 9-12.

[106] Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103), pp. 13-49.

[107] Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103), pp. 50-58.

[108] Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103), pp. 59-135.

[109] Davies, Elements of Geometry and Trigonometry, 1862 (cit. n. 103), pp. 136-259.

and Trigonometry appeared. Georgetown and Transylvania University were two

institutions which turned to the textbook in the 1860s. Furthermore, A. S. Barnes &

Co. no longer marketed the textbook merely as part of a series but as a component of

"Davies' national course of mathematics."[110] The publishing house found seven

justifications for adding "national" to a set of books "now rounded to their perfect

fruition": the use of the system in the national military and naval academies; a

"*quasi*" endorsement by Congress; the system's use in the schools in Washington,

D.C.; its utility as a reference in resolving governmental questions about

mathematics; its role as educator of the nation's great soldiers, sailors, and scientists;

its larger role as educator of the greatest number of citizens; and the fact that the

system was used in every state.[111]

Davies and Mathematical Styles

"*Davies's Legendre*" ultimately dominated the market so much that it became

its own brand name, and yet it can be difficult at first glance to discern any

philosophical intent behind the textbook. The series was never reviewed in a journal

nor mentioned in the reviews of other mathematical textbooks, Davies never

mentioned "analysis" or "synthesis" in his few surviving letters, and Davies did not

provide readers with a direct glimpse into his philosophy of mathematics education

until he was well into the prolific production of textbooks; this work was the 1850

Logic and Utility of Mathematics.[112] Once he published *Logic and Utility*, Davies did use

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[110] Advertising supplement to Charles Davies, *The Logic and Utility of Mathematics, With the Best Methods of Instruction Explained and Illustrated* (New York: A. S. Barnes & Co., 1850), p. 11. This list was organized topically, while the series was termed "the West Point course" and divided into three parts of "common school," "academic," and "collegiate" (including "*Davies' Legendre's Geometry*") textbooks in Davies, *Elements of Geometry and Trigonometry*, 1862 (cit. n. 103), p. ii.

[111] Advertising supplement to Davies, *Logic and Utility* (cit. n. 110), p. 11.

[112] Davies, *Logic and Utility* (cit. n. 110). A shortened version of *Logic and Utility* was published in Great Britain as: Charles Davies, *Mathematical Science: Its Logic and Utility*, London: W. Kent and Co., n.d. Although there were no reviews of Davies's series in any of the American review journals,

"analysis" in print in several ways. Most often, he distinguished analysis as associated with algebra and the differential and integral calculus.[113] He saw geometry as separate from algebra, analytical geometry, and differential and integral calculus, which were all part of analysis, since these subjects dealt with quantities represented by letters.[114] Davies also mentioned that the analytical type of reasoning in logic was better for introducing science, but synthesis was better for memorization, and he raised analysis and synthesis again when he spent the third part of *Logic and Utility* arguing that the usefulness of mathematics lay in its role in training the intellect.[115]

Still, it cannot be said that Davies did not consider the three understandings of analysis and synthesis in earlier years; one such example was his desire to propagate the so-called "West Point system of mathematical instruction." As his textbook series became the backbone of Academy mathematics instruction throughout the nineteenth century, Davies claimed public credit for the success of Academy graduates, ultimately titling his series "The West Point Course" in the 1862 edition of *Elements of Geometry and Trigonometry*.[116] By this phrase, which

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there were two announcements of "new books" by Davies in the short-lived mathematics journal conducted by Charles Gill, *Mathematical Miscellany*. Gill noted in the first number that he had received *Elements of Geometry and Trigonometry* and *Elements of Algebra* from Davies, and he "trust[ed] they will become as popular as their merits deserve" in a phrase indicating the unease with which other mathematics writers seem to have viewed Davies's production of textbooks. In the second number, Gill announced the appearance of *Elements of Analytical Geometry* and *Elements of Differential and Integral Calculus* without further comment; *Mathematical Miscellany* 1 (1836-1838): 52ff.

[113] See, for example, Davies, "Inaugural Address" (cit. n. 59), p. 124; Davies, *Logic and Utility* (cit. n. 110), pp. 117-221; Charles Davies and William Guy Peck, *Mathematical Dictionary and Cyclopedia of Mathematical Science* (New York: A. S. Barnes & Burr, 1855), pp. 22-23. The second edition of *Dictionary* appeared in 1859 and was used for this study.

[114] Davies, *Logic and Utility* (cit. n. 110), pp. 261-292.

[115] Davies, *Logic and Utility* (cit. n. 110), pp. 41-97, 293-340.

[116] Davies, *Elements of Geometry and Trigonometry*, 1862 (cit. n. 103), p. ii.

turned up frequently in the advertisements and introductions to Davies's textbooks, Davies meant generally the foundation of French and mathematics taught in highly structured recitation sessions which was established at the Military Academy under Thayer.[117] In case the contributions of this educational system over the decades were not obvious, Davies reminded readers that Academy graduates were desired "wherever science of the highest grade has been needed" and touted the Academy for "scattering science and knowledge over the nation."[118] Davies's description of Academy instruction highlighted the feature of generality which was still most often associated with the French treatment of mathematics in the middle of the nineteenth century:

> It is of the essence of that system that a principle be taught before it is applied to practice; that general principles and general laws be taught, for their contemplation is far more improving to the mind that the examination of isolated propositions; and that when such principles and such laws are fully comprehended, their applications be then taught as consequences or practical results. . . . In that system Mathematics is the basis — Science precedes Art — Theory goes before Practice — the general formula embraces all the particulars.[119]

The typically "French" aspects of mathematics teaching at the Academy encompassed what subjects were included in the curriculum as well as the mode in which they were discussed. For example, Davies noted that French authors had devoted much labor and talent to elementary textbooks on analytical geometry,

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[117] Charles Davies, *Elements of Analytical Geometry* (New York: Wiley & Long, 1836).

[118] Davies, *Logic and Utility* (cit. n. 110), pp. 3–4; Davies, "Address," *Sixth Annual Reunion* (cit. n. 69), p. 9.

[119] Davies, *Logic and Utility* (cit. n. 110), pp. 3–4.

while descriptive geometry was taught in most public schools in France as a discipline essential for architects and engineers.[120] Similarly, these disciplines were a part of the Academy curriculum. In contrast, Harvard and Yale were typical of American liberal arts colleges in not providing instruction in analytical geometry until more than ten years after Thayer enforced the 1816 *Regulations* at the Academy and in never including descriptive geometry in the college course. In addition, Academy professors and former professors were often described by nineteenth-century observers as the first Americans capable of introducing the modern, usually algebraic techniques gradually perfected during the eighteenth century by Continental mathematicians. For instance, an anonymous reviewer praised Ferdinand Hassler — who was popularly associated with the Academy long after he left West Point for the Coast Survey in 1810 — for making analytical trigonometry understandable for students through an algebraic approach because the geometrical method was obsolete.[121]

However, thanks largely to Davies, the "West Point system" was not based solely on "French" characteristics. Davies regularly described the Academy's method of mathematical instruction as "the union of the French and English systems of mathematics."[122] By this, as Helena Pycior argued, Davies meant that the French tendency toward speculation was combined with an English bent for practical results.[123] Note that Davies thus saw English contributions to mathematics in a

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[120] Davies, *Analytical Geometry* (cit. n. 117); Davies, *Descriptive Geometry* (cit. n. 77).

[121] Review of *Elements of Analytic Trigonometry, Plane and Spherical;* by Ferdinand R. Hassler, *American Quarterly Review* 1 (1827): 38-54, on pp. 51-52.

[122] Davies, *Logic and Utility* (cit. n. 110), p. 3. See also, for instance, Davies, *Elements of Algebra* (cit. n. 79), p. iv.

[123] Pycior, "British Synthetic" (cit. n. 73), p. 137.

rather different and more positive light than did someone like Day, who rejected both voluminous and concise English books as unsuited for American students and who appreciated Legendre for including applications in *Éléments*. Pycior correctly attributed Davies's perception to his experience with Hutton's *A Course of Mathematics* as the archetype of English mathematics.[124] It may be recalled that the second volume was devoted heavily to the "useful" subjects more commonly found at the Royal Military Academy than at Oxford or Cambridge: mensuration of superficies and the circle, gauging, heights and distances, surveying, navigation, dialling, and spherical astronomy.

In other words, as someone responsible for training soldiers and engineers, Davies sympathized with those filling similar roles in England. Further, as a whole, English higher education represented greater devotion to mixed mathematics than French institutions did — by the time Davies became active, the *École Polytechnique* would have appeared to be dominated by pure mathematicians who almost excluded applications in their zeal for the fruits of abstract theory. Although instructors such as Cauchy and Poisson created students able to invent new mathematics, the intellectual independence they fostered was not necessarily a good thing for the success of Academy cadets who would be more concerned with putting their knowledge to use in building the growing American infrastructure than with following developments in higher mathematics. Davies thus felt justified in combining his understandings of the advantages of the French and English styles. Davies added features that he considered "English," or practical, to his series of translated French mathematics textbooks because he believed that, "Practical examples cannot fail to point out the generality and utility of abstract science."[125]

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[124] Pycior, "British Synthetic" (cit. n. 73), p. 152.

[125] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. iv.

Note lastly that Davies modeled his books on features of mathematics generally found only in England. But although Davies did not acknowledge the influence of Scottish mathematics, it was present through the sources on which he relied.

While there is no evidence that Davies was even capable of mastering the original mathematical research of his day, he could recognize trends that were currently in favor. Even though Legendre and Playfair both included sections on isoperimetry, Davies removed this topic, noting that problems of maxima and minima properly belonged to the calculus of variations.[126] Davies could also evoke the priorities of eighteenth-century mathematicians such as Euler. In one instance, he experimented with basing *Elements of Geometry and Trigonometry* on the property of extension.[127] Yet, Davies made this connection between geometry and mechanics in the same era that Benjamin Peirce used "geometer" in the French sense of a mathematician of highest order in his textbook of mechanics.[128]

One case study of Davies's efforts to combine "French" and "English" aspects of mathematics is to observe some of the modifications he introduced through the various editions of *Elements of Geometry and Trigonometry*.[129] For example, Davies went from admitting he had made some alterations to Legendre's *Éléments* in 1834 to firmly pointing out that he had departed from the original in 1851: "In the preparation of the present edition of the Geometry of A. M. LEGENDRE, the original

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[126] See Davies and Peck, *Mathematical Dictionary* (cit. n. 113), p. 279, where Davies grouped isoperimetry with analytical geometry, calculus, and the calculus of variations as part of "higher geometry."

[127] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 9-12.

[128] Benjamin Peirce, *Physical and Celestial Mechanics: A System of Analytic Mechanics* (Boston: Little, Brown, and Co., 1855).

[129] See also Helena Pycior's discussion of Davies's presentation of negative numbers in *Elements of Algebra*, where she concluded that Davies gutted the analytical method of Bourdon in the act of incorporating his own practical concerns; Pycior, "British Synthetic" (cit. n. 73), p. 145.

has been consulted as a model and guide, but not implicitly followed as a standard."[130] In 1834, Davies replaced Carlyle's essay on proportion with a more substantial treatment from *Encyclopaedia Metropolitana*. He substituted the key proposition in the book on quadrature, on making a circle and polygon coincide, with a theorem from William Wallace's Book V in the *Edinburgh Encyclopaedia*, which was modeled on Euclid's *Elements* rather than on Legendre's textbook. In other areas including regular polygons and polyedrons, though, Davies rearranged the chain of reasoning so much that the influences upon him are no longer recognizable. He also tried to establish himself as an influence upon others by renaming the book on spheres "spherical triangles and polygons" and, later, "spherical geometry," but this material never really took hold with other professors as a part of elementary geometry despite the market dominance of *Elements of Geometry and Trigonometry*. Ironically, Davies backed off from noting other influences upon him and the modifications he had made to Legendre's *Éléments* in the 1862 edition of the textbook, by which time the departure from Legendre's work was at its greatest level.

Davies and Educational Technique

Although Davies emphasized the practical benefits of a mathematical education in the style of that offered at the Military Academy, the reason for teaching geometry and the rest of elementary mathematics which he always valued at least as much as applications was mental discipline. As he explained in *Elements of Descriptive Geometry*, mathematics was worthy of attention whether it was studied as

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[130] Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. iii; Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. iii; emphasis in source.

an introduction to mechanics or in order to exercise the mind.[131] Davies devoted the

most space to describing the benefits of mental discipline in *Logic and Utility*. There,

he explained that mathematical science was the preeminent subject to "best develop

and steady the intellect of the young" while "at the same time lay the foundations of

all that is truly great in the Practical."[132] Assuming that the importance of practical

mathematical results was self-evident to his readers, Davies prepared *Logic and*

Utility "to point out and note the mental faculties which [mathematical science] calls

into exercise; to show why and how it develops those faculties; and in what respect

it gives to the whole mental machinery greater power and certainty of action than

can be attained by other studies."[133]

Further, Davies exhorted in favor of mental discipline regardless of the

setting. For instance, he focused most of his inaugural address at Columbia on

explaining how the study of mathematics accomplished mental training, by filling

the mind with clear ideas expressed in a certain language.[134] Davies returned to

these themes in a broader sense when he delivered a lecture at the Normal School in

Ypsilanti, Michigan, in 1852.[135] In all of education, he reminded his audience, the

faculties of the young mind must be strengthened and developed along with the

physical and moral natures.[136] And, as he had said of mathematics in *Logic and*

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[131] Charles Davies, *Elements of Descriptive Geometry With Their Application to Spherical Trigonometry, Spherical Projections, and Warped Surfaces*, rev. ed. (New York: A. S. Barnes & Co., 1866), pp. ii-viii.

[132] Davies, *Logic and Utility* (cit. n. 110), p. 13.

[133] Davies, *Logic and Utility* (cit. n. 110), p. 17.

[134] Davies, "Inaugural Address" (cit. n. 59), pp. 126-129.

[135] Charles Davies, *Lecture on the Duties and Relations of Parents, Teachers and Pupils, as Connected With Education* (Detroit: Free Press Book and Job Office Print., 1852).

[136] Davies, *Lecture on the Duties* (cit. n. 135), p. 7.

Utility, "Teach one thing at a time — teach that thing thoroughly — and as far as possible, teach all its connections with other things!"[137] Throughout the speech, Davies advocated the thorough education of children in public schools, urging their future teachers to be faithful to their calling. This was one of the few recorded times that Davies spoke more generally of education rather than concentrating on mathematics. His emphasis on mental discipline, though, was remembered even well after his death. For instance, one of Davies's assistant professors at Columbia, J. Howard Van Amringe, edited a posthumous edition of *Elements of Geometry and Trigonometry* in which he described "*Davies's Legendre*" as superior to any work for training the logical powers of pupils and instruction in geometrical truth.[138]

What about mathematics made it the vehicle to mental discipline? Davies explained that no one could master systematic knowledge of any subject without disciplining the mind.[139] This mental training ought to fill the mind with "clear and distinct ideas" expressed in unambiguous language, and the only candidate for a common language free from error was mathematics.[140] He described mathematical knowledge as founded on the concepts of number and quantity, and he argued that the study of mathematics forced students to develop habits of "close attention, nice discrimination, and certain judgment."[141] These habits enabled students to grasp abstract concepts, so Davies defined the purpose of *Elements of Geometry and Trigonometry* as strengthening the "faculty of abstraction" through "the intellectual

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[137] Davies, *Lecture on the Duties* (cit. n. 135), p. 14. See also Davies, *Logic and Utility* (cit. n. 110), p. 13.

[138] [Van Amringe], *Elements of Geometry From Davies' Legendre* (cit. n. 103), p. v.

[139] Davies, *Logic and Utility* (cit. n. 110), p. 15.

[140] Davies, *Logic and Utility* (cit. n. 110), p. 106; Davies 1858, pp. 126-127.

[141] Davies, *Logic and Utility* (cit. n. 110), pp. 14, 342.

labor" of learning propositions without reference to particular diagrams.[142] In other writings, Davies also appealed to two different definitions of "analysis" to emphasize the relationship between mathematics and mental discipline. First, he used "analysis" to denote the process of dividing a problem, mathematical or otherwise, into its constituent parts. The process of classifying these elements sharpened the mind.[143] Second, Davies contrasted the "Analytical method [which] is best adapted to investigation, and the presentation of subjects in their general outlines" with "the Synthetical method [which] is best adapted to instruction, because it exhibits all the parts of a subject separately, and in their proper order and connection."[144] Davies believed that the analytical method was currently employed to develop all branches of mathematics besides arithmetic and geometry, but it could potentially help students understand symbols as based upon the notion of quantity in the entirety of mathematics.

But to maximize the value of mathematics in mental training, Davies urged that students be asked to deal with only one subject at a time. First came arithmetic, "the most useful and simple branch of mathematical science."[145] Then, Davies taught algebra, the universal arithmetic, up to quadratic equations before turning to geometry. Most of the next subjects were "applications": trigonometry, the application of arithmetic, algebra, and geometry to the measurement of triangles; surveying and levelling, applications of trigonometry; descriptive geometry, which projected lines, surfaces, and solids onto paper; and shades, shadows, and

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[142] Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. iii.

[143] Davies, *Lecture on the Duties* (cit. n. 135), p. 18.

[144] Davies, *Logic and Utility* (cit. n. 110), p. 106.

[145] Davies, *Logic and Utility* (cit. n. 110), p. 345.

perspective, the application of descriptive geometry.[146] Lastly, students ought to learn analytical geometry, which revealed the full power of algebra and geometry to the mind, and the differential and integral calculus, which gave a new and even greater view of the power of mathematics.[147] Once the pupil had mastered each method of mathematics, he was equipped "to compare different methods with each other."[148] There was also a proper order to follow in presenting each subject. As Davies explained in his essay on geometry, students had to be exposed to geometrical objects first, and then definitions and axioms, before they were ready to comprehend the architecture of a proof.[149] Unlike Day's reviewer and probably because many of his textbooks were for a very youthful audience, Davies argued that the content of mathematics could be simplified without sacrificing the development of mental discipline, as will be seen in his approach to the foundations of geometry.

Davies and Method of Proof

While American professors rarely alluded to the ancient meanings of "analysis" and "synthesis" as the two directions of proof, Davies did do so in the mathematical dictionary he published with his son-in-law, William Guy Peck, in 1855.[150] Although Davies first described analysis as he generally did, as embracing "all of that portion of the science of mathematics in which the quantities considered are denoted by letters, and the operations to be performed upon them are indicated

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[146] Davies, *Logic and Utility* (cit. n. 110), pp. 346-350.

[147] Davies, *Logic and Utility* (cit. n. 110), pp. 350-352.

[148] Davies, *Logic and Utility* (cit. n. 110), p. 345.

[149] Davies, *Logic and Utility* (cit. n. 110), p. 257.

[150] On Peck, see "Peck, William Guy," in *NCAB* (cit. n. 7), vol. 5, p. 520.

by signs,"[151] he then distinguished between ancient analysis, which was the process of reasoning discussed by Pappus, and modern analysis, which denoted the algebraic means used in contemporary mathematical investigation and invention.[152] Davies quoted Pappus' definition of "analysis" and gave an example of the analytical and synthetical steps involved in determining a geometrical construction, also taken from the works of Pappus.

Davies was more normally concerned, however, with the types of proof appealed to in geometry, such as direct and indirect demonstrations. He gave standard definitions for these in the dictionary:

> In the *direct* method the premises are definitions, axioms, and previous propositions, and by a process of logical argumentation, the magnitudes of which something is to be proved, are shown to bear the mark by which that something may always be inferred; or, in other words, they are shown to fall under some definition, axiom, or proposition previously laid down. . . . In the indirect demonstration, therefore, the conclusion is compared with the truths known antecedently to the proposition in question.[153]

Davies illustrated the indirect method, or *reductio ad absurdum*, with a proposition from Legendre's *Éléments* and explained that this process of reasoning was just as conclusive as a direct argument. He had written these definitions for *Logic and Utility* and reported them in a note which appeared only in the 1851 edition of *Elements of Geometry and Trigonometry*; his acceptance of indirect proof represented a departure

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[151] Davies and Peck, *Mathematical Dictionary* (cit. n. 113), p. 22.

[152] Davies and Peck, *Mathematical Dictionary* (cit. n. 113), pp. 22-23.

[153] Davies and Peck, *Mathematical Dictionary* (cit. n. 113), p. 281; emphasis in source.

from the viewpoint that *reductio ad absurdum* should be appealed to as rarely as possible, which was held by Carlyle in the essay on proportion that Davies removed from *Elements of Geometry and Trigonometry* in 1834.[154]

More significantly, Davies tried to avoid reference to particular diagrams throughout the editions of *Elements of Geometry and Trigonometry*. After noting the responsibility accompanying the alteration of a work as celebrated as Legendre's *Éléments*, Davies explained that the most regrettable departure from the method of Euclid in versions of *Éléments*, including "the original work, as well as in the translations of Dr. Brewster and Professor Farrar, [was that] the propositions are not enunciated in general terms, but with reference to, and by the aid of, the particular diagrams used for the demonstrations."[155] Davies believed that this practice prevented beginners from exerting enough intellectual labor to develop their faculties of abstraction. In other words, reference to particular diagrams prevented the development of mental discipline, which was the main reason for teaching geometry. Thus, Davies removed any reference to diagram labels from the statements of propositions. For instance, while Carlyle gave, "If, from a point C assumed within the triangle ABC, straight lines OB, OC, be drawn to the extremities of a side BC, the sum of these straight lines will be less than that of the two other sides AB, AC," for the ninth proposition of Book I, Davies wrote, "If, from any point within a triangle, two straight lines be drawn to the extremities of either side, their sum will be less than the sum of the two other sides of the triangle."[156] This was the

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[154] Davies, *Logic and Utility* (cit. n. 110), pp. 241-246; Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 245-246; [Thomas Carlyle, trans.], *Elements of Geometry and Trigonometry; With Notes. Translated From the French of A. M. Legendre*, ed. David Brewster (Edinbrugh: Oliver & Boyd, 1824), p. ix.

[155] Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. iii.

[156] [Carlyle], *Elements of Geometry and Trigonometry* (cit. n. 154), p. 9; Davies, *Elements of Geometry and Trigonometry*, 1834 (cit. n. 90), p. 18. This was proposition 8 in Davies's work because he

style of the propositions in versions of Euclid's *Elements*, including Simson's *The Elements of Euclid* and Playfair's *Elements*.

It was a style apparently not received as well in the United States, though. Recall that Timothy Walker, who prepared an abridged version of Legendre's *Éléments*, had argued in 1828 that one reason Legendre's work was superior to Simson's *The Elements of Euclid* or Playfair's *Elements* was that Legendre rendered each proposition "specific and definite by the introduction of letters, referring each part immediately to the diagram; whereas in Euclid the enunciations are all general and without letters."[157] From Walker's point of view, Legendre's method prevented learners from having to engage in the difficult activities of generalization and abstraction too soon. Farrar, the subject of Walker's review, followed Legendre's method of referring to the diagram in the statement of the proposition, but one reason Walker raised this point was that he had heard that Davies was preparing *Elements of Geometry and Trigonometry*, "in which one of the chief alterations will be the omission of the letters," and Walker wanted to register his preference for retaining the letters.[158] Davies himself allowed reference to particular diagrams when he felt the situation warranted it. For example, in *Practical Geometry*, Davies decided that general readers could not understand geometrical truths and their exactness unless he "omit[ted] the demonstrations altogether, and rel[ied] on the accuracy of the enunciation and the illustrations of the diagram."[159] By 1862, Davies

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converted the first proposition into an axiom in this revision of *Elements of Geometry and Trigonometry* ("All right angles are equal to each other").

[157] [Timothy Walker], review of *Elements of Geometry*; by Adrien-Marie Legendre, trans. John Farrar, *North American Review* 27 (1828): 191-214, on p. 198.

[158] [Walker], review of *Elements of Geometry* (cit. n. 157), p. 198.

[159] Charles Davies, *Practical Geometry: With Selected Applications in Mensuration, in Artificers' Work and Mechanics* (Philadelphia: A. S. Barnes & Co., 1839), p. vi.

was even defending reference to the diagram within the proofs of the propositions in *Elements of Geometry and Trigonometry*. Instead of rejecting reference to particular diagrams outright, he allowed each proposition to be "first enunciated in general terms, and afterwards, with reference to a particular figure, that figure being taken to represent any one of the class to which it belongs."[160] Davies felt he had to reiterate that the truths communicated in his propositions and proofs were indeed general, implying that a study of them promoted mental discipline.

Finally, in addition to raising the directions of analysis and synthesis and proof techniques, Davies put his stamp on the organization of geometry as a system by adding to the subject's first principles throughout the editions of *Elements of Geometry and Trigonometry*. Instead of reducing his dependence on assumptions, Davies asked students to accept more and more without proof. He began by adding axioms from Euclid's *Elements* in 1834. (See Figure 5.1 for a comparison of Playfair's, Legendre's, and Davies's axioms.) The examples given by Davies in the chapter on geometry in *Logic and Utility* indicate that Davies probably believed that equality of lines or angles could not be asserted without referring to these axioms.[161] Davies also more often than not defined additional terms in later printings of the textbook. Davies not only founded all geometrical reasoning on the definitions and axioms from which the propositions were deduced but concluded that this basis for reasoning had to be enlarged to establish sufficient grounds for determining the truth of propositions.[162] By setting elementary geometry upon a maximized

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[160] Davies, *Elements of Geometry and Trigonometry*, 1862 (cit. n. 103), p. iii.

[161] See, for example, Davies, *Logic and Utility* (cit. n. 110), pp. 237-239.

[162] Davies, *Logic and Utility* (cit. n. 110), p. 257. To Davies, the principles of arithmetic and algebra were also deduced from definitions and axioms. See Davies, *Logic and Utility* (cit. n. 110), p. 276.

a)

AXIOMS.

I.

N. THINGS which are equal to the same thing are equal to one another.

II.

If equals be added to equals, the wholes are equal.

III.

If equals be taken from equals, the remainders are equal.

IV.

If equals be added to unequals, the wholes are unequal.

V.

If equals be taken from unequals, the remainders are unequal.

VI.

Things which are doubles of the same thing are equal to one another.

VII.

Things which are halves of the same thing are equal to one another.

*VIII.

Magnitudes which coincide with one another, that is, which exactly fill the same space, are equal to one another.

IX.

The whole is greater than its part.

X.

All right angles are equal to one another.

" Two straight lines, which intersect one another, cannot be
" both parallel to the same straight line."

b)

Axioms.

22. Two quantities, each of which is equal to a third, are equal to one another.

23. The whole is greater than its part.

24. The whole is equal to the sum of all its parts.

25. Only one straight line can be drawn between two points.

26. Two magnitudes, whether they be lines, surfaces, or solids, are equal, when, being applied the one to the other, they coincide with each other entirely, that is, when they exactly fill the same space.

c)

Axioms.

1. Things which are equal to the same thing, are equal to each other.

2. If equals be added to equals, the wholes will be equal.

3. If equals be taken from equals, the remainders will be equal.

4. If equals be added to unequals, the wholes will be unequal.

5. If equals be taken from unequals, the remainders will be unequal.

6. Things which are double of the same thing, are equal to each other.

7. Things which are halves of the same thing, are equal to each other.

8. The whole is greater than any of its parts.

9. The whole is equal to the sum of all its parts.

10. All right angles are equal to each other.

11 From one point to another only one straight line can be drawn.

12. Through the same point, only one straight line can be drawn which shall be parallel to a given line.

13. Magnitudes, which being applied to each other, coincide throughout their whole extent, are equal.

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Figure 5.1. Davies pasted Euclid's axioms into Legendre's textbook. a) From John Playfair, *Elements of Geometry* (Philadelphia, 1806), pp. 6-7. b) From [John Farrar, trans.], *Elements of Geometry* (Cambridge, 1819), p. 3. c) From Charles Davies, ed., *Elements of Geometry and Trigonometry* (Philadelphia, [1834]), p. 13.

foundation, Davies hoped he could accommodate *Elements of Geometry and Trigonometry* to an audience younger than the college students who normally studied the textbook and thus continue to sell textbooks even though American colleges gradually ceased offering instruction in geometry and required prospective students to have already mastered the basic tenets of elementary geometry in a secondary school after the Civil War.

In 1851, Davies furthermore introduced the three postulates from Simson's *The Elements of Euclid*: "Let it be granted, that a straight line may be drawn from one point to another point," "That a terminated straight line may be prolonged, in a straight line, to any length," and "Let it be granted that the circumference of a circle may be described from any centre, and with any radius" (the third postulate was placed in Book III, on circles).[163] More importantly, Davies foreshadowed several constructions with postulates, asserting that it was possible to accomplish these necessary tasks:

> 3. That if two straight lines are unequal, the length of the less may always be laid off on the greater.
>
> 4. That a given straight line may be bisected: that is, divided into two equal parts.
>
> 5. That a straight line may bisect a given angle.
>
> 6. That a perpendicular may be drawn to a given straight line, either from a point without the line, or at a point of a line.
>
> 7. That a straight line may be drawn, making with a given straight line, an angle equal to a given angle.[164]

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[163] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), pp. 20, 58.

[164] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. 20.

1. [in Book VI] Let it be granted, that from a given point of a plane, a line may be drawn perpendicular to that plane.

2. Let it be granted, that from a given point without a plane, a perpendicular may be let fall on the plane.[165]

In other words, Davies was not content merely to do something such as, say, bisect a straight line. First, he had to convince his readers that it was permissible to bisect any line. Additionally, these postulates were not essential presuppositions or hypotheses as mathematicians usually understand them.[166] Rather, Davies's definition stated that a "*postulate* grants the solution of a self-evident problem."[167] Postulates were constructions which were not solved; students were not exposed to the steps to follow in order to accomplish any of the above tasks. Readers did, however, eventually reach the constructions of these objects, in their original location in the text following Book III. If anything, Davies may actually have confused the younger students he hoped to reach by requiring them to accept so much before beginning to master the propositions and the deductive connections between them.

Conclusion

Davies was the dominant American mathematics textbook author of the nineteenth century, with his name especially synonymous with Legendre's *Éléments*. The popularity of these textbooks even outlived him. For example, Van Amringe published a revision of *Elements of Geometry and Trigonometry* in 1882, in which he made only cosmetic changes, except for student exercises he inserted at the end of

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[165] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. 157.

[166] See, for example, Thomas L. Heath, trans. and intro., *The Thirteen Books of the Elements*, 2d ed., vol. 1 (New York: Dover Publications, Inc., 1956), pp. 119-120.

[167] Davies, *Elements of Geometry and Trigonometry*, 1851 (cit. n. 94), p. 18; emphasis in source.

each book and in an appendix containing ninety propositions for the student to demonstrate.[168] The three editions of this version of the geometry textbook were published by the American Book Company, which was formed from A. S. Barnes and Co. and the textbook divisions of a number of other nineteenth-century publishers.[169] Indeed, from his early years enforcing Thayer's regimented system and introducing translations of French textbooks considered modern at that time through his stops at Trinity College, the University of the City of New York, and Columbia, Davies conducted his career as a teacher. His political connections, which were largely created as he taught the military figures of the nineteenth century at the United States Military Academy, enabled him to serve as a scientific consultant to the federal government, while his professional activity took place in organizations such as the New York Teachers' Association and not the American Academy of Arts and Sciences or similar societies. Davies was certainly never a candidate for the American Association for the Advancement of Science founded by Benjamin Peirce and the Lazzaroni and devoted to professional science and university research.

Although scientific elites tended to look upon Davies as a hack writer, Davies shaped the geometry education of thousands of Americans and imparted his understandings of analysis and synthesis through *Elements of Geometry and Trigonometry*. He combined French, English, and, implicitly, Scottish styles of presenting geometry into his own version, which he considered the "West Point system of mathematical instruction." Even though Davies taught mathematics as preparation for engineering at the Military Academy, he placed greater emphasis on teaching mathematics and geometry to develop mental discipline. To him, the

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[168] [Van Amringe], *Elements of Geometry From Davies' Legendre* (cit. n. 103).

[169] Pratt, *Century of Book Publishing* (cit. n. 48), pp. 12-15.

analytical process of dividing a problem into parts helped strengthen the young mind, which should only be exposed to one subject at a time and in proper order. Davies knew of analysis and synthesis as directions of proof. He accepted *reductio ad absurdum* as a method of proof but persistently rejected reference to particular diagrams. Finally, Davies eventually based geometry upon a large number of postulates and axioms because he believed that accepting the obvious without proof made geometry more readily understood by beginning students.

Davies's choices in geometry reflected the fact that teaching and business were like two internally touching circles to him. He genuinely cared for the students he was remembered as gently correcting in recitation, but at the same time he never lost sight of the need to actively market his textbooks. His skill at maximizing his reputation as a textbook author obscured the more pleasant aspects of his personality in public life. Nevertheless, Davies's "national course" carried on elements common to Day's and Farrar's series and helped ensure that the traditional goals for college liberal education were a part of American life long past 1840.

CHAPTER SIX

EPILOGUE: A GENTLEMAN'S ART IN A CHANGING WORLD

From the time American periodicals began to appear in the early decades of

the nineteenth century, Americans voiced a number of concerns about college

education in these publications. For example, writers were continually evaluating

the quality of instruction offered by colleges. In response to *Edinburgh Magazine's*

low opinion of the American state of learning in the 1810s, Sidney Willard defended

some colleges for producing good scholars, although twenty years later, he then

argued that there were too many colleges in the United States and that the practice

of one generation of students teaching the next gradually lowered the quality of

colleges.[1] Rufus Ellis, on the other hand, complained that too many new

schoolbooks were being produced even though the old texts were still satisfactory.[2]

To maintain the level of instruction, other authors considered the structure of the

college course. For instance, William Ware and Francis Bowen both reviewed Josiah

Quincy's 1841 pamphlet, *Remarks on the Nature and Probable Effects of Introducing the*

Voluntary System in the Studies of Latin and Greek.[3] The Harvard president had been

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[1] Sidney Willard, "On the Means of Education, and the State of Learning in the United States,"*North American Review* 9 (1819): 240-259; Sidney Willard, review of *Thoughts on the Present Collegiate System in the United States;* by Francis Wayland, *North American Review* 55 (1842): 302.

[2] Rufus Ellis, "School and College Text-Books,"*Christian Examiner* 53 (1852): 424-450, on p. 425.

[3] William Ware, review of *Remarks on the Nature and Probable Effects of Introducing the Voluntary System in the Studies of Latin and* Greek; by Josiah Quincy, *Christian Examiner* 30 (1841): 140-143; Francis Bowen, review of *Remarks on the Nature and Probable Effects of Introducing the Voluntary System in the Studies of Latin and Greek;* by Josiah Quincy, and *Report and Resolutions of the President and Fellows of Harvard University Respecting the Introduction of the Voluntary System in the Studies of the Mathematics,*

won over to reducing the required study of Greek and Latin by Benjamin Peirce's

successful experiment with making mathematics optional in the junior and senior

years. More students continued to study mathematics than Peirce and Quincy had

expected, and additionally time was made for subjects which had not been taught at

Harvard previously, such as natural history, civil history, chemistry, geology,

geography, and modern languages. Ware thought Quincy's plan for expanding the

voluntary system was a good idea, but Bowen criticized Harvard for trying to keep

up with what was popular and for becoming too much like a common school

system.

Throughout this period, it was also a regular practice to compare American

colleges with European institutions. One extended discussion appeared in *American

Quarterly Review* in 1831, where the anonymous author argued that college

instruction must suit the nation in which the colleges were located.[4] Like Willard,

the writer believed that too many colleges had been established in the United States

without attention to preparing the students who entered them for advanced study.

The author listed his version of the essential characteristics of college education,

which were based mainly upon features of Scottish and English universities. For

instance, the author thought American universities were most similar to those in

Scotland, which served as both schools and colleges due to the young age of their

students. He wanted American professors to be paid with student fees, like Scottish

professors, rather than by salaries. He would give the professors authority to set

their own course material, but he wanted them to combine lectures with written

examinations in the style of English universities rather than to examine orally only,

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Latin, and Greek, and *The Report of the Committee to Whom Was Referred the Report and Resolutions, North American Review* 54 (1842): 35-73.

[4] "College Instruction and Discipline," *American Quarterly Review* 9 (1831): 283-314.

as in Scotland. The written examinations ensured that it was not too easy to earn a bachelor's degree. Yet, the author believed that professors who adhered too closely to a textbook lost sight of recent improvements, especially in science. Finally, the reviewer would require study of both ancient languages and the English language, which, he noted, was being taught so well in the new University of London. All these characteristics were, not coincidentally, features of the University of Virginia; indeed, much of the review was written to promote the educational system in place there.

This study has found that mathematical textbooks printed in the United States between 1790 and 1840, including geometry textbooks, appeared in response to the same types of issues. In order to determine the best structure for college education — or to defend the existing structure — American professors asked themselves what they should teach, who their students were, and what textbooks enabled them to impart the level of knowledge they desired. In the preceding chapters, it has been observed that Day's *Mensuration*, Farrar's *Elements of Geometry*, and Davies's *Elements of Geometry and Trigonometry* set the standard for teaching geometry as the textbooks printed most often between 1790 and 1840. These books suited the needs of American colleges, where geometry was considered essential because it modeled proper reasoning for law, medicine, and theology, as well as for mathematics. Students experienced the material within a formally structured course where they memorized proofs under the guidance of a tutor or section leader. In other words, these three textbooks served as tools for creating American gentlemen. In addition, the textbooks have been shown to reflect concerns about mathematics commonly held in the early nineteenth century through an examination of the different understandings of "analysis" and "synthesis." Geometry also had potential as a useful art to Day, Farrar, and Davies, and they played a role in making

incremental adjustments to the college curriculum which broadened teaching in mathematics and science. What remains, then, in the dissertation is to bring together the various elements raised with respect to Day and *Mensuration*, Farrar and *Elements of Geometry*, and Davies and *Elements of Geometry and Trigonometry*, and observe them as a whole.

Day, Farrar, Davies: A Review

There were a number of gradual shifts in American college geometry teaching during the period between 1790 and 1840. Around 1790, geometry education was in a relatively embryonic state, as the college mathematics course had only recently expanded beyond the final year of study. Americans saw mathematics as part of the classical liberal education which trained students for the ministry. They depended upon compendium textbooks and various versions of Euclid's *Elements*. Professors acted as parents and administrators, while they and their former students taught by rote. By 1840, college geometry education was well developed, and instruction was beginning to shift into the expanding and formalizing forms of secondary education, such as the new institution of the American high school. By 1860, algebra and geometry were solidly established as high school subjects — although some colleges offered elementary geometry until the twentieth century.[5] Long before then, however, the college course had become more thorough with mathematics studied in every year, and professors had introduced textbooks for each subject. The years between 1810 and 1830 were especially active in the preparation of texts in the United States. Mathematicians were increasingly dissatisfied with Euclid's *Elements* as an elementary textbook, while new subjects were also established in the 1810s and 1820s, such as mensuration, descriptive geometry, and other applications. For

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[5] John Donald Wilson, "An Analysis of Plane Geometry Content of Geometry Textbooks Published in the United States Before 1900" (Ed.D. diss., University of Pittsburgh, 1959), pp. 1-14.

geometry, the textbook of choice in Northern colleges and the University of Virginia was some version of Legendre's *Éléments* and increasingly Davies's *Elements of Geometry and Trigonometry*. Other Southern colleges were outside the discussion of textbooks: except for William and Mary, institutions were generally younger in the South and tended to be aristocratic in form and less advanced in mathematical curriculum; then, schools founded in the middle of the century were usually patterned on the Military Academy and led by Academy graduates who relied on Davies's textbook series. While professors and tutors often continued to teach geometry by memorization, they were beginning to include more student problems in textbooks and asking students to prove propositions on their own. Americans saw colleges as the route to a variety of professions, such as the law, theology, and medicine, but also business, while professors started to view themselves as professionals in their fields and to become tired of conducting research on their own time with their own funds.

The careers of Day, Farrar, and Davies typified many of these gradual changes. All three men prepared series of textbooks so that there would be separate works for each subject in the college mathematics course. They believed that liberal education should include substantial amounts of science and mathematics, and Day and Farrar suggested small modifications to the classical curriculum. Although Day and Farrar originally intended to become ministers when they were students, they took their responsibilities to the next generation of students seriously when they were chosen to become professors at a young age and read mathematics and science on their own. Similarly, Davies began to teach at a young age and overcame the deficiencies of his abbreviated Military Academy education under Thayer's guidance. All three proved to be successful in the classroom, beloved by their students even though American students generally despised studying mathematics.

Day, though, also conducted his career as Yale's president according to the tenets of paternalism, with kindly but firm authority. Finally, Day, Farrar, and Davies attempted to accommodate contemporary understandings of mathematics to the collegiate tutor system and the traditional reliance on memorization of proofs. They thought geometry had useful purposes, but the subject's importance in training the intellect remained its primary function.

The appreciation Day, Farrar, and Davies showed for French mathematics textbooks was shaped in part by their exposures to Scottish influences. While the three men were paid by salary and worked within the tutor system, they modeled themselves on Scottish professors by building close relationships with groups of students and by presenting public lectures on mathematics and science. Hyman Kuritz has also argued that the interests of professors such as Day, Farrar, and Davies in incorporating modern mathematics and science while refusing to allow students to pursue studies of specialized topics stemmed from the influence of Scottish moral philosophy.[6] In addition, Scottish intellectuals from John Playfair to David Brewster were among the leaders in the English-speaking world in publicizing French mathematics and advocating educational reform. As an avid reader of the *Edinburgh Review*, Day would have been the first of the three professors to encounter Playfair's thoughts on the so-called "British decline" in mathematics. As a close friend and colleague of Benjamin Silliman, he also would have heard about what Silliman learned during a trip to the University of Edinburgh, where he heard Playfair's lectures on geology.[7] One of Farrar's tutors, Levi Hedge, was a

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[6] Hyman Kuritz, "The Popularization of Science in Nineteenth-Century America," *History of Education Quarterly* 21 (1981): 259-274.

[7] Chandos Michael Brown, *Benjamin Silliman: A Life in the Young Republic* (Princeton: Princeton University Press, 1989).

proponent of Common Sense philosophy. Although Farrar's writing did not openly reflect Hedge's influence, all three professors became familiar with the *Edinburgh Encyclopaedia* as it appeared and drew upon the mathematical articles. In general, whenever Day, Farrar, or Davies cited a foreign publication written in English, it invariably was from a journal or book prepared in Scotland. Whether they admitted it or not, they relied on Scottish writers to convey European mathematical culture.

In addition, the understandings of "analysis" and "synthesis" provided the lens for viewing the professors' contributions in this study and are a means for comparing Day, Farrar, and Davies even more directly. For example, all three men accepted the distinction between French and British mathematical styles — Americans usually did not distinguish the institutional and philosophical differences between England and Scotland when they weighed the British style of mathematics against their understandings of the French style. Americans thought that French mathematicians mostly worked with algebra, while the British tended toward geometry and specifically the fluxional notation for the calculus. Day, Farrar, and Davies all considered mathematics done in France to have led to the superior theories and results of their time, but the three differed when they balanced the importance of algebra against that of geometry. Day kept algebra and geometry separate, and he, like John Leslie, believed that calculations done in geometrical measurement or in the theory of proportions were arithmetical rather than algebraic. He allowed symbols to make learning easier for his students and because he did not expect his students to spend their lives doing mathematics. Farrar supported the intermingling of algebra and geometry, but Davies wanted all mathematical subjects to be treated individually and in sequential order for educational purposes.

With respect to analysis and synthesis as method of proof, Farrar and Davies disagreed on what proof techniques ought to be allowed to geometry students.

Farrar did not question Legendre's acceptance of reference to particular diagrams, and he was unique in trying to further explain Legendre's attention to geometrical vocabulary. Davies, on the other hand, vehemently rejected reference to particular diagrams and resurrected the defense of *reductio ad absurdum*. As he grew older and as he wrote more textbooks for younger audiences, Davies also expanded the foundations of geometry by adding postulates and axioms to the last revisions of *Elements of Geometry and Trigonometry*. He wanted students to accept statements which intuitively sounded true before leading them into proofs which constructed the geometrical objects in those statements.

Day and Farrar raised the issue of whether geometry could be taught analytically, through discovery. While they both thought that it was important to demonstrate the practical utility of geometry in the physical sciences and to treat geometry as foundational knowledge for further study in mathematics, they both also maintained the recitation mode of teaching. Day was concerned that presenting original discoveries to students would confuse them since none of them would ever make advancements in mathematics themselves. He preferred to show mathematics and geometry as a completed system, emphasizing the most valuable rules. Although he was more open to his students' possibilities with higher mathematics, Farrar argued that the analytical method of teaching was too complicated, given the time constraints there were on the college course followed by the entire student body.

Returning full circle to the central issue of college education in the antebellum United States, all three professors considered mental discipline to be the chief reason for teaching geometry. As Day helped set the agenda for American mathematics education, placing mental discipline at the center and taking inspiration for textbooks from Scottish and French sources, he argued that the concept of mental

discipline encompassed all of the skills necessary for a successful life. Farrar and

Davies also believed that geometry was vital for honing the habit of careful

reasoning, for strengthening young minds. Davies marketed his textbooks,

including *Elements of Geometry and Trigonometry*, as an illustration of how the goal of

mental discipline could be implemented. In the "West Point system of mathematical

instruction," he mixed elements from his conceptions of the generality denoted by

the "French" mode with the practicality typical of the "English" style in order to

train the intellect.

American College Geometry Education After 1840

Only two college geometry textbooks challenged Davies's and Playfair's

books for popularity in the middle of the nineteenth century — numerous geometries

were published only once. The first was a revision of Carlyle's translation of

Legendre's *Éléments* by James Bates Thomson (1808-1883). An 1834 graduate from

Yale, Thomson got his start in textbook publishing when he was asked by Day to

revise his algebra textbook when Thomson was unable to teach due to ill health in

1842.[8] Thomson's *Elements of Geometry* was meant to be an addition to Day's series,

which was subsequently renamed "Day and Thomson's series for schools and

academies." The first printing, in 1844, claimed to be the third edition, but likely the

printers meant that Thomson had edited the 1824 Edinburgh printing of the so-

called "Brewster's Legendre," Thomas Carlyle's translation of *Elements of Geometry*

and Trigonometry. That version was called a second edition even though only proof

copies were made when David Brewster first tried to have the book published in

1822, as the reader may recall. In any event, Thomson's work was published three

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[8] "Presidents of New York State Teachers' Association," *American Journal of Education* 15 (1865): 477-487, on p. 487.

more times, in 1846, 1848, and 1850.[9] Although fewer than ten copies exist today,

booksellers in Boston and New York advertised it in 1849, along with Farrar's and

Davies's textbooks.[10] It was favorably reviewed for retaining the spirit of Legendre's

Éléments in *American Journal of Science* in 1845.[11]

The other textbook was also a revision of Legendre's *Éléments*, this time by

Elias Loomis (1811-1889) in 1847.[12] It was printed three times before 1850 and a total

of twenty-four times by 1864. Loomis then added a section on trigonometry to the

textbook, which appeared in nineteen more editions between 1869 and 1895. Loomis

graduated from Yale in 1830, and, after teaching at Mount Hope Academy and

entering Andover Theological Seminary, was chosen by Day for the next open

tutorship.[13] He taught recitations in Latin, mathematics, and natural philosophy at

Yale from 1833 to 1836. In the meantime, his chief interest proved to be astronomy,

and he made observations with Alexander Twining and Denison Olmsted. He was

then appointed to the professorship of mathematics and natural philosophy at

Western Reserve College and spent a year studying in Paris under Arago, Biot, and

others before assuming his duties. In 1844, he moved to the University of the City of

New York, which is where he wrote the geometry textbook. He returned to Yale to

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[9] Louis C. Karpinski, *Bibliography of Mathematical Works Printed in America Through 1850* (Ann Arbor: The University of Michigan Press, 1940), p. 459; *The National Union Catalog: Pre-1956 Imprints* (hereinafter cited *NUC*),vol. 240 (London: Mansell, 1976), p. 241.

[10] Karpinski, *Bibliography* (cit. n. 9), p. 229.

[11] Review of *Elements of Geometry; On the Basis of Dr. Brewster's Legendre;* by James B. Thomson, *American Journal of Science* 48 (1845): 210.

[12] Elias Loomis, *Elements of Geometry and Conic Sections* (New York: Harper & Brothers, 1847). There was also George Roberts Perkins (1812-1876), *Elements of Geometry With Practical Applications*, which appeared in eight printings between 1847 and 1872, but no further information about this text was available. *NUC* (cit. n. 9), vol. 340, pp. 505-506; vol. 450, p. 672.

[13] Jeremiah Day to Elias Loomis, 18 August 1832 and 24 April 1833, Loomis Papers, Vol. C-D (3), Beinecke Rare Book and Manuscript Library, Yale University.

succeed Olmsted in 1860, holding this professorship until his death. Even though he was an active researcher in astronomy throughout his life, biographer David Eugene Smith felt that Loomis "exerted his greatest influence" through his textbooks.[14] The proceeds from the books enabled him to leave an enormous gift of $300,000 to Yale.

In *Elements of Geometry and Conic Sections*, Loomis attempted to combine the best of Euclid's *Elements* and Legendre's *Éléments* by filling out proofs from the French textbook with Euclidean logic.[15] Then, Loomis included two more books than versions of Legendre's *Éléments* generally did, one on ratio and proportion and one separating out all the problems that Legendre had put in appendices throughout the books making up the text. Loomis also made a number of small changes throughout the textbook, mainly simply adding and subtracting corollaries and scholia, although he additionally removed several theorems from the later books and rearranged the remaining propositions. Finally, Loomis put in forty-five pages on conic sections. He appended four pages of notes to further clarify the textbook in the 1850 third edition.[16]

One exception to textbooks drawn from Legendre's *Éléments* was Benjamin Peirce's 1837 *Elementary Treatise on Plane and Solid Geometry*.[17] Like Legendre, Peirce separated his book into plane and solid geometry and placed all the figures at the end. Although Peirce also included an explanation of signs and definitions of geometrical statements which were similar to those in Farrar's and Carlyle's

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[14] David Eugene Smith, "Loomis, Elias," in *Dictionary of American Biography*, ed. Allen Johnson, vol. 11 (New York: Charles Scribner's Sons, 1927-1936), pp. 398-399.

[15] Loomis, *Elements of Geometry and Conic Sections* (cit. n. 12), pp. 1-8.

[16] Elias Loomis, *Elements of Geometry and Conic Sections* (New York: Harper & Brothers, 1850).

[17] Benjamin Peirce, *An Elementary Treatise on Plane and Solid Geometry* (Boston: James Munroe and Company, 1837).

translations, he departed from other textbooks by treating each geometrical object in its own chapter: the point, the straight line, the angle, parallel lines, perpendicular and oblique lines, sides and angles of polygons, the circle and the measure of angles, proportional lines, similar polygons, regular polygons, areas, isoperimetrical figures, planes and solid angles, surface and solidity of solids, similar solids, the sphere, and regular polyedrons. Although his theorems and problems covered the standard properties, Peirce abbreviated his proofs to the minimum facts. The two reviews of *Elementary Treatise* were in sympathetic journals, *North American Review* and *Christian Examiner*, and generally approved of this conciseness. The more laudatory review, in *North American Review* and by an anonymous reviewer, contained praise for Peirce's ability to bridge ancient geometry and modern analysis in order to show students that the calculus followed from more familiar mathematics.[18] Joseph Lovering, Peirce's colleague as the other professor of mathematics at Harvard, wrote the other review.[19] He considered *Elementary Treatise* to be a superior textbook but too abstract for beginning students, who he thought were increasingly found in high schools rather than in colleges. Tellingly, though, *Elementary Treatise* was not reviewed in the leading American scientific journal, the Yale-centered *American Journal of Science*, despite that periodical's publication of reviews of the 1834 *Young's Elements of Geometry* and Thomson's 1844 *Elements of Geometry*.[20] With its summary proofs and reliance on infinitesimals, *Elementary Treatise* departed too far from the familiar form and content of Davies's *Elements of Geometry and Trigonometry* to find a

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[18] Review of *An Elementary Treatise on Plane and Solid Geometry*; by Benjamin Peirce, *North American Review* 45 (1837): 500-502.

[19] Joseph Lovering, review of *Elementary Treatise on Sound* and *Elementary Treatise on Plane and Solid Geometry*; by Benjamin Peirce, *Christian Examiner* 20 (1838): 388-390.

[20] Review of *Dr. Young's Elements of Geometry, &c.*, *American Journal of Science* 25 (1834): 206-208; Review of *Elements of Geometry*; by James B. Thomson (cit. n. 11).

substantial American audience and thus experienced only a brief existence as a college textbook.

Indeed, the overwhelming dominance of Davies's textbook dictated that other American geometry textbooks would have to be based upon Legendre's *Éléments* to sell many copies, and it helped cause American college geometry education to lose much of its ability to change by 1840. For example, Davies and others preached the creation of gentlemen through college geometry and mathematics education throughout their lives even though the social and cultural context in which young men were educated and pursued employment was completely transformed between the first decades of the nineteenth century and the Reconstruction era. One of Davies's protégés, J. Howard Van Amringe, was still trying to shape Columbia as an old-style college as late as 1894. Along with the stagnation that resulted from the misinterpretation of Day's Yale Report, the stance of professors with this mindset helped lead to the tension between pedagogists and research mathematicians and scientists so prevalent during the rise of the university. They failed to recognize that even though geometry was still considered necessary knowledge for an educated person, its intellectual role was changing by the time of the Civil War. Teachers began to believe that geometry gave the student different tools besides those for argumentative reasoning. Yet, although authors began to prepare geometry books with new features to communicate the applications of geometry and to improve student retention of its principles — some in fact also adopted by Davies in textbooks other than *Elements of Geometry and Trigonometry* — it took the rest of the nineteenth century for the geometry textbooks which were lists of propositions and proofs to fade away. First, after 1858, some theorems were presented without proof for the students to solve.[21] This in turn led to the

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[21] Wilson, "Analysis of the Plane Geometry Content" (cit. n. 5), p. 148.

preparation of keys containing the solutions. Second, geometry textbooks were increasingly written for a younger audience, with simpler language and an increased number of illustrations.[22]

Furthermore, although the terms, "analysis" and "synthesis" passed from international mathematical discourse by the 1830s and 1840s and ultimately became the forte of historians and philosophers when ancient analysis re-emerged at the end of the nineteenth century with the renewed attention paid to translations of Greek mathematical documents, American mathematics educators broke off from the mathematical mainstream by retaining the senses of analysis and synthesis as approaches to teaching. For example, an 1890 Bureau of Education survey of mathematical teaching at all educational levels directed a question on analysis and synthesis to respondents from universities and colleges.[23] When asked, "How does analytical mathematics compare in disciplinary value to synthetical?" 71 of the professors and presidents preferred analytical mathematics, 23 chose the synthetical subjects, 25 placed equal weight on analytical and synthetical mathematics, and 14 gave no answer. From their comments, it is not entirely clear what exactly analytical and synthetical mathematics were understood to be at the end of the nineteenth century. Apparently, analysis still most often denoted algebra, which was taught so that students discovered the material on their own, while synthetic geometry was presented systematically as a finished product. A pamphlet published by Professor Beebe at Yale shortly after the turn of the twentieth century returned to defining

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[22] See, for example, W. E. Byerly, ed., *Chauvenet's Treatise on Elementary Geometry* (Philadelphia: J. B. Lippincott Co., 1887); and Andrew Wheeler Phillips and Irving Fisher, *Elements of Geometry* (New York: American Book Co., 1896).

[23] Florian Cajori, *The Teaching and History of Mathematics in the United States* (Washington, DC: Government Printing Office, 1890). pp. 307-311. Perhaps, though, it is telling that the questionnaire for normal schools had no questions about whether analysis and synthesis were taught to future teachers.

analysis and synthesis as directions of reasoning and simultaneously associated

analysis with algebra and synthesis with geometry and argued that analysis and

synthesis were needed together to solve geometrical problems.[24] Even in the 1930s,

some geometry teachers experimented with the "analytical method," which to them

was a process by which students wrote their own proofs through trial and error.[25]

Others still believed that the principal reason for teaching geometry was the

development of mental discipline.[26] Thus, American geometry teaching in the late

nineteenth century did not follow the path of geometrical education in Great Britain,

where contention over non-Euclidean geometry coincided with belated interest in

projective geometry as an alternative to the singular value of Euclidean geometry

with respect to mental discipline.[27]

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[24] Professor Beebe, "Outline of Analytical Geometry," 1907, Outlines, Syllabi, Etc. Used in the Teaching of Geometry and Trigonometry in Yale University [Y65Q14], Manuscripts and Archives, Yale University Library, New Haven.

[25] W. S. Schlauch, "The Analytic Method in the Teaching of Geometry," in *The Teaching of Geometry*, ed. W. D. Reeve, National Council of Teachers of Mathematics 5th Yearbook (New York: Columbia University Bureau of Publications, 1930), pp. 134-144. See also W. D. Reeve, ed., *The Teaching of Mathematics in the Secondary School*, 8th Yearbook (New York: Columbia University Bureau of Publications and National Council of Teachers of Mathematics, 1933); and Lemuel Pitts and Robert A. Davis, "A Comparison of the Analytic and Synthetic Method of Teaching Geometry," *School Science and Mathematics* 31 (1931): 333-339, which also reflects the mania for intelligence tests in the 1920s and 1930s. A mix of this kind of analysis with a synthetical approach was considered conventional in the 1970s; Charles Brumfiel, "Conventional Approaches Using Synthetic Euclidean Geometry," in *Geometry in the Mathematics Curriculum*, ed. Kenneth B. Henderson, 36th Yearbook (Reston, VA: National Council of Teachers of Mathematics, 1973), pp. 95-115.

[26] George D. Birkhoff and Ralph Beatley, "A New Approach to Elementary Geometry," and William Betz, "The Transfer of Training, With Particular Reference to Geometry," both in Reeve, *Teaching of Geometry* (cit. n. 25), pp. 86-95, 149-198.

[27] On James M. Wilson, the Association for the Improvement of Geometrical Teaching, and the attempts to replace Euclid's *Elements* as an elementary textbook, see G. Howson, *A History of Mathematics Education in England* (Cambridge: Cambridge University Press, 1982), pp. 123-140; Joan L. Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England* (Boston: Academic Press, Inc., 1988), pp. 161-198; and W. H. Brock, "Geometry and the Universities: Euclid and His Modern Rivals, 1860-1901," *History of Education* 4, no. 2 (1975): 21-35. Some of the relevant primary sources are: Augustus De Morgan, review of "Elementary Geometry;" by J. M. Wilson, *Athenaeum* 2125 (1868): 71-73; 2129 (1868): 216; 2130 (1868): 241-242; and A. J. G. Barclay, "On the Teaching of Elementary Geometry," *Proceedings of the Edinburgh Mathematical Society* 2 (1883-1884): 24-25.

Final Thoughts

In summary, there were two influences serving as "givens" for American mathematics professors in the early nineteenth century: French mathematics and the analysis/synthesis distinction. French mathematicians were widely viewed as superior at the end of the eighteenth century, most notably for the achievements of Laplace and Lagrange. Then, the textbook-writing projects during the French Revolution made their contributions more accessible to non-elite readers. Legendre wrote *Éléments* at this time; the book was viewed in France as a rigorous alternative to Euclid's *Elements* and as suited for advanced rather than elementary readers. Second, the terms "analysis" and "synthesis' were rooted in ancient ideas about the direction of reasoning in proof. By the end of the eighteenth century, mathematicians also understood the terms as references to algebra and geometry. The words could also denote modes of doing and teaching mathematics, through invention and discovery or by presenting a completed system to students.

These three understandings of "analysis" and "synthesis" were some of the issues behind the development of geometry textbooks in Scotland between 1750 and 1825. Robert Simson used the techniques of the ancient Greeks to reconstruct proofs for *The Elements of Euclid*. He also helped direct British mathematics toward a focus on Euclidean geometry as the only wholly sound discipline within mathematics and to rely on Euclid's *Elements* for teaching schoolchildren and university students. An advocate of French mathematics, Playfair tried to accommodate Euclid's *Elements* to modern developments with his *Elements*. For instance, he took some steps toward treating proportions as numbers rather than as geometric magnitudes. While John Leslie used the same sort of symbols in *Elements of Geometry* that Playfair did, Leslie was careful to note that he did not consider them to be satisfactorily rigorous. The last geometry textbook to appear during this time period was the translation of

Legendre's *Éléments* that Thomas Carlyle prepared under David Brewster's direction. Like Leslie's *Elements of Geometry*, *Elements of Geometry and Trigonometry* failed to find a significant audience either in Scotland or England. These textbooks were rather diverse and separate parts when they were written, but taken together, they formed a whole of influence upon American mathematics professors.

As in Great Britain, Playfair's *Elements* was the piece which loomed the largest in American colleges, such as at Yale during Jeremiah Day's career as professor and president there. To accompany that textbook, Day introduced a mathematics series to replace the compendia books. The third volume, *Mensuration*, supplemented the reasoning skills students developed by mastering Playfair's *Elements* with examples of how to apply geometry to measurement. Following Timothy Dwight's example, Day and his colleagues trained Yale students to enter professional schools, become businessmen, or carry on Yale's paternal and thorough course as professors. As with his vision of the ideal college curriculum, Day always strove to strike balances or to draw parallels. Without a balance between literature and science or a curriculum which evolved gradually with the times, he believed, students were not prepared to be successful adult men. Similarly, Day balanced his early interests in studying discoveries in mathematics and science with his pastoral, parental, and administrative responsibilities to the students at Yale. He was always committed to preserving college liberal education, including an emphasis on teaching students the mental discipline they would need to conduct the rest of their lives.

John Farrar, on the other hand, brought French influences directly to his students as translations of entire textbooks. Although his active mathematical years were almost as few as Day's, perhaps Farrar would have desired to make the transition from exposition into research under other circumstances. He did, after all,

restore the prestige of the Hollis Professorship and was an active member of the American Academy of Arts and Sciences. Farrar tended, though, to be just as ambivalent about his goals as the institution he served was with respect to major adaptations in the curriculum, as was shown by the convoluted process he went through before producing a series of textbooks. In addition, the gradual curriculum changes he proposed were not pursued. Farrar treated geometry as one of the essential arts, but his influence through textbooks such as *Elements of Geometry* was not long lasting in the United States or even at Harvard.

Charles Davies conducted his career as the professor and businessman who successfully combined the French and English approaches to mathematics. Like Day and Farrar, Davies learned much of his mathematics on his own, but he was not as careful to pass along the intellectual depths reached by, say, Legendre to others. Yet, while his textbooks appeared superficial to mathematicians later in the nineteenth century, Davies's series proved to be wildly popular with those who purchased textbooks. Furthermore, aside from the professional contacts he cultivated, his experience in a military environment did not distinguish him from Day or Farrar as much as one might think. The form of the mathematical portion of Thayer's system resembled recitations at Harvard and Yale, although Academy cadets were divided into sections by ability. Davies also advocated mental discipline, for he valued geometry both for its applicability and its intellectual rewards. He departed from Day and Farrar by openly marketing the large number of textbooks he prepared. "*Davies's Legendre*" did become the model for geometry teaching for the rest of the nineteenth century.

Throughout, the dissertation has demonstrated the need for historical research into mathematics education and the mathematical community in the early American republic by highlighting several key issues. For example, Day, Farrar, and

Davies were among the professors who were an essential step between the well-remembered colonial independent amateurs like David Rittenhouse and Benjamin Franklin and the professional researchers first typified by Benjamin Peirce and then brought to greatness by J. J. Sylvester, Felix Klein, E. H. Moore, and their students. The professors' concern for improving approaches to teaching mathematics influenced American colleges to shed their colonial identities by the early nineteenth century and to develop into a unique set of institutions which were friendly to mathematics and science. Professors at the colleges created a mathematical community in which they were interconnected with many of the other professors. They pushed for incremental adjustments to the curriculum, which helped make their colleges a mixture of tradition and innovation, as Jurgen Herbst described them.[28] By beginning the process of picking and choosing among foreign sources — in other words, by employing a quite active form of reception — and by disseminating mathematics in a form students could comprehend, Day, Farrar, and Davies typified a sort of republic-building in American higher education.

In addition, the dissertation has extended Helena Pycior's reinterpretation of the "British, then French" thesis popularized by Florian Cajori and demonstrated that the two influences were even more commingled than she believed. It has discussed the ways in which understandings of the various senses of "analysis" and "synthesis" were a part of geometry teaching in the American college. Throughout this period, professors placed the major importance of geometry in its role in training the intellect, and this use of geometry accompanied the ideal of liberal education, which was the creation of gentlemen. American mathematics professors drew upon the European context, with even French influences often being

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[28] Jurgen Herbst, "American Higher Education in the Age of the College," *History of Universities* 7 (1988): 37-59.

channeled through Scotland first. Scottish mathematicians bequeathed a background of influence with respect to geometry textbooks and their tradition of liberal education with a pragmatic cast. Finally, the dissertation has brought attention to the biographical details of professors who have been obscured by time. Similar biographies or studies of textbooks would cast further light upon the history of mathematics education and illustrate the popular reception of mathematics. Research into specific information on mathematics instruction at individual colleges is also lacking, as are pursuits into the links between well-known figures such as Nathaniel Bowditch and their institutional involvements.

To conclude, fuller accounts of the careers and geometry textbooks of Day, Farrar, and Davies were overdue. The three men were representative of mathematics professors in the early republican scientific community in the northern United States as well as influential authors/editors of geometry textbooks. Their opinions on contemporary issues such as the quality of college instruction and the relative merits of British and French mathematics—especially Day's definition of the nature of Yale and Davies on Legendre's *Éléments* as a textbook—helped set the standard for American college geometry teaching in the nineteenth century, although the subtle ability to change of the early century was largely lost to successors of the three professors. As they viewed themselves in reference to European mathematics and Scottish sources, treated geometry as not necessarily the ideal mathematical subject but certainly the best for training reasoning, and never lost sight of that proper reasoning even though they accommodated recent mathematics and science, Day, Farrar, and Davies modeled mathematics as a gentleman's art for their students.

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